

Problem Set 7

1- If the production function is $f(x, y) = x + \min\{x, y\}$, then there are constant returns to scale. T/F?

2- A firm has two variable factors and a production function $f(x_1, x_2) = (2x_1 + 4x_2)^{1/2}$. Find the technical rate of substitution between x_1 and x_2 .

3- A firm has the production function $f(X, Y) = X^{3/4}Y^{1/4}$, where X is the amount of factor x used and Y is the amount of factor y used. On a diagram we put X on the horizontal axis and Y on the vertical axis. We draw some isoquants. Now we draw a straight line on the graph and we notice that wherever this line meets an isoquant, the isoquant has a slope of -9. Write down the equation of this straight line.

4- A firm has the production function $f(x_1, x_2) = (x_1^b + x_2^b)^c$, where $b > 0$ and $c > 0$. Find a relation between b and c (such as an inequality) for which this firm will have increasing

5- Draw the isoquants for the following production functions: a. $f(x, y) = \max\{2x, y^3\}$

b. $f(x, y) = (\max\{x, y\})^2 - (\min\{x, y\})^2$

c. $f(x, y) = 3x - \min\{2x, y\}$

6- Suppose a firm's production function is given as

$$Q(K, L) = 150K^2L^2 - K^3L^3$$

Assume $K=5$ is fixed.

- Derive the firm's total product of labor, marginal product of labor and average product of labor.
- Plot MPL and APL on the same graph.
- Find the amount of labor at which MPL is maximized.
- Find the amount of labor necessary for this production process for $MPL=APL$.
- What is the maximum number of output the firm can produce in the short run?

7- Let $TP(L)$ denote a total product function. Derive a mathematical relationship between AP and MP to show

- If $MP > AP$, AP is increasing.
- If $MP < AP$, AP is decreasing.

c. At an interior maximum of AP , $AP=MP$.

8- A firm's production function is $Q = 20KL/(K+L)$.

a. Are there diminishing returns to each input? Explain.

b. Are there increasing, constant or decreasing returns to scale for this function?

9- Find the profit maximizing quantity of output for the following cases:

a. A competitive firm's production function is $f(x_1, x_2) = 6x_1^{1/2} + 8x_2^{1/2}$. The price of factor 1 is \$1 and the price of factor 2 is \$4. The price of output is \$8.

b. Firm's production function is given by $f(x,y)=\ln(xy)$.

10- A competitive firm uses a single input x to produce its output y . The firm's production function is given by $y = x^{3/2}$ for quantities of x between 0 and 4. For quantities of x greater than 4, the firm's output is $y = 4 + x$. If the price of the output y is \$1 and the price of the input x is \$3, how much x should the firm use to maximize its profit?

11- A competitive firm has a production function described as follows. "Weekly output is the square root of the minimum of the number of units of capital and the number of units of labor employed per week." Suppose that in the short run this firm must use 16 units of capital but can vary its amount of labor freely.

a. Write down a formula that describes the marginal product of labor in the short run as a function of the amount of labor used. (Be careful at the boundaries.)

b. If the wage is $w = \$1$ and the price of output is $p = \$4$, how much labor will the firm demand in the short run?

c. What if $w = \$1$ and $p = \$10$?

d. Write down an equation for the firm's short-run demand for labor as a function of w and p .

12- Suppose that a firm produces using only one input. When a firm uses x units of input, the marginal product of that input is $10-(x/100)$ units of product. If the price of the product is 5 TL and the price of the input is 1.5 TL, then what is the input demand of this firm assuming that it aims to maximize profits?

13- A competitive firm has the three-factor production function $f(x, y, z) = (x + y)^{1/2}z^{1/2}$. The factor prices used to be $w_x = \$1$, $w_y = \$2$, and $w_z = \$3$. Suppose that the price of factor y doubled while the other two prices stayed the same. Then by how much does the cost of production change?