

# Problem Set 4

Department of Economics

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**Instructions.** When using the Lagrangian or Kuhn–Tucker conditions, briefly justify why the method is valid (concavity, convexity of budget set, monotonicity, or active constraints).

**Q1.** Consider the utility maximization problem

$$\max_{x,y \geq 0} u(x,y) = \ln x + \ln y \quad \text{s.t.} \quad p_x x + p_y y \leq M.$$

(a) Derive the Marshallian demands. (b) Check and state whether the first-order conditions are sufficient.

**Q2.** A consumer has utility  $u(x,y) = \sqrt{x} + 2\sqrt{y}$  and income  $M > 0$  with prices  $p_x, p_y > 0$ . (a) Find the demand  $(x^*, y^*)$ . (b) State whether the Kuhn–Tucker sufficiency conditions are satisfied and justify briefly.

**Q3.** Find the Marshallian demand functions for the following utility function using the Kuhn–Tucker conditions. Discuss whether the Kuhn–Tucker sufficiency conditions are satisfied.

$$U(x,y) = xy + 20y \quad \text{s.t.} \quad 4x + y \leq 40.$$

**Q4.** Use the Kuhn–Tucker conditions to derive demand for general prices and wealth levels (i.e.  $p_i, w$  should appear in the demand functions) for the following utility maximization problems and discuss whether the Kuhn–Tucker sufficiency conditions are satisfied.

(a)  $u(x_1, x_2) = \ln(x_1 + 1) + \ln(x_2 + 1)$  s.t.  $p_1 x_1 + p_2 x_2 \leq m$ ,  $x_1 \geq 0$ ,  $x_2 \geq 0$ ,  $m > 0$ .

(b)  $u(x_1, x_2) = x_1^2 + (x_2 - 5)^2$  s.t.  $x_1 \geq 0$ ,  $x_2 \geq 0$ ,  $2x_1 + x_2 \leq 4$ ,  $m > 0$ .

(c)  $u(x_1, x_2) = x_1 x_2$  s.t.  $x_1 + x_2 \leq 100$ ,  $x_1 \leq 40$ ,  $x_1 \geq 0$ ,  $x_2 \geq 0$ .

**Q5.** A consumer has utility  $u(x,y) = x + y + xy$  and faces  $p_x x + p_y y \leq M$ ,  $x, y \geq 0$ ,  $p_x, p_y, M > 0$ . (a) Derive the demand using Kuhn–Tucker conditions. (b) Discuss whether the Kuhn–Tucker sufficiency conditions hold.

**Q6.** Consider  $u(x,y) = \min\{x, 2y\}$  with income  $M > 0$  and prices  $p_x, p_y > 0$ . (a) Solve for the demand. (b) Explain why the Lagrangian derivative method is not valid, and discuss sufficiency conditions.

**Q7.** Let  $u(x,y) = x^2 + y$  and  $p_x x + p_y y = M$ ,  $x, y \geq 0$ . (a) Derive the demand. (b) Show that the optimum is a corner solution and explain why first-order conditions alone are not sufficient. (c) Discuss Kuhn–Tucker sufficiency conditions for this problem.