HMW. 3 Solutions

5.1.12 $f(x,y) = \begin{cases} x \cdot e^{-x(1+y)} & x \geq 0 \text{ and } y \geq 0 \\ 0 & \text{o.w.} \end{cases}$

a) $P(X>3) = \int_3^\infty \int_0^\infty x e^{-x(1+y)} \, dy \, dx = \int_3^\infty \left[x \cdot \frac{e^{-x(1+y)}}{-x} \right]_0^\infty dx = \int_3^\infty e^{-x} dx$

$= \left[-e^{-x} \right]_3^\infty = e^{-3}$

b) $f_X(x) = \int_0^\infty x e^{-x(1+y)} dy = e^{-x}$ for $x \geq 0$. So $f_X(x) = \begin{cases} e^{-x} & \text{for } x \geq 0 \\ 0 & \text{o.w.} \end{cases}$

$f_Y(y) = \int_0^\infty x e^{-x(1+y)} dx = \frac{1}{(1+y)^2}$ for $y \geq 0$. So $f_Y(y) = \begin{cases} \frac{1}{(1+y)^2} & \text{for } y \geq 0 \\ 0 & \text{o.w.} \end{cases}$

by integration by parts
let $u = x$ and $dv = e^{-x(1+y)} dx$
$\Rightarrow du = dx$ $\Rightarrow v = \frac{e^{-x(1+y)}}{-(1+y)}$

Since $f(x,y) \neq f_X(x) \cdot f_Y(y)$, X and Y are not independent.

c) Let L = the lifetime of at least one component exceeds 3

$P(L) = 1 - P(X \leq 3 \text{ and } Y \leq 3) = 1 - \int_0^3 \int_0^3 x e^{-x(1+y)} dy \, dx$

$= 1 + \int_0^3 \left[e^{-x(1+y)} \right]_{y=0}^{y=3} dx = 1 + \int_0^3 \left(e^{-4x} - e^{-x} \right) dx$

$= 1 + \left[\frac{e^{-4x}}{-4} - \frac{e^{-x}}{-1} \right]_0^3 = 1 + \left[\frac{e^{-12}}{-4} - \frac{e^{-3}}{-1} \right] - \left[\frac{1}{-4} - \frac{1}{-1} \right]$

$= -\frac{e^{-12}}{4} + e^{-3} + \frac{1}{4} = 0.3$

5.2.29 When X and Y are independent,

$E(XY) = E(X) \cdot E(Y)$. Hence $Cov(X,Y) = E(XY) - E(X)E(Y)$

$= E(X) \cdot E(Y) - E(X) \cdot E(Y)$

$= 0$.

Also, $Corr(X,Y) = \frac{Cov(X,Y)}{\sigma_X \cdot \sigma_Y} = \frac{0}{\sigma_X \sigma_Y} = 0$.