

ECON 205

Quiz 1 Solutions

1) Y is hypergeometric with $N=15$, $M=4$ and $n=3$

$$a) P(Y=1) = \frac{\binom{4}{1} \binom{11}{2}}{\binom{15}{3}} = \frac{44}{91}$$

$$b) P(Y>2) = P(Y=3) = \frac{\binom{4}{3} \binom{11}{0}}{\binom{15}{3}} = \frac{4}{455}$$

$$c) E(Y) = n \cdot \frac{M}{N} = 3 \cdot \frac{4}{15} = \frac{4}{5}$$

$$d) V(Y) = \frac{N-n}{N-1} \cdot n \cdot \frac{M}{N} \left(1 - \frac{M}{N}\right) = \frac{12}{14} \cdot 3 \cdot \frac{4}{15} \cdot \left(1 - \frac{4}{15}\right) \\ = \frac{88}{175}$$

2) a) $\int_{-\infty}^{\infty} f(x) dx = 1$ must be satisfied.

$$\int_{-\infty}^{\infty} f(x) dx = \int_1^2 cx dx = \left[\frac{cx^2}{2} \right]_1^2 = 2c - \frac{c}{2} = \frac{3c}{2} = 1 \Rightarrow c = \frac{2}{3}$$

$$b) E(X) = \int_{-\infty}^{\infty} x f(x) dx = \int_1^2 cx^2 dx = \left[\frac{cx^3}{3} \right]_1^2 = \frac{c}{3} \cdot 7 = \frac{14}{9}$$

$$c) P\left(\frac{3}{2} \leq X \leq \frac{5}{2}\right) = \int_{\frac{3}{2}}^2 cx dx = \left[\frac{cx^2}{2} \right]_{\frac{3}{2}}^2 = 2c - \frac{9c}{8} = \frac{7c}{8} = \frac{7}{12}$$

$$\begin{aligned}
 3) \quad V(X) &= E[(X-\mu)^2] = E(X^2 - 2\mu X + \mu^2) = E(X^2) - 2\mu E(X) + E(\mu^2) \\
 &= E(X^2) - 2\mu^2 + \mu^2 = E(X^2) - \mu^2 = E(X^2) - [E(X)]^2
 \end{aligned}$$

by using the given approximation for $h(x)$

$$\begin{aligned}
 4) \quad V(h(X)) &= \int_{-\infty}^{\infty} [h(x) - E(h(x))]^2 f(x) dx \approx \int_{-\infty}^{\infty} [h(\mu) + h'(\mu)(x-\mu) - E(h(\mu) + h'(\mu)(x-\mu))]^2 f(x) dx \\
 &= \int_{-\infty}^{\infty} [\cancel{h(\mu)} + h'(\mu)(x-\mu) - \cancel{h(\mu)} - E(h'(\mu)(x-\mu))]^2 f(x) dx \\
 &= \int_{-\infty}^{\infty} (h'(\mu))^2 [x-\mu - E(x-\mu)]^2 f(x) dx = (h'(\mu))^2 \int_{-\infty}^{\infty} [X - E(X)]^2 f(x) dx = (h'(\mu))^2 V(X)
 \end{aligned}$$