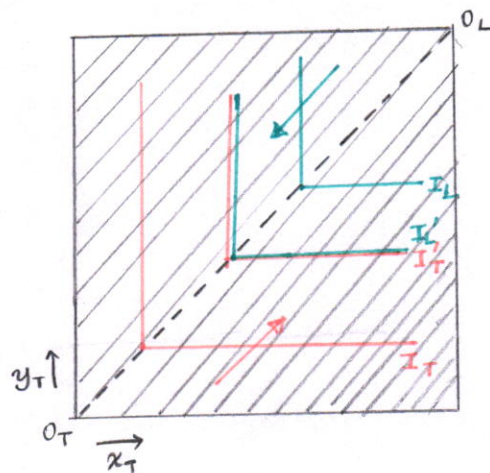


PS#9 suggested solutions

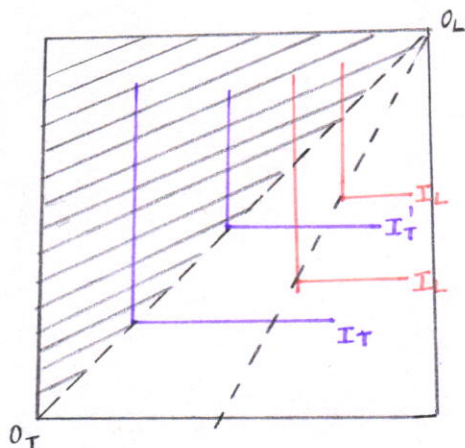
① (a) $u_T = \min \{x_T, y_T\}$

$u_L = \max \{x_L, y_L\}$

- Total endowment of both goods are identical.
- Indifference curves overlap so P.O. allocations are equal to the whole (shaded area).



(b)



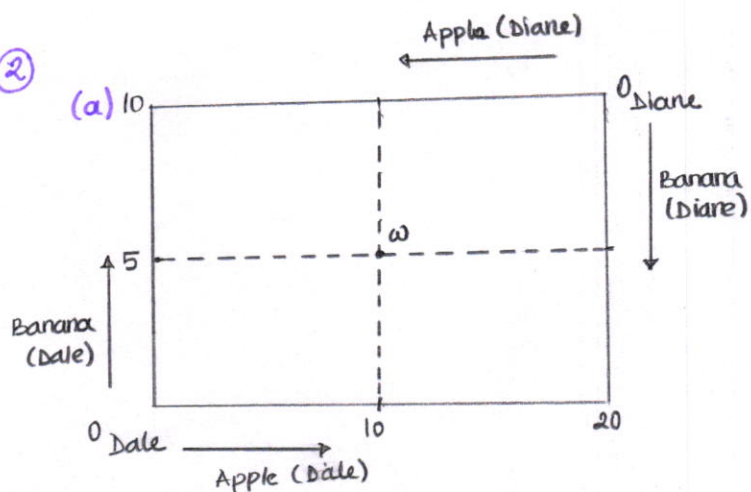
$u_T = \min \{x_T, y_T\}$

$u_L = \max \{2x_L, y_L\}$

- Set of PO allocations: shaded area.

②

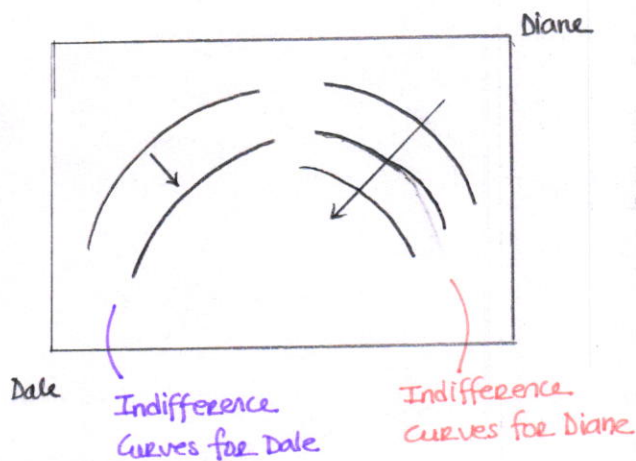
(a)



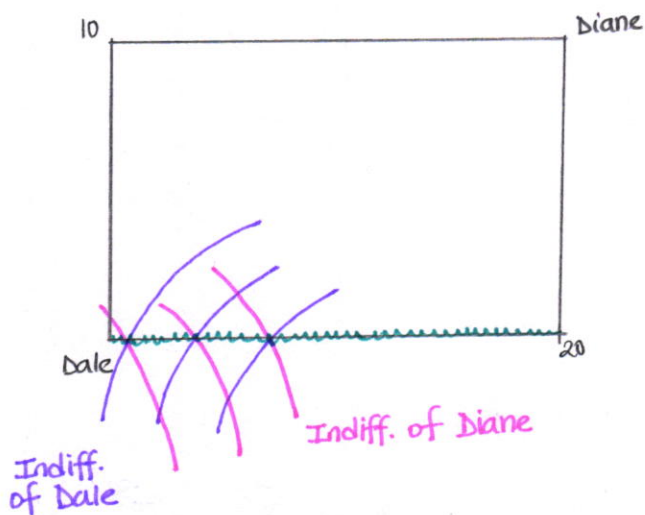
• $\omega_{Diane} = (10, 5)$

$\omega_{Diane} = (10, 5)$

so, total initial endowment:
20 apples & 10 bananas

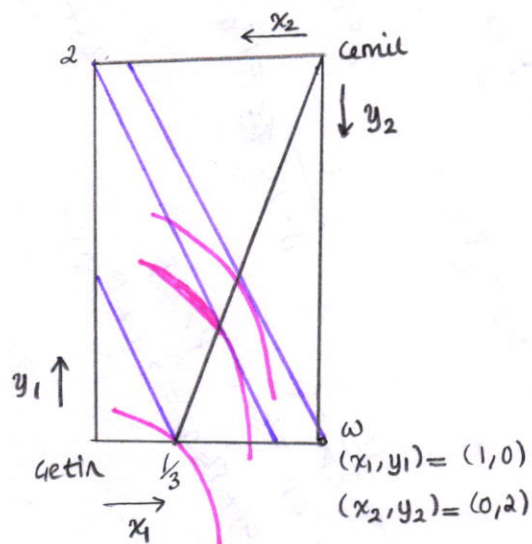


(c)



- PO allocations are those at which Dale's banana consumption is zero (otherwise, he can give his banana to Diane for nothing, and they both will be better off)
- So, allocations along the bottom line of the Edgeworth box are PO.

③ $u_1(x, y) = 3x + y$; $x_1 = 1$ $y_1 = 0$ (Getin)
 $u_2(x, y) = xy$; $x_2 = 0$ $y_2 = 2$ (Cemil)

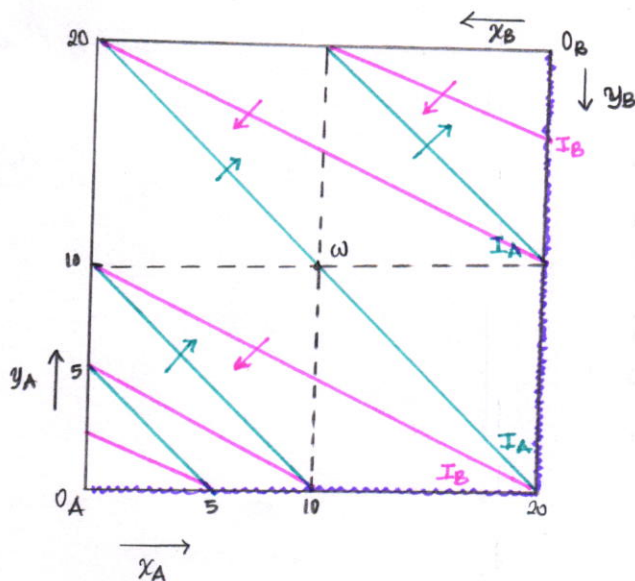


- At P.O. allocations,

$$MRS_1 = \frac{3}{1} = \frac{y_2}{x_2} = MRS_2$$

$$y_2 = 3x_2 \text{ for Cemil.}$$

④ $u_A = x_A + y_A$; $(x_A, y_A) = (10, 10)$
 $u_B = x_B + 2y_B$; $(x_B, y_B) = (10, 10)$



- Contract curve consists of the bottom & right border of Edgeworth box.

You can see it by fixing one agent's indifference curve and trying to make the other better off.

5

$$u_A = x_A^\alpha y_A^{1-\alpha} ; \bar{x}_A = 1 \wedge \bar{y}_A = 2$$

$$u_B = x_B^\beta y_B^{1-\beta} ; \bar{x}_B = 2 \wedge \bar{y}_B = 1$$

- First, find the Marshallian demands of agents for both goods.

Notice that the utility fncs are Cobb-Douglas and for the incomes of the agents:

$$I_A = P_X x_A + P_Y y_A = 1 \cdot P_X + 2 \cdot P_Y$$

$$I_B = 2 \cdot P_X + 1 \cdot P_Y$$

the demand fncs are:

$$x_A^* = \frac{\alpha(P_X + 2P_Y)}{P_X}$$

$$y_A^* = \frac{(1-\alpha)(P_X + 2P_Y)}{P_Y}$$

$$x_B^* = \frac{\beta(2P_X + P_Y)}{P_X}$$

$$y_B^* = \frac{(1-\beta)(2P_X + P_Y)}{P_Y}$$

- To find the competitive eqm, mkts should clear & a market is cleared if the sum of excess demand for a good is equal to zero.

$$\begin{aligned} ED_X &= ED_X^A + ED_X^B = x_A^* - \bar{x}_A + x_B^* - \bar{x}_B = 0 \\ &= \frac{\alpha(P_X + 2P_Y)}{P_X} - 1 + \frac{\beta(2P_X + P_Y)}{P_Y} - 2 = 0 \end{aligned}$$

Take P_X as a numeraire ; i.e. $P_X = 1$ (only relative price matters)

Then P_Y will be: $P_Y = \frac{3-\alpha-2\beta}{2\alpha+\beta} (*)$

Do the same for good y (mkt clearing)

$$ED_Y = \frac{(1-\alpha)(P_X + 2P_Y)}{P_Y} - 2 + \frac{(1-\beta)(2P_X + P_Y)}{P_Y} - 1 = 0$$

When we take $P_X = 1$, P_Y will be: $P_Y = \frac{3-\alpha-2\beta}{2\alpha+\beta} (*)$

Notice that (*) are identical. You do not need to do it twice. When the first mkt is cleared, the second will be cleared for the same relative prices as well.

- To find the eqm allocation, plug the value of P_X and P_Y to the demand functions we derived.

$$(P_X, P_Y) = \left(1, \frac{3-\alpha-2\beta}{2\alpha+\beta} \right)$$

$$x_A^* = \frac{\alpha(6-3\beta)}{2\alpha+\beta}$$

$$y_A^* = \frac{(1-\alpha)(6-3\beta)}{3-\alpha-2\beta}$$

$$x_B^* = \frac{3\beta(\alpha+1)}{2\alpha+\beta}$$

$$y_B^* = \frac{3(1-\beta)(\alpha+1)}{3-\alpha-2\beta}$$

- ⑥ (a) The competitive (Walrasian) eqm. is defined as the allocation $(x_1^A, x_2^A, x_1^B, x_2^B)$ and the prices (p_1, p_2) such that both ms. A and Mr. B are solving their consumer problem with endowments and mkt's clear for both goods.

For A $\Rightarrow \max \sqrt{x_1^A} + x_2^A$
s.t. $p_1 x_1^A + p_2 x_2^A = p_1 \omega_1^A + p_2 \omega_2^A$ — take $p_2 = 1$ as numeraire
 $p_1 x_1^A + x_2^A = 2p_1 + 1$

If we substitute $x_2^A = 2p_1 + 1 - p_1 x_1^A$ in ms. A's utility max. problem:

$$\max \sqrt{x_1^A} + 2p_1 + 1 - p_1 x_1^A$$

FOC yields: $\frac{1}{2\sqrt{x_1^A}} - p_1 = 0$ so $\hat{x}_1^A = \frac{1}{4p_1^2}$

$$x_2^A = 2p_1 + 1 - p_1 \cdot \frac{1}{4p_1^2} = 2p_1 + 1 - \frac{1}{4p_1} \Rightarrow \text{Here, } x_2^A \geq 0 \text{ iff } p_1 \geq \frac{1}{4}(\sqrt{3}-1)$$

$$\hat{x}_2^A = \begin{cases} 2p_1 + 1 - \frac{1}{4p_1} & \text{if } p_1 \geq \frac{1}{4}(\sqrt{3}-1) \\ 0 & \text{if } p_1 < \frac{1}{4}(\sqrt{3}-1) \end{cases}$$

For B $\Rightarrow \max x_1^B + \sqrt{x_2^B}$
s.t. $p_1 x_1^B + p_2 x_2^B = p_1 \omega_1^B + p_2 \omega_2^B$
 $p_1 x_1^B + x_2^B = 2$

By substituting $x_1^B = \frac{2-x_2^B}{p_1}$, we have that Mr. B is maximizing the

expression $\frac{2-x_2^B}{p_1} + \sqrt{x_2^B}$. The FOC yields

$$-\frac{1}{p_1} + \frac{1}{2\sqrt{x_2^B}} = 0 \Rightarrow \hat{x}_2^B = \frac{p_1^2}{4}$$

and so, $x_1^B = \frac{2 - p_1^2/4}{p_1} = \frac{2}{p_1} - \frac{p_1}{4}$

Here $x_1^B \geq 0$ iff $p_1 \leq 2\sqrt{2}$, so $\hat{x}_1^B = \begin{cases} \frac{2}{p_1} - \frac{p_1}{4} & \text{if } p_1 \leq 2\sqrt{2} \\ 0 & \text{if } p_1 > 2\sqrt{2} \end{cases}$

• mkt clearing condition for good 1: $\hat{x}_1^A + \hat{x}_1^B = \bar{\omega}_1 = 2$

check if $p_1 > 2\sqrt{2}$:

$$\text{if } p_1 > 2\sqrt{2} \quad \hat{x}_1^B = 0; \text{ so } \left. \begin{aligned} \hat{x}_1^A + 0 &= 2 \\ \frac{1}{4p_1^2} &= 2 \end{aligned} \right\} p_1 = \sqrt[4]{8} < 2\sqrt{2}$$

contradiction ∇
 $p_1 > 2\sqrt{2}$ is not possible.

$$p_1 \leq 2\sqrt{2} \Rightarrow \frac{1}{4p_1^2} + \frac{2}{p_1} - \frac{p_1}{4} = 2$$

This eqn. actually does have three solutions: $(p_1=1), p_1 = -\frac{1}{2}\sqrt{77}, p_1 = \frac{1}{2}\sqrt{77} - \frac{9}{2}$ but only one of them ($p_1=1$) satisfies our requirements that $p_1 \geq 0$ and $p_1 \leq 2\sqrt{2}$. Substituting

$$p_1=1 \quad \hat{x}_1^A = \frac{1}{4}; \quad \hat{x}_2^A = \frac{11}{4}; \quad \hat{x}_1^B = \frac{7}{4}; \quad \hat{x}_2^B = \frac{1}{4} \quad \text{to } p_1=p_2=1$$

(b) First solve the set of Pareto efficient allocations in the interior of the Edgeworth Box by setting $MRS_{12}^A = MRS_{12}^B$:

$$MRS_{12}^A = \frac{MU_1^A}{MU_2^A} = \frac{\left(\frac{1}{2\sqrt{x_1^A}}\right)}{1} = \frac{1}{\left(\frac{1}{2\sqrt{x_2^B}}\right)} = \frac{MU_1^B}{MU_2^B} = MRS_{12}^B$$

$$\frac{1}{2\sqrt{x_1^A}} = 2\sqrt{x_2^B}$$

$$x_2^B = \frac{1}{16x_1^A} \Rightarrow$$

In the competitive eqm. allocation in the first problem, we found that $x_2^B = \frac{1}{4}$ and $x_1^A = \frac{1}{4}$, which satisfies this eqn., and so yes, the eqm. allocation is indeed Pareto efficient.

(c) Recall that all points in the interior of the Edgeworth Box which satisfy $x_2^B = \frac{1}{16x_1^A}$ is P.E. Because $x_2^B = \frac{1}{2}$ and $x_1^A = \frac{1}{8}$ satisfy, we know that this allocation is Pareto efficient.

⑦ 3 person 2 good economy:

Competitive eqm ① each agent solves ump
② mkt clear

$$A: x_1^{A*} = \frac{I^A}{2P_1} \quad x_2^{A*} = \frac{I^A}{2P_2}$$

$$\text{Take } P_2 = P \wedge P_1 = 1; \quad \omega_A = (0, 1) \\ I^A = P$$

$$\text{Hence; } x_1^{A*} = \frac{P}{2} \text{ and } x_2^{A*} = \frac{1}{2}$$

$$B: x_1^{B*} = x_2^{B*} = \frac{I^B}{P_1 + P_2} \text{ and } I^B = 1 + P$$

$$\text{Hence; } x_1^{B*} = x_2^{B*} = 1$$

$$C: x_1^{C*} = \frac{3}{4} \frac{I^C}{P_1} \text{ and } x_2^{C*} = \frac{1}{4} \frac{I^C}{P_2} \text{ and } I^C = 1$$

$$x_1^{C*} = \frac{3}{4} \text{ and } x_2^{C*} = \frac{1}{4P}$$

• Each agent maximized their utility w.r.t the BC.

Now, mkts clear (if one clears, the other will be cleared too)

$$x_1^{A*} + x_1^{B*} + x_1^{C*} = 2 \text{ (total endowment of } x_1)$$

$$\frac{P}{2} + 1 + \frac{3}{4} = 2 \Rightarrow P = \frac{1}{2}$$

$$\text{Walrasian Eqm: } \{(P_1, P_2), (x_1^A, x_2^A), (x_1^B, x_2^B), (x_1^C, x_2^C)\}$$

$$W.E. = \{(1, \frac{1}{2}), (\frac{1}{4}, \frac{1}{2}), (1, 1), (\frac{3}{4}, \frac{1}{2})\}$$

⑧ Let $p = p_1/p_2$ so that values are measured in terms of good 2.

(a) Alf: Alf's budget constraint is: $p_1 x_1^a + x_2^a = I^a$

Therefore, given the information in the question, the demand functions are:

$$x_1^a = x_2^a = \frac{I^a}{p+1}$$

This behavior is consistent with the following utility fnc:

$$u^a(x_1^a, x_2^a) = \min \{x_1^a, x_2^a\}$$

Bill: Budget constraint is $p_1 x_1^b + p_2 x_2^b = I^b$ and the expenditure on good 1 is $p \cdot x_1^b$; the expenditure on good 2 is x_2^b

From the question, we know that $p \cdot x_1^b = 2 x_2^b$

$$p \cdot x_1^b + x_2^b = p \cdot x_1^b + \frac{p x_1^b}{2} = I^b$$

$$\underbrace{\frac{3p \cdot x_1^b}{2}} = I^b$$

$$x_1^b = \frac{2I^b}{3p} \quad \text{and} \quad x_2^b = \frac{I^b}{3} \Rightarrow$$

• Notice that this is consistent with the demand functions derived from Cobb-Douglas utility func. maximization.

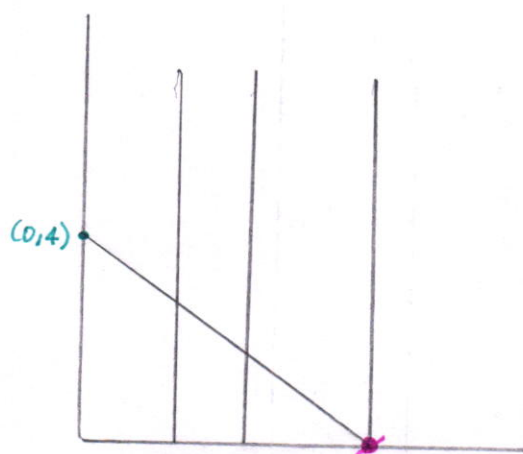
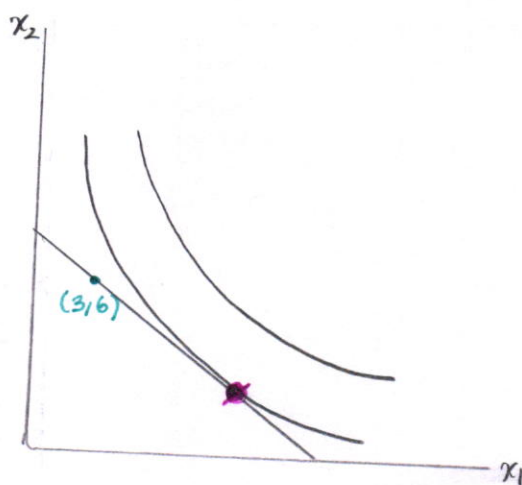
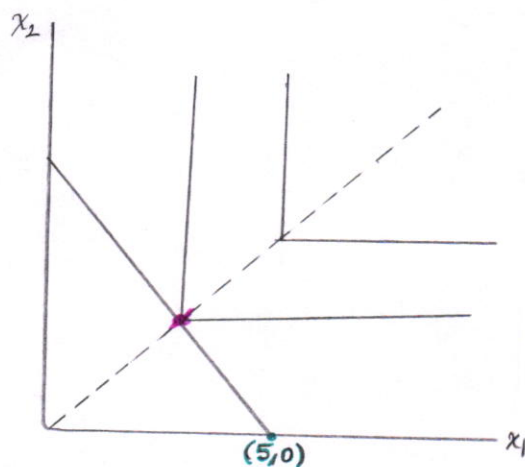
$$u^b = (x_1^b)^{2/3} (x_2^b)^{1/3}$$

Charlie: Budget constraint: $p_1 x_1^c + x_2^c = I^c$

By given information $x_2^c = 0 \wedge x_1^c = \frac{I^c}{p}$

The utility fnc. consistent w/ this behavior is $u^c = x_1^c$.

• Hence, indifference maps are:



• initial point
• optimum pt.

(b) Since $w_A = (5, 0)$, $w_B = (3, 6)$, $w_C = (0, 4)$ by using the results found in part (a):

• Alf's demand: $I^A = 5P \Rightarrow x_1^A = x_2^A = \frac{5P}{P+1}$

• Bill's demand: $I^B = 3P + 6 \Rightarrow x_1^B = 2 + \frac{4}{P} \quad x_2^B = P + 2$

• Charlie's demand: $I^C = 4 \Rightarrow x_1^C = 4/P \quad x_2^C = 0$

Hence, excess demand for good 2 is:

$$E_2 = \frac{5P}{P+1} + P + 2 - 10 \quad \text{and if } E_2 = 0 \Rightarrow P = -2 \text{ or } P = 4 \checkmark$$

Hence, eqm. price ratio is equal to 4.

utility levels are: $u^A = 4$, $u^B = 3.78$ and $u^C = 1$.

(i) Excess demand is now ($w_A = (9, 0)$)

$$E_2 = \frac{9P}{P+1} + P + 2 - 10 \quad \text{and the eqm. price is } 2.$$

Then, the utility levels are: $u^A = 6$, $u^B = 4$ and $u^C = 2$

(ii) $w_C = (4, 4) \Rightarrow I^C = 4P + 4$

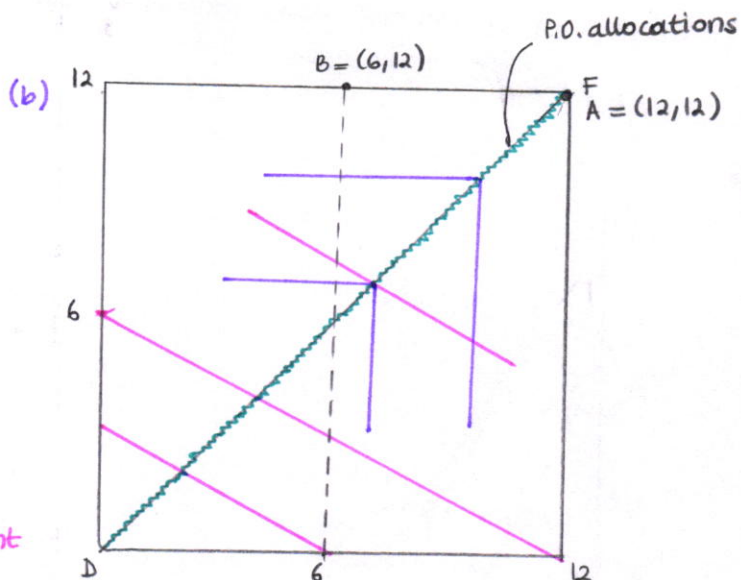
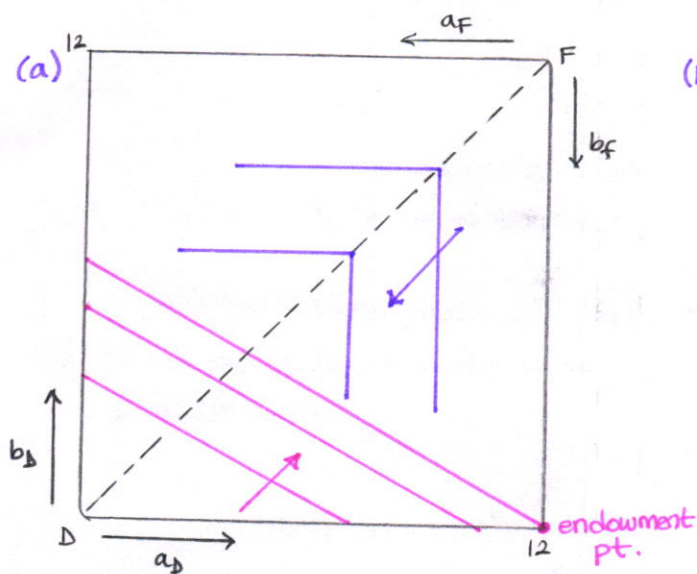
$$x_1^C = P + 1 \quad \text{and} \quad x_2^C = 0$$

$$u^C = x_1^C = P + 1$$

Excess demand for good 2 is $E_2 = \frac{5P}{P+1} + P + 2 - 10$. and the price is 4.

utility levels: $u^A = 4$, $u^B = 3.78$ and $u^C = 4 + 1 = 5$.

⑨ $u_D = a + 2b \quad \bar{a}_D = 12, \bar{b}_D = 0$
 $u_F = \min\{a, b\} \quad \bar{a}_F = 0, \bar{b}_F = 12$



• The point $A = (a_D, b_D) = (12, 12)$ is on line of P.O. allocations, so, it is Pareto efficient

• The point $B = (a_D, b_D) = (6, 12)$ is not.

(c) $MRS_D = \frac{1}{2} = \frac{P_a}{P_b} \Rightarrow$ At eqm. $P_b = 2P_a$
 Take $P_a = 1 \Rightarrow P_b = 2$

• Fatma choose $a_f = b_f \Rightarrow 1 \cdot a_f + 2b_f = I = 12 \cdot 2$
 $b_f = a_f = 8$
 $a_D = 12 - 8 = 4$ and $b_D = 12 - 8 = 4$

(d) (i) $a_D = 12, b_D = 12, a_f = 0, b_f = 0 \Rightarrow w = \max \{36, 0\} = 36$

(ii) $w = \min \{u_D, u_f\} \Rightarrow u_D = u_f$
 $a_D + 2b_D = a_f = b_f$ and $a_D + a_f = 12$
 $b_D + b_f = 12$
 $a_f = 9, a_D = 3, b_f = 9, b_D = 3$

(iii) $w = 120u_D + 120u_f$
 $\left. \begin{array}{l} a_D = 12, b_D = 12 \\ a_f = 0, b_f = 0 \end{array} \right\} w = 4320$

- (10) (i) 100 of the traders' initial endowment is $(10, 0, 0)$, each of them
 (2) 50 " " is $(0, 5, 0)$ each
 (3) 50 " " is $(0, 0, 20)$ each

so, the total initial endowment is $\bar{e}_{total} = (1000, 250, 1000)$

All the traders have the same utility function: $u = x_1^{1/2} x_2^{1/4} x_3^{1/4}$ and a monotonic transformation of this function is $u' = \log(u) = \frac{1}{2} \log x_1 + \frac{1}{4} \log x_2 + \frac{1}{4} \log x_3$ which can be used for maximization instead of u .

Then, for each group of traders the Lagrangian:

$$\omega = \frac{1}{2} \log x_1^t + \frac{1}{4} \log x_2^t + \frac{1}{4} \log x_3^t + \lambda^t [\omega^t - p_1 x_1^t - p_2 x_2^t - p_3 x_3^t]$$

where $t = 1, 2, 3$ and ω^t is the budget of traders.

$$\omega^1 = 10P_1; \omega^2 = 5P_2; \omega^3 = 20P_3$$

• From FOCs; $\partial \omega / \partial x_1^t = 0, \partial \omega / \partial x_2^t = 0, \partial \omega / \partial x_3^t = 0, \partial \omega / \partial \lambda^t$ we find demand functions for a trader of type t :

$$x_1^t = \frac{\omega^t}{2P_1}; x_2^t = \frac{\omega^t}{4P_2}; x_3^t = \frac{\omega^t}{4P_3}$$

• Mkt clearing: total demand should be equal to total supply (initial endowment) so, there should not be excess demand for any good.

For Good 1: $100x_1^1 + 50x_1^2 + 50x_1^3 = 1000$

$$100 \cdot \frac{10P_1}{2P_1} + 50 \cdot \frac{5P_2}{4P_2} + 50 \cdot \frac{20P_3}{4P_3} = 1000 \Rightarrow \frac{1}{4} \frac{P_2}{P_1} + \frac{P_3}{P_1} = 1 \quad (i)$$

For Good 2: $100x_2^1 + 50x_2^2 + 50x_2^3 = 250$

$$100 \cdot \frac{10P_1}{4P_2} + 50 \cdot \frac{5P_2}{4P_2} + 50 \cdot \frac{20P_3}{4P_2} = 250 \Rightarrow \frac{P_1}{P_2} + \frac{P_3}{P_2} = \frac{3}{4} \quad (ii)$$

• (i) and (ii) implies $\frac{P_2}{P_1} = 2$ and $\frac{P_3}{P_1} = \frac{1}{2}$

using good 1 as numeraire, we immediately see that

$\omega^1 = \omega^2 = \omega^3 = 10$, then all are equally well off.