

Non-Nested Tests

Non-Nested Models

$$1) Y_t = a_0 + a_1 X_t + u_t$$

$$2) Y_t = b_0 + b_1 X_t + b_2 W_t + u_t$$

$$3) Y_t = c_0 + c_1 Z_t + c_2 Q_t + u_t$$

Nested Models: nested within a general model

(1) & (2): Nested models. Under $b_2 = 0$ on (2) we obtain (1)

(2) & (3) or (1) & (3): Non-nested. Under no restriction imposed on one of the models we can obtain the other

Model Comparison: Given that the models are theory-consistent and data-acceptable, AIC, SC or adjusted R^2 may be used. However, all these maintain that only one of the models is correct. But: the models may be observationally equivalent or neither of the models may be correctly specified

Encompassing

- * A preferred model should encompass all the alternative models to explain the variable of interest
- * the preferred model should explain why the alternative models cannot better explain the variable of interest
- * Unless new models are always encompassing, research progress in the area of interest will be non-systematic

Non-Nested Models & Testing

$$\text{Model A: } Y_t = b_0 + b_1 X_t + b_2 W_t + U_t$$

$$\text{Model B: } Y_t = c_0 + c_1 X_t + c_2 Q_t + U_t$$

consider a hybrid/nested model

$$\text{Model C: } Y_t = d_0 + d_1 X_t + d_2 W_t + d_3 Q_t + U_t$$

Model C encompasses/nests Models A & B

Under $d_3 = 0$: Model A is better

Under $d_2 = 0$: Model B is better

Encompassing F-test

$$\text{Model A: } Y_t = b_0 + b_1 X_t + b_2 W_t + U_t$$

$$\text{Model B: } Y_t = c_0 + c_1 X_t + c_2 Q_t + U_t$$

Comparing Models A & B

The Nested model

$$\text{Model C: } Y_t = d_0 + d_1 X_t + d_2 W_t + d_3 Q_t + U_t$$

H_{01} : Model A encompasses Model B: test $d_3 = 0$

H_{02} : Model B encompasses Model A: test $d_2 = 0$

$$\text{Model A: } Y_t = b_0 + b_1 X_t + b_2 W_t + b_3 Z_t + U_t$$

$$\text{Model B: } Y_t = c_0 + c_1 X_t + c_2 Q_t + c_3 M_t + U_t$$

Comparing Models A and B: Construct the nested model

$$\text{Model C: } Y_t = d_0 + d_1 X_t + d_2 W_t + d_3 Z_t + d_4 Q_t + d_5 M_t + U_t$$

↓ H_{01} : Model A encompasses Model B: Test $d_4 = d_5 = 0$
F-test. Unrestricted Model: C. Restricted Model: A

H_{02} : Model B encompasses Model A: Test $d_2 = d_3 = 0$

F-test. Unrestricted model: C.

Restricted model B

Davidson & MacKinnon J-test

Model A: $Y_t = b_0 + b_1 X_t + b_2 W_t + U_t$

Model B: $Y_t = c_0 + c_1 X_t + c_2 Q_t + U_t$

Suppose we nest models A & B in a general model

Model D: $Y_t = \lambda(\text{Model A}) + (1-\lambda)(\text{Model B})$

Estimate Model D

If $\lambda = 1$ choose Model A

If $\lambda = 0$ choose Model B

However Model D cannot be estimated

Model A: $Y_t = b_0 + b_1 X_t + b_2 W_t + U_t$

Model B: $Y_t = c_0 + c_1 X_t + c_2 Q_t + U_t$

Estimate Model A to obtain the fitted values of $Y_t: Y_t^{FA}$

• Estimate Model B to obtain the fitted values for $Y_t: Y_t^{FB}$

H_{0A} : Model B does not encompass Model A.

estimate:

$$Y_t = d_0 + d_1 X_t + d_2 W_t + d_3 Y_t^{FB} + U_t$$

test H_{0A} : $d_3 = 0$. R_{H0A} : Choose Model A

J-test

H_{0b} : Model A does not encompass Model B

Estimate:

$$Y_t = e_0 + e_1 X_t + e_2 W_t + e_3 Y_t^{FA} + U_t$$

Test $H_{0a}: e_3 = 0$. $R_{H_{0b}}$: Choose model B

Some possible outcomes

Reject H_{0a} and do not Reject $R_{H_{0b}}$: Choose model B

Do not Reject H_{0a} and Reject $R_{H_{0b}}$: Choose model A

Reject H_{0a} and Reject $R_{H_{0b}}$: An encompassing general model may be better

Do not Reject H_{0a} and do not Reject $R_{H_{0b}}$: the models are observationally equivalent