

Handout on Multipliers
(Rev. 4 – October 2013)

In economics, a multiplier is a factor of proportionality that measures how much an **endogenous** variable changes in response to a change in some **exogenous** variable. For example, suppose a one-unit change in some variable x causes another variable y to change by a factor M . Then the multiplier is M . To obtain multiplier in a model whatever the complexity of model is we should solve the model. Solving a model means that obtaining endogenous variables in terms of exogenous variables and parameters. You can find definitions of endogenous and exogenous variable and parameter in the glossary at the end of the handout.

1. A Generic Case

$$Y = C + I \quad (1)$$

$$C = C_0 + cY \quad (2)$$

$$I = I_0 \quad (3)$$

The above 3 equations describe a Classical model that consists of households and firms. Eq. 1 is called as “national income identity” where Y is GDP; C is private consumption expenditures; I is investment expenditures; eq. 2 is consumption function where C_0 is autonomous part of consumption and c is marginal propensity to consume (MPC) where $c \in (0,1)$. (The value of c is equal to the slope of AE function in this simple model).

To solve a model and to get **unique** solutions for each endogenous variable, we need equality of number of equations and number of endogenous variables. So, to solve this model, we need 3 endogenous variables. However, since 3rd equation states that investment expenditures are exogenous, when we substitute I in equation 1 by I_0 ; we will have 2 equations and need to 2 endogenous variables to solve the model.

In this two by two system (2 unknowns in 2 equations), Y and C are endogenous variables; C_0 and I_0 are exogenous variables and c is a parameter. We need to solve Y and C in terms of C_0 , I_0 and c .

To do this, plug equation 2 into 1' and rearrange.

$$Y = C_0 + cY + I_0 \quad \Rightarrow \quad Y_{Eq} = \frac{C_0}{1 - c} + \frac{I_0}{1 - c}$$

We obtain income in terms of exogenous variables and parameter. The subscript (**Eq**) denotes the equilibrium value of the national income. For given exogenous variables and parameters, we can determine the value of national income. By plugging this into equation 2, we will get private consumption expenditures in terms of exogenous variables and parameters, i.e. we will solve it.

$$C = C_0 + \frac{cC_0}{1-c} + \frac{cI_0}{1-c} \quad \Rightarrow \quad C_{Eq} = \frac{C_0}{1-c} + \frac{cI_0}{1-c}$$

The value of an endogenous variable can change by an exogenous shock. This type of shock can occur by a change in exogenous variables or parameters. A change in parameters is called a “structural change” and we will not deal with such changes. Our concern will be limited to shocks that occurred in exogenous variables; i.e. in C_0 and I_0 .

To see the effects of an exogenous shock in I_0 on endogenous variables; Y and C , we should take total derivative of equations under the “ceteris paribus” assumption; when I_0 is changed, C_0 stays constant. Taking total derivative is equivalent to writing an equation in difference form;

$$\begin{aligned} \Delta Y_{Eq} &= \frac{1}{1-c} \Delta C_0 + \frac{1}{1-c} \Delta I_0 \quad ; \quad \Delta Y_{Eq} = \frac{1}{1-c} \Delta I_0 && \text{Since } \Delta C_0 = 0 \text{ by} \\ &&& \text{an assumption} \\ \Delta C_{Eq} &= \frac{1}{1-c} \Delta C_0 + \frac{c}{1-c} \Delta I_0 \quad ; \quad \Delta C_{Eq} = \frac{c}{1-c} \Delta I_0 \end{aligned}$$

The solution says that if I_0 is increased, let say 10%, the national income will increase by 10% times the value of the multiplier; $1/(1-c)$. Since $c \in (0,1)$; the multiplier is greater than unity. This means that, the national income will increase more than an increase in the exogenous investment expenditures. Similarly, the private consumption expenditures will increase by 10% times MPC times the value of multiplier; $1/(1-c)$.

In this model, the multiplier is simply equal to the reciprocal of marginal propensity to save (MPS). Higher MPC lowers MPS which leads to a higher multiplier. Logic is simple; since consumers spend higher part of their income as it increases; this leads to an increase in aggregate demand and higher increases in national income. If multiplier is high, a little increase in exogenous variables leads to greater increases in endogenous variables.

2. A Complicated Case

$$Y = C + I + G + X - M \quad (1)$$

$$C = a + cY_d \quad (2)$$

$$I = I_0 - ki + fY \quad (3)$$

$$G = G_0 \quad (4)$$

$$X = X_0 \quad (5)$$

$$M = M_0 + mY \quad (6)$$

$$Y_d = Y - T + Tr \quad (7)$$

$$T = T_0 + tY \quad (8)$$

$$Tr = Tr_0 \quad (9)$$

Here, an open economy model with government which includes tax function with both autonomous and induced components and autonomous transfer payments will be solved and multipliers will be obtained.

We have eight exogenous variables; I_0 ; autonomous investment expenditures, G_0 ; government expenditures on goods and services, X_0 ; export demand, Tr_0 ; transfer payments from government to households, C_0 ; autonomous consumption demand, M_0 ; autonomous import demand, T_0 ; autonomous taxes and i is nominal interest rate. The parameters of the model are; c is marginal propensity to consume; m is marginal propensity to import; t is marginal tax rate; k and f are sensitivity of investment demand with respect to nominal interest rate and national income, respectively. All parameters are $\in (0,1)$.

By replacing equations (4), (5) and (9) into other equations where G, X and Tr appear, we have 6 equations and need 6 endogenous variables; Y, C, I, M, Y_d and T . Y is national income; C is private consumption expenditures which is a function of disposable income; I is investment demand which is a function of national income and nominal interest rate; M is import demand which is a function of national income; Y_d is disposable income which is a function of national income, taxes and transfer payments; T is taxes which has both autonomous and induced components.

To solve the model, plug equations from 2 to 9 into equation 1 and rearrange.

$$Y = C_0 + cY - cT_0 + ctY + cTr_0 + I_0 - ki + fY + G_0 + X_0 - M_0 - mY$$

$$Y_{Eq} = \frac{C_0 + I_0 + G_0 + X_0 - M_0}{1 - c + ct - f + m} - \frac{cT_0}{1 - c + ct - f + m} + \frac{cTr_0}{1 - c + ct - f + m} - \frac{k i}{1 - c + ct - f + m}$$

$$\Delta Y_{Eq} = \frac{\Delta C_0 + \Delta I_0 + \Delta G_0 + \Delta X_0 - \Delta M_0}{1 - c + ct - f + m} - \frac{c \Delta T_0}{1 - c + ct - f + m} + \frac{c \Delta Tr_0}{1 - c + ct - f + m} - \frac{k \Delta i}{1 - c + ct - f + m}$$

In this model, simple multiplier is equal to $1/(1 - c + ct - f + m)$. As MPC and responsiveness of investment demand to national income increases, simple multiplier will be larger; as

marginal tax rate and marginal propensity to import increase, simple multiplier will be lower.

National income will respond to changes in autonomous private consumption expenditures; autonomous investment demand; government expenditures; exports demand and autonomous import demand by a change in these items times simple multiplier; ceteris paribus. (Remember that, we can analyze the effects of an exogenous shock on endogenous variables under “ceteris paribus” assumption).

Exogenous Shock (an increase in)	Magnitude and Direction of Change in Y_{Eq}
C_0	$1/(1 - c + ct - f + m) * \Delta a$
I_0	$1/(1 - c + ct - f + m) * \Delta I_0$
G_0	$1/(1 - c + ct - f + m) * \Delta G_0$
X_0	$1/(1 - c + ct - f + m) * \Delta X_0$
M_0	$-1/(1 - c + ct - f + m) * \Delta M_0$
T_0	$-c/(1 - c + ct - f + m) * \Delta T_0$
Tr_0	$c/(1 - c + ct - f + m) * \Delta Tr_0$
i	$-k/(1 - c + ct - f + m) * \Delta i$

Shortly, $-c/(1 - c + ct - f + m)$ is called as “lump – sum tax multiplier” (the sixth row). Adding this to the government spending multiplier, third row, we obtain “balanced budget multiplier”. It is equal to $(1 - c)/(1 - c + ct - f + m)$. Remember that, balanced budget multiplier is equal to 1 if there are no induced taxes and/or transfers.

By substituting Y_{Eq} into equations (2), (3), (6), (7) and (8), you can get equilibrium values of other endogenous variables C_{Eq} , I_{Eq} , M_{Eq} , Y_{dEq} and T_{Eq} .

3. Other Cases

a) A closed economy with government where taxes and transfer payments are autonomous.

System of equations;

$$Y = C + I + G \quad (1) \quad Y_d = Y - T + Tr \quad (5)$$

$$C = C_0 + cY_d \quad (2) \quad T = T_0 \quad (6)$$

$$I = I_0 \quad (3) \quad Tr = Tr_0 \quad (7)$$

$$G = G_0 \quad (4)$$

Simple multiplier: $1/(1 - c)$ where (c) is the slope of AE function

Lump-Sum Tax multiplier: $-c/(1 - c)$

Transfer payments multiplier: $c/(1 - c)$

Balanced budget multiplier: $(1 - c)/(1 - c) = 1$

b) A closed economy with government where tax function has both autonomous and induced components

System of equations;

$$Y = C + I + G \quad (1) \quad G = G_0 \quad (4)$$

$$C = C_0 + cY_d \quad (2) \quad Y_d = Y - T \quad (5)$$

$$I = I_0 \quad (3) \quad T = T_0 + tY \quad (6)$$

Simple multiplier: $1/(1 - c + ct)$ where $(c - ct)$ is the slope of AE function

Lump-Sum Tax multiplier: $-c/(1 - c + ct)$

Balanced budget multiplier: $(1 - c)/(1 - c + ct)$

c) An open economy with government where taxes are autonomous

System of equations;

$$Y = C + I + G + X - M \quad (1) \quad X = X_0 \quad (5)$$

$$C = C_0 + cY_d \quad (2) \quad M = M_0 + mY \quad (6)$$

$$I = I_0 \quad (3) \quad Y_d = Y - T \quad (7)$$

$$G = G_0 \quad (4) \quad T = T_0 \quad (8)$$

Simple multiplier: $1/(1 - c + m)$ where $(c - m)$ is the slope of AE function

Lump-Sum Tax multiplier: $-c/(1 - c + m)$

Balanced budget multiplier: $(1 - c)/(1 - c + m)$

d) An open economy with government where taxes and transfer payments are autonomous.

System of equations;

$$Y = C + I + G + X - M \quad (1) \quad M = mY \quad (6)$$

$$C = C_0 + cY_d \quad (2) \quad Y_d = Y - T + Tr \quad (7)$$

$$I = I_0 \quad (3) \quad T = T_0 \quad (8)$$

$$G = G_0 \quad (4) \quad Tr = Tr_0 \quad (9)$$

$$X = X_0 \quad (5)$$

Simple multiplier: $1/(1 - c + m)$ where $(c - m)$ is the slope of AE function

Lump-Sum Tax multiplier: $-c/(1 - c + m)$

Transfer payments multiplier: $c/(1 - c + m)$

Balanced budget multiplier: $(1 - c)/(1 - c + m)$

e) An open economy with government where tax function has both autonomous and induced components

System of equations;

$$Y = C + I + G + X - M \quad (1) \quad X = X_0 \quad (5)$$

$$C = C_0 + cY_d \quad (2) \quad M = mY \quad (6)$$

$$I = I_0 \quad (3) \quad Y_d = Y - T + Tr \quad (7)$$

$$G = G_0 \quad (4) \quad T = T_0 + tY \quad (8)$$

Simple multiplier: $1/(1 - c + ct + m)$ where $(c - ct - m)$ is the slope of AE function

Lump-Sum Tax multiplier: $-c/(1 - c + ct + m)$

Balanced budget multiplier: $(1 - c)/(1 - c + ct + m)$

Glossary

Endogenous Variable: A variable is called endogenous if it is explained within the model. For example, in a simple linear consumption function that is used above, induced part of consumption, i.e. cY or cY_d , is endogenous variable since it is dependent on the level of the national income; as GDP increases, consumption expenditures increases by MPC, or vice versa. Since the level of the total consumption expenditures could not be determined from outside of the model, it is an endogenous variable.

Exogenous Variable: In an economic model, an exogenous change is one that comes from outside of the model and is unexplained by the model. For example, in a simple linear consumption function that is used above, autonomous part of consumption, i.e. C_0 , is not a function of any variable and it's a given value. Since it is independent from the state of the economy, e.g. the level of the national income, it is an exogenous variable.

Parameter: Parameters have values which do not change with time. In this respect, these can be called as “structural parameters” which are obtained from the data. However, as they change, not only endogenous variables change but also dynamics of the model change. Therefore, in a model, changes in parameters are ruled out.