

PROBLEM SET III SOLUTIONS

Section I: Chapter 5

Question 1

1. Consider a one-period economy that consists of many households that are identical with the following preferences, where c is the consumption, l is leisure and γ is a preference shock.

$$U(c, l) = \log(c) + \gamma \log(l)$$

The households have a limited amount of time at their disposal, h , and have the choice to use it doing leisurely activities that give them utility (going out, reading a book etc.) or work (let the labor supply of household be N^s) and get paid the market wage w . Each household also own an equal share of the firms in this economy and get profit income π from them. There is also a government that uses lump-sum taxation to extract T from the households, to finance its expenditures G . Production of the consumption good in this economy is done by a large number of identical firms that have the following technology, where K is the fixed capital stock of the economy and N^d is the labor demand by the firm and z is the exogenous TFP shock.

$$Y = zf(K, N^d) = zK^\alpha N^{1-\alpha}$$

Households and firms interact in the labor market exchanging labor in return for wages (note that this is a real-economy and wages are paid in terms of consumption good, i.e. $w = 2$ means one hour of labor will get you 2 units of consumption good).

- (a) What are the exogenous variables in this model? What are the endogenous variables? What are the parameters of the model?
- (b) Define the competitive equilibrium of this economy; be precise as in your textbook.
- (c) Write down the budget and time constraint of the household (you can assume the total number of hours available is normalized to 1, i.e. $h = 1$). Write down the utility maximization problem of the representative household. What are the choice variables for the household?
- (d) Write down the profit maximization problem of the representative firm, what does the firm choose?
- (e) Solve the problem of the household to get optimal consumption level, c^* , and labor supply N^{s*} for a given level of wage, w (i.e. the consumption and labor supply functions you find will only be functions of the exogenous variables, parameters and w).
- (f) Solve the problem of the firm to get the labor demand, N^{d*} .

(g) What is the equilibrium level of w that will clear the labor market, i.e. $N^s = N^d$? What is the equilibrium level of employment, N^* (and leisure, l^*)? Equilibrium level of output, Y^* , and aggregate consumption C^* ? (The starred equilibrium variables will only be functions of exogenous variables and parameters of the model).

(h) Using the government budget $G - T$, and budget constraint of the representative household, show that when labor market is in equilibrium, i.e. $N^s = N^d = N^*$, aggregate resource constraint holds, i.e. $Y^* = C^* + G$. Now consider this country comes under the rule of a benevolent dictator. This dictator can dictate inputs to production by the representative firm, can force the consumer to supply the appropriate quantity of labor, and then distributes consumption goods and leisure hours to the consumer; all in a way that makes the consumer as well off as possible. There is no need for the existence of decentralized markets and prices for allocation mechanism as the dictator claims it can do a better job. The problem of the dictator is to maximize the utility of household subject to production technology, $Y = zf(K, N)$ and aggregate resource constraint $Y = C + G$.

(i) Write down the maximization of social welfare problem of the benevolent dictator.

(j) Solve this problem to get the socially optimal level of consumption, leisure, employment and output. Is this allocation Pareto optimal?

(k) Is the dictator's claim of being able to do a better job than markets supported? If you were an economic advisor to this dictator, which economic theorem would you have cited to convince him his claim would not hold?

PS #2 Suggested Solutions

① $u(c, l) = \log(c) + \gamma \log(l)$

(a) Exogenous variables: z, K, G

Endogenous variables: C, N^S, N^d, w, T, Y

Parameters: γ, α

* Exogenous varb: a factor whose value is determined by factors or varbs. outside the causal system under study.

* Endogenous varb: a varb. is said to be endogenous within the causal model if its value is determined or influenced by one or more of the independent variables.

(b) A competitive equilibrium is a set of endogenous quantities C, N^S, N^d, T, Y and w such that given the exogenous variables z, K, G the following are satisfied:

① The representative consumer chooses C and N^S to make herself as well off as possible subject to her budget constraint given w, T, π .

② Representative firm chooses N^d to maximize profits with maximized output $Y = z \cdot f(K, N^d)$ and maximized profit $\pi = z \cdot f(K, N^d) - w \cdot N^d$. In eqm., the profits that the representative firm earns must be equal to the dividend income that is received by the consumer.

labour supply equals labor demand

③ labor mkt clears ($N^d = N^S$)

④ The government budget constraint is satisfied ($T = G$)

government purchases equal taxes

(c) Budget constraint of the household:

$$C = \underbrace{w \cdot N^S}_{\text{labor income}} + \underbrace{\pi}_{\text{dividend income}} - \underbrace{T}_{\text{tax}}$$

amount spent on consumption

Time constraint of the household:

$$h = \underbrace{l}_{\text{leisure}} + \underbrace{N^S}_{\text{labor supply}} \Rightarrow h=1, \text{ then } 1 = l + N^S$$

amount of time at disposal

• utility maximization problem of the household:

$$\max_{c, l} u(c, l)$$

$$\text{subject to } C = w \cdot N^S + \pi - T \quad \text{and} \quad h = l + N^S$$

If we substitute time constraint into the budget constraint and write the utility max. problem in an open form:

$$\begin{aligned} \max_{c, l} \quad & \log c + \gamma \log l \\ \text{s.t.} \quad & c = w(1-l) + \pi - T \end{aligned}$$

choice vars — can be set by the agent to many different values. one of these values is the optimal (i.e. best or efficient) solution for problem at hand.

(d) Profit maximization problem of the representative firm

$$\max_{N^d} \quad z \cdot f(K, N^d) - w \cdot N^d$$

If we write the production function explicitly:

$$\max_{N^d} \quad z \cdot L^\alpha N^{1-\alpha} - w \cdot N^d$$

L choice var. for the firm

(e) Substitute budget constraint into the utility function:

$$\max_l \quad u(w(1-l) + \pi - T, l)$$

— now, the only choice var. is l

$$\text{FOC wrt } l: -w \cdot u_c(c, l) + u_l(c, l) = 0$$

$$u_l(c, l) = w \cdot u_c(c, l)$$

$$u_l(c, l) = \frac{\gamma}{l} = w \cdot \frac{1}{c} = u_c(c, l)$$

$$\frac{\gamma}{l} = w \cdot \frac{1}{w(1-l) + \pi - T} \quad \text{— since } c = w(1-l) + \pi - T$$

$$\gamma w(1-l) + \gamma \pi - \gamma T = w l$$

$$\gamma w + \gamma \pi - \gamma T = l w (1 + \gamma)$$

$$l^* = \frac{\gamma(w + \pi - T)}{w(1 + \gamma)}$$

From the time constraint; $l = l^* + N^S$

$$N^S = 1 - l^* = 1 - \frac{\gamma(w + \pi - T)}{w(1 + \gamma)} = \frac{w - \gamma(\pi - T)}{w(1 + \gamma)}$$

From the budget constraint; $c^* = w \cdot N^S + \pi - T$

$$= w \cdot \frac{w - \gamma(\pi - T)}{w(1 + \gamma)} + \pi - T$$

$$c^* = \frac{w + \pi - T}{1 + \gamma}$$

(2)

(f) obtain the FOCs from profit maximization problem of the representative firm

$$\text{FOC w.r.t } N^d: z \cdot f_{N^d}(K, N^d) - w = 0$$

$$w = z \cdot f_{N^d}(K, N^d)$$

$$f_{N^d} = \frac{\partial y}{\partial N^d} = z \cdot (1-\alpha) K^\alpha N^{d-\alpha} = z \cdot \frac{\partial f(K, N^d)}{\partial N^d}$$

$$w = z \cdot (1-\alpha) K^\alpha N^{d-\alpha}$$

$$N^{d*} = K \left[\frac{z(1-\alpha)}{w} \right]^{1/\alpha}$$

(g) From $N^s = N^{d*}$, we can find the real wage.

from the utility
max. problem

from the profit
max. problem

$$\frac{u_L(c, l)}{u_C(c, l)} = z \cdot f_{N^d}(K, N^d)$$

$$\frac{w - \delta(\pi - T)}{w(1+\delta)} = K \left[\frac{z(1-\alpha)}{w} \right]^{1/\alpha}$$

— From this eqn., we can obtain w^* . Then, by substituting w^* into either N^{s*} or N^{d*} , we can find N^* in terms of exogenous vars & parameters.

— We can find L^* from the time constraint:

$$L^* = 1 - N^*$$

— We can obtain C^* from the budget constraint of the consumer & by substituting N^* into the production function, we can find Y^* .

(h) We need to show that if $N^s = N^d = N^*$ and $G = T$, then $Y^* = C^* + G$.

$$C^* = w^* N^* + \pi - T$$

$$\pi = Y^* - w^* N^* \quad T = G$$

$$C^* = w^* N^* + Y^* - w^* N^* - G$$

$$Y^* = C^* + G$$

(i) Social planner solves:

$$\max_{C, \ell} u(C, \ell) \quad \text{subject to} \quad C = z \cdot F(K, N) - G$$

$$\begin{aligned} L &= N + \ell \\ 1 &= N + \ell \\ N &= 1 - \ell \end{aligned}$$

Substitute $N = 1 - \ell$ into the utility function:

$$\max_{\ell} u(z \cdot F(K, 1 - \ell) - G, \ell)$$

(j) From the social planner's problem, we can obtain FOC with respect to ℓ :

$$\underbrace{-\frac{1}{C}(1-\alpha)zK^{\alpha}(1-\ell)^{-\alpha}}_{\frac{\gamma}{C}} + \underbrace{\frac{\gamma}{\ell}}_{\frac{\gamma}{\ell}} = 0$$

$$\frac{\gamma}{\ell} = \frac{1}{z \cdot K^{\alpha} (1-\ell)^{1-\alpha} - G} (1-\alpha)zK^{\alpha}(1-\ell)^{-\alpha}$$

We can find ℓ^* from this eqn.
and $N^* = 1 - \ell^*$ by substituting ℓ^* in this eqn.

• Since the social planner has all the information about the economy, the allocation of social planner is always PO.

(k) Dictator's claim is not supported. Because the allocation of competitive eqm. equals the allocation of social planner. According to the FBWT, a competitive eqm. is PO.

What could cause a competitive eqm. to fail to be PO?

(1) Externalities – any activity for which an individual firm or consumer does not take into account of all associated costs & benefits.

(2) Existence of distorting taxes – If wage income is equal to $w(1-t)(h-\ell)$ so the effective wage is equal to $w(1-t)$.

When the consumer optimizes, he/she sets $MRS_{\ell, C} = w(1-t)$ while the firm optimizes by setting $MP_N = w$.

Therefore, in competitive eqm. $MRS_{\ell, C} < MP_N = MRT_{\ell, C}$

(3) Firms may not be price-takers, they may have monopoly power.

Question 2

Consider the following economy. A government finance its exogenous spending g through a labor income tax. In particular, the government spending g is equal to the amount of tax collected T . Hence, $g = T$. A representative firm produces goods using a linear technology:

$$y = zn$$

where y is the output, z is a constant exogenous technology, and n is the labor input. A representative consumer

$$\begin{aligned} \max_{c,l} \quad & u(c,l) \\ \text{s.t.} \quad & c = (1 - \tau_w)w(1 - l) \end{aligned} \tag{1}$$

Hence, the tax imposed by the government is a fraction τ_w of the wage-bill $w(1 - l)$ of the household. The government collects only labor taxes, i.e. $T = \tau_w w(1 - l)$.

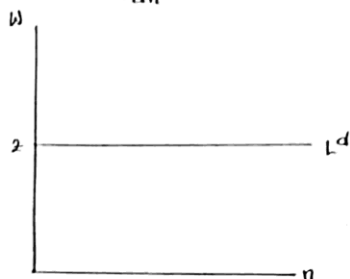
- (a) Find the optimal demand schedule for labor for the firm.
- (b) Find the optimal supply schedule of labor for the households.
- (c) Define accurately a competitive equilibrium for this economy.
- (d) Write the equivalent social planner problem.
- (e) Find the equilibrium that yields the optimal level of labor in the economy.
- (f) Are the competitive equilibrium and the solution to the social planner problem equivalent? Explain.
- (g) Use a graph in the space c (y-axis) and l (x-axis). Display the social planner equilibrium and the competitive equilibrium in the same graph. Explain carefully all the curves you plot.

Solution:

- ② (a) In order to find the demand for labor (N^d), we need to solve the firm's max. problem:

$$\max_n \pi = \underbrace{z\eta}_{\text{Revenue}} - \underbrace{wn}_{\text{Cost}}$$

$$\frac{d\pi}{dn} : z - w = 0 \Rightarrow z = w$$



• Thus, we have that the labor demand schedule (L^d) is infinitely elastic at $w = z$.

• more formally;

$$L^d = \begin{cases} 0 \leq n \leq \infty & \text{if } z = w \\ 0 & \text{if } z < w \\ \infty & \text{if } z > w \end{cases}$$

- (b) This time, we need to focus on the maximization problem of the household in order to find (N^s):

$$\max_{c, l} u(c, l)$$

$$\text{subject to } c = (1 - \tau w) w (1 - l)$$

$$\mathcal{L} = u(c, l) + \lambda [(1 - \tau w) w (1 - l) - c]$$

$$\text{FOCs: } \frac{\partial \mathcal{L}}{\partial c} : u_c(c, l) - \lambda = 0$$

$$\frac{\partial \mathcal{L}}{\partial l} : u_l(c, l) - \lambda (1 - \tau w) w = 0$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} : (1 - \tau w) w (1 - l) - c = 0$$

substitute

$$\frac{u_l(c, l)}{u_c(c, l)} = (1 - \tau w) w$$

$$\frac{u_l((1 - \tau w) w (1 - l), l)}{u_c((1 - \tau w) w (1 - l), l)} = (1 - \tau w) w$$

↳ defines the optimal value of leisure for the household

- If we rewrite $l = 1 - n$:

$$L^s : \frac{u_l((1 - \tau w) w n, 1 - n)}{u_c((1 - \tau w) w n, 1 - n)} = (1 - \tau w) w$$

optimal labor supply schedule
optimality condition expressed as a function of labor.

- As we can see, the optimal labor supply decision for the household is to equate the ratio of the marginal utilities (of leisure and consumption) to the perceived price ratio. Note that the price of goods = 1.

Furthermore, the wage that the household takes into account in his maximization decision is the after tax wage. (tax rates becomes distortionary as they affect the labor supply decision of the hh.)

(c) A competitive eqm. is a set of quantities c, ℓ, n and prices w , which satisfies the following properties:

- Consumers choose c and ℓ optimally to maximize their utility given w and τw .
- Firm optimally chooses inputs (n) to maximize profits given w .
- Markets clear. Hence both labor and goods mkt have to clear.
- The government budget constraint is satisfied ($g = T$).

(d) The social planner tries to maximize the overall welfare by achieving the max. utility. However, in contrast with the household problem taxes are not taken as given, on the contrary they adjust to the optimal level of govt. spending.

- The objective function of the social planner:

$$\begin{aligned} & \max_{c, \ell} u(c, \ell) \\ \text{s.t. } & c + g = z(1 - \ell) \end{aligned}$$

↗ substitute & the problem will be translated into an unconstrained max. problem.

$$\max_{\ell} u(z(1 - \ell) - g, \ell)$$

$$\frac{du}{d\ell}: u_{\ell}(z(1 - \ell) - g, \ell) = z \cdot u_c(z(1 - \ell) - g, \ell)$$

$$\frac{u_{\ell}(c, \ell)}{u_c(c, \ell)} = z \quad \text{--- Ratio of the marginal utilities of leisure and consumption will be equal to the marginal productivity of labor. (taxes are not part of the decision rule) } \Rightarrow \text{ this max. will lead to the PO eqm., in contrast with the max. of the hh.}$$

(e) To find the eqm. that yields the optimal level of labor, use the eqn. above to replace c from the BC:

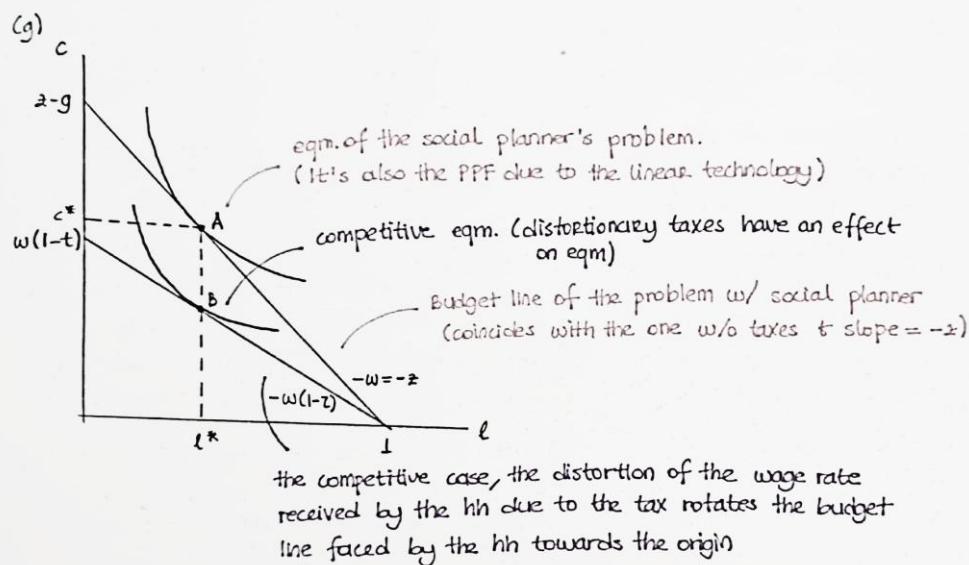
$$\frac{u_{\ell}(z(1 - \ell^*) - g, \ell^*)}{u_c(z(1 - \ell^*) - g, \ell^*)} = z$$

(f) The problem of social planner will lead to a Pareto optimal equilibrium in contrast with the competitive eqm. in the presence of distortionary labor taxes.

As taxes increase, the perceived budget line of the household rotates towards the origin as the received wage by the hh is lower than the eqm. wage ($w = z$ reduced by $(1 - \tau w)$) and thus the choices are distorted.

As such the marginal productivity of labor is not equal to the ratio of marginal utilities. On the other hand the social planner equates marginal productivity of labor with the ratio of the marginal utilities and thus the optimal labor supply will be equal with the case of competitive eqm. without the presence of taxes.

In the CE, prices play the role of the equilibrating mechanism. This implies that any distortion on the prices will appear in eqm. On the contrary the social planner equates the slope of the PPF with the one of the indifference curve of the hh and such distortions due to prices disappear. (the graph in part (g)).



Question 3

Suppose that the substitution effect of an increase in the real wage is always larger than the income effect for the representative consumer. Also assume that the economy is always in the low tax-rate equilibrium on the good side of the Laffer curve. Determine the effects of an increase in total factor productivity, z , on the Laffer curve, on the equilibrium tax rate, and on consumption, leisure, the quantity of labor supplied, and output.

Solution:

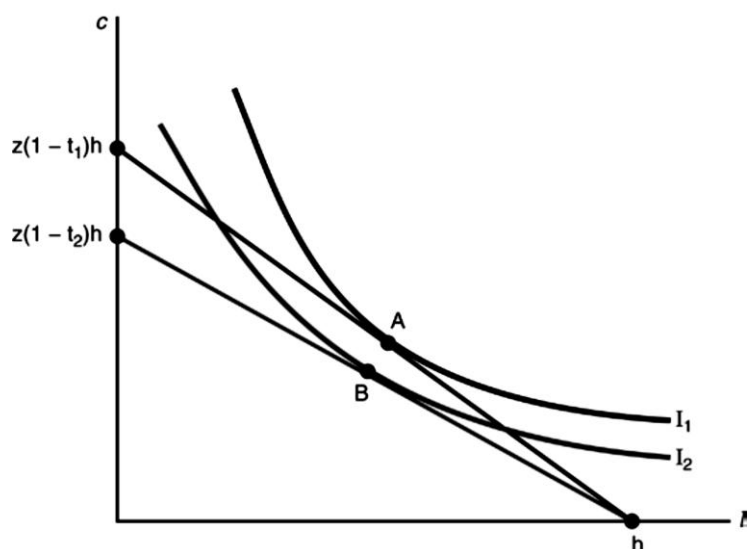
We know from previous analysis that an improvement in total factor productivity pushes up the *PPF*, and thus leads to an increase in consumption, a decrease in leisure, and thus an increase in the quantity of labor supplied. This increases the tax base, and thus allows a reduced tax rate to achieve the same tax revenue, or in other words, it pushes the left portion of the Laffer curve to the left. The reduction in the tax rate has then a further impact on the variables of interest: as we saw in question 7, first part with a reversal of all signs: consumption increases even more and leisure decreases yet more, leading to an even higher quantity of labor. All in all, as both labor and total factor productivity increase, output increases.

Question 4

In the simplified model proportional taxation there can be two equilibria, one with a high tax rate and one with a low tax rate. Now, suppose that government spending increases determine the effects of an increase in G on consumption, leisure, labor supply, real output and the tax rate in a high tax rate equilibrium and in a low tax rate equilibrium. How do your results differ? Explain why

Solution:

We need to analyze each case separately. Start with the good equilibrium. As government expenses increase, more tax revenue needs to be raised, and thus the tax rate needs to be increased. As shown in the figure below, this tilts down the linear production possibility frontier (*PPF*). The new equilibrium leads to a lower indifference curve. This results in a negative income effect and a lower after-tax wage (recall that the wage is given by $z(1 - t)$, which in turn generates a substitution effect. The income effect lowers both consumption and leisure, while the substitution effect decreases consumption and increases leisure. All in all, consumption is lower and leisure is higher, since the substitution effect dominates the income effect. This implies that labor supply is reduced, which in turn reduces equilibrium labor and output.



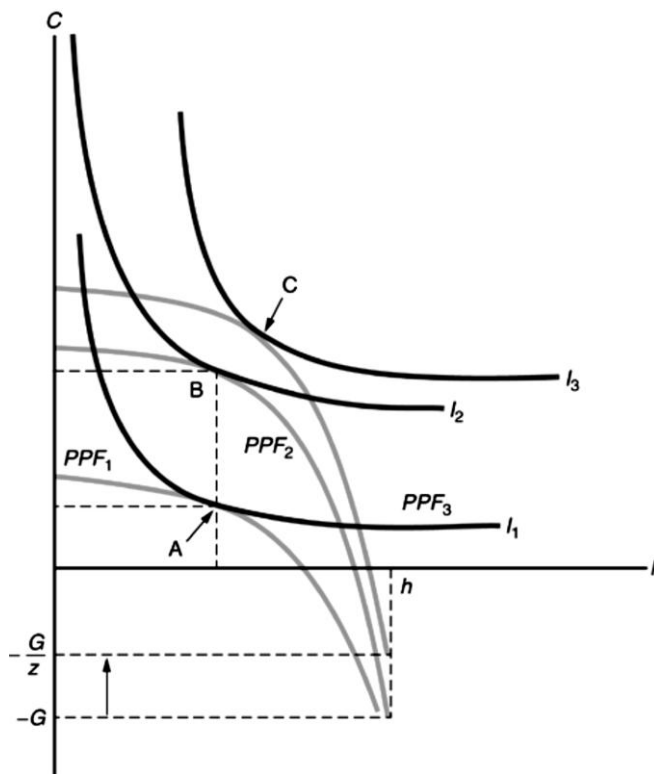
The story is different in the bad equilibrium. To increase tax revenue, one needs to reduce the tax rate. Then all the changes discussed above are exactly in the opposite direction.

Question 5

Suppose that total factor productivity, z , affects the productivity of government production just as it affects private production. That is, suppose that when the government collects taxes, it acquires goods that are then turned into government-produced goods according to $G = zT$ so that z units of government goods are produced for each unit of taxes collected. With the government setting G , an increase in z implies that smaller quantity of taxes are required to finance the given quantity of government purchases G . Under these circumstances, using a diagram determines the effects of an increase in z on output, consumption, employment, and the real wage, treating G as given. Explain your results.

Solution:

Government Productivity. First consider the benchmark case in which $z = 1$, and there is no effect of changes in z on government activities. Now suppose that z increases. This case of an increase in z is depicted in the figure below. The original production possibilities frontier is labeled PPF_1 and the competitive equilibrium is at point A. If the increase in z only affects the economy through the change in $zF(K, N)$, then the new production possibilities frontier is PPF_2 . The diagram shows a case in which the income and substitution effects on leisure exactly cancel out, and the economy moves to point B. The equation for the production possibilities frontier is $C = zF(K, h - l) - T$. In the benchmark case, $T = G$ and so we have $C = zF(K, h - l) - G$. For this problem, $T = G/z$ and so the production possibilities frontier is given by $C = zF(K, h - l) - G/z$. When $z = 1$, the two PPF s coincide. When z increases, the vertical intercept of the PPF increases by $G/\Delta z$. Therefore, the new PPF is PPF_3 in the figure below. The competitive equilibrium is at point C. There is an additional income effect that provides an additional increase in equilibrium consumption, and a reinforced income effect that tends to make leisure increase. Therefore, relative to the benchmark case, there is a larger increase in consumption, and either a smaller decrease in leisure or a larger increase in leisure.



Question 6

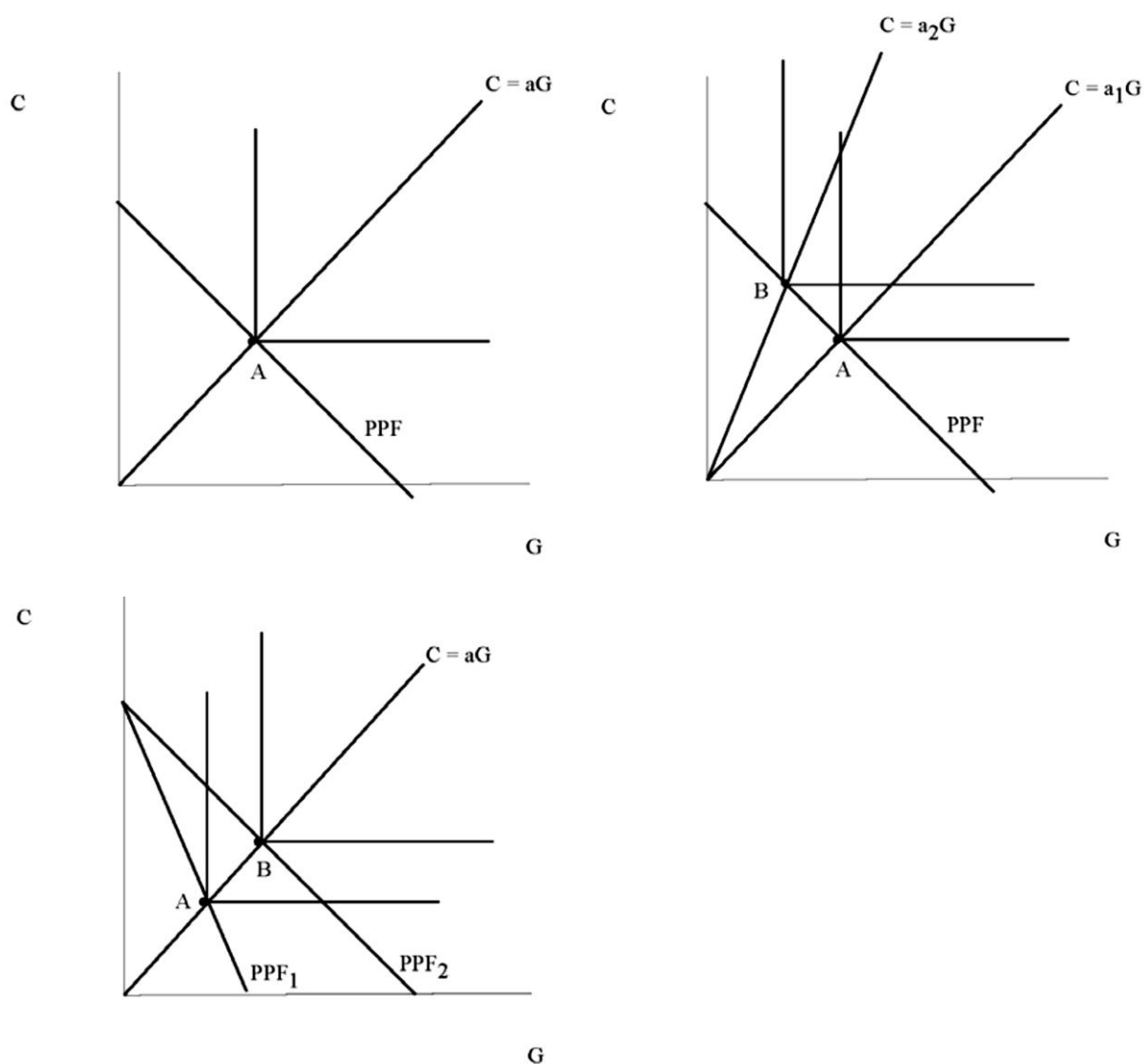
Consider the model of public goods in the last section of this chapter.

- Suppose that preferences over private consumption C and public goods G are such that these two goods are perfect substitutes; that is, the marginal rate of substitution of public goods for private goods is a constant $b > 0$. Determine the optimal quantity of public goods that the government should provide, and interpret your results. Make sure you show all of the relevant cases. What happens when b changes, or when q changes?
- Repeat part (a), except with perfect complements preferences; that is, for the case where the representative consumer always wishes to consume private consumption goods and public goods in fixed proportions, or $C = aG$, with $a > 0$.

Solution:

- With perfect substitutes preferences, indifference curves are straight lines with slope $-b$, where b is the marginal rate of substitution. If $b > 1/q$, so that the indifference curves are steeper than the PPF, then the optimal choice for the government is $G=qY$, so that $C = 0$. Thus, if b is relatively large (the consumer cares relatively more about public goods relative to private goods) and q is relatively large (the government is relatively efficient), then all production should be carried on by the government. Alternatively, if $b < 1/q$, then $G = 0$ and $C = Y$, so that government is inactive. Thus, if b increases or q increases, this makes it more likely that $b > 1/q$ and we have the first case, where all production comes from the government.
- With perfect complements, indifference curves are as depicted in Figure 10.1, and the initial equilibrium is at point A. If a increases, then the equilibrium shifts from A to B

in Figure 10.2. An increase in a represents a greater preference for private goods relative to public goods, and in Figure 10.2, this results in less public goods and more private consumption in equilibrium. If q increases, this shifts the PPF out as in Figure 10.3, and the equilibrium shifts from A to B. Both C and G increase, driven by income effects.



Section II: Chapter 9

Question 7

Suppose that there is a household who lives for two periods. Its lifetime utility function is:

$$U = u(C_t) + \beta u(C_{t+1})$$

$u(\cdot)$ is a function that is increasing and concave (the first derivative is positive and the second derivative is negative). The household faces two within period budget constraints:

$$C_t + S_t = Y_t$$

$$C_{t+1} = Y_{t+1} + (1 + r_t)S_t$$

r_t is the interest rate on the stock of savings brought from period t into period $t + 1$. The household would like to pick consumption and saving to maximize lifetime utility subject to the two budget constraints.

- a) Solve for S_t from either the t or $t + 1$ budget constraints, plug it into the other, and derive the unified, intertemporal budget constraint. In words, describe what the intertemporal budget constraint says.
- b) Solve for C_{t+1} in terms of C_t , Y_t , Y_{t+1} , and r_t in the intertemporal budget constraint you derived on part (a). Plug this into the objective function, which turns the problem into an unconstrained maximization problem just over C_t . Derive the Euler equation (first order necessary condition for a maximum). Provide an economic interpretation of the Euler equation.
- c) Define an indifference curve and derive an expression for the slope of the indifference curve.
- d) Combine the indifference curve with a graphical representation of the intertemporal budget constraint (what we call in the graph the “budget line”). Graphically show the optimal consumption bundle.
- e) Suppose that there is an increase in Y_t . Graphically show how this ought to affect C_t and C_{t+1} .
- f) Suppose that there is an increase in Y_{t+1} . Graphically show how this ought to affect C_t and C_{t+1} .
- g) How does first period saving, S_t , react differently to an increase in Y_t relative to an increase in Y_{t+1} ?

Solution:

lifetime utility: $u = u(c_t) + \beta u(c_{t+1})$

Budget constraints: $c_t + s_t = y_t$

$$c_{t+1} + s_{t+1} = y_{t+1} + (1+r_t)s_t$$

(a) The household want to set $s_{t+1} = 0$

Since the consumer lives for only two periods, there is no point saving in the second period.

(b) setting $s_{t+1} = 0$

$$c_t + s_t = y_t$$

$$c_{t+1} = y_{t+1} + (1+r_t)s_t$$

$$s_t = y_t - c_t$$

└ plug it in future BC $\Rightarrow c_{t+1} = y_{t+1} + (1+r_t)(y_t - c_t)$

$$c_{t+1} = y_{t+1} + (1+r_t)y_t - (1+r_t)c_t$$

$$c_t + \frac{c_{t+1}}{1+r_t} = y_t + \frac{y_{t+1}}{1+r_t} \Rightarrow \text{lifetime budget constraint}$$

• It states that the present value of lifetime consumption equals the present value of lifetime income

(c) From part (b); $c_{t+1} = y_{t+1} + (1+r_t)y_t - (1+r_t)c_t$

Plug it in the utility fnc:

$$\max_{c_t} u(c_t) + \beta u(y_{t+1} + (1+r_t)y_t - (1+r_t)c_t)$$

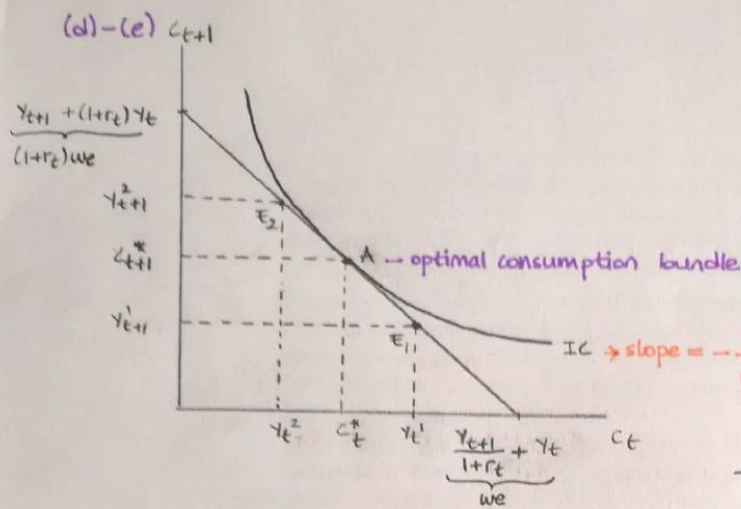
$$\partial u / \partial c_t: u'(c_t) + \beta u'(c_{t+1}) [-(1+r_t)] = 0$$

$$u'(c_t) = (1+r_t) \cdot \beta u'(c_{t+1})$$

$$\frac{u'(c_t)}{\beta u'(c_{t+1})} = (1+r_t)$$

At the optimal consumption path, the MRS, $u'(c_t) / \beta u'(c_{t+1})$ is equal to the gross interest rate $(1+r_t)$

Graphically, the slope of the indifference curve is equal to the slope of the intertemporal budget constraint.

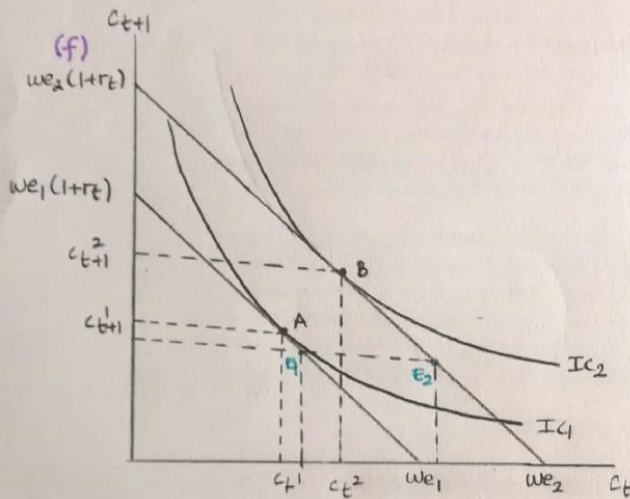


E_1 & E_2 - endowment pts

- If E_1 is the endowment point of the consumer, the s/he is a lender, as the consumption bundle chosen implies positive savings.

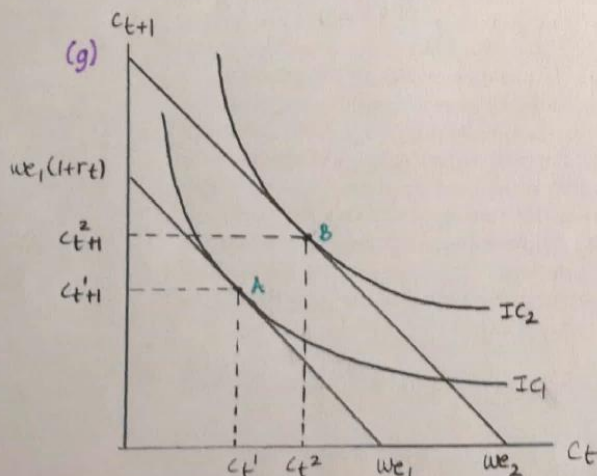
(in the current period s/he consumes less than the endowment & in the future period s/he consumes more than the endowment)

- If E_2 is the endowment point of the consumer, s/he is a borrower.



• When current income increases, lifetime wealth increases from w_1 to w_2 . The lifetime budget constraint shifts out, and the slope of the constraint remains unchanged, because the real interest rate does not change. Initially, the consumer chooses A, he or she chooses B after current income increases. Current and future consumption both increase (both goods are normal), and current consumption increases by less than the increase in current income.

(suppose that the consumer is under and the endowment point is E_1 initially, after the increase in y_t , the endowment of c_t also increase from e_1 to e_2 . However, the consumer does not spend all of his/her income to the current consumption. s/he also makes savings to increase future consumption)



• An increase in future income increases lifetime wealth from w_1 to w_2 , shifting the lifetime budget constraint to the right and leaving its slope unchanged. The consumer initially chooses point A, and s/he chooses B after the budget constraint shifts. Future consumption increases by less than the increase in future income, saving decreases, and current consumption increases.

Question 8

Assume a consumer who has the current period income $y=200$, future period income $y'=150$, current and future taxes $t=40$ and $t'=50$, respectively, and faces a market real interest rate of $r=0.05$, or 5% per period. The consumer would like to consume equal amounts in both periods, that is, he or she would like to set $c=c'$ if possible. However, this consumer is faced with a credit market imperfection, in that he or she cannot borrow at all, that is $s \geq 0$.

- Show the consumer's lifetime budget constraint and indifference curves in a diagram.
- Calculate his or her optimal current period and future period consumption and optimal saving and show this in your diagram.
- Suppose that everything remains unchanged, except that now $t = 20$ and $t' = 71$. Calculate the effects on current and future consumption and optimal saving, and show this in your diagram.
- Now, suppose alternatively that $y = 100$. Repeat parts (a) to (c), and explain any differences.

Solution:

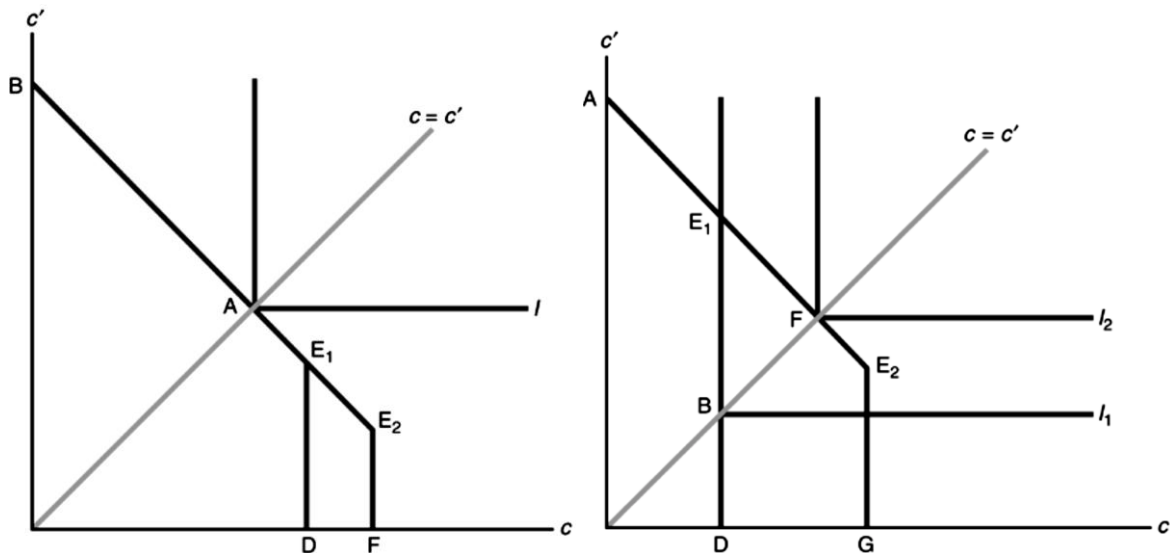
- a) If the consumer could borrow and lend at the real interest rate, $r = 0.05$, then the consumer's lifetime budget constraint would be given by:

$$c + \frac{c'}{(1+r)} = y - t + \frac{y' - t'}{(1+r)}$$

Plugging in the numbers from this problem, we obtain:

$$c + 0.95c' = 255.2.$$

In the figure below, the initial budget constraint is given by BE_1D . The budget constraint has a kink at the initial endowment point $E_1=(160,100)$, because the consumer cannot borrow, and therefore cannot consume more than 160 in the first period. Because the consumer has perfect-complements preferences, the indifference curves are kinked at $c = c'$.



- b) With perfect-complements preferences, the consumer picks point A in the figure on the previous page. Plugging in $c = c'$ into the budget constraint and solving, we find that $c = c' = 130.7$ and $s = y - t - c = 160 - 130.7 = 29.3$. In this case, the fact that the consumer cannot borrow does not matter for the consumer's choice, as the consumer decides to be a lender.
- c) When $t = 20$ and $t' = 71$, the consumer's lifetime wealth remains unchanged at 255.2. However, the budget constraint shifts to BE_2F , figure on the previous page, with the new endowment point at $E_2 = (180, 79)$. This change does not matter for the consumer's choice again because he or she chooses to be a lender. Consumption is still 130.7, but now savings is $s = y - t - c = 180 - 130.7 = 49.3$.
- d) Now first-period income falls to 100. Wealth is now equal to $w = 155.2$. In the figure above, the budget constraint for the consumer is AE_1D , so when the consumer chooses the point on his or her budget constraint that is on the highest indifference curve, any point on the line segment BE_1 will do. Suppose that the consumer chooses the endowment point E_1 , where $c = 60$ and $c' = 100$. This implies that $s = 0$, and the consumer is credit-constrained in that he or she would like to borrow but cannot. Now with the tax change, the budget constraint shifts to AE_2G , with the endowment point $E_2 = (80, 79)$. Thus, the consumer can choose $c = c'$ on the new budget constraint, and solving for consumption in each period using the budget constraint:

$$c + 0.95c' = 155.2$$

we get $c = c' = 79.5$, and $s = 0.5$. Here, notice that first-period consumption increased by almost the same amount as the tax cut, although lifetime wealth remains unchanged at 155.2. Effectively, the budget constraint for the consumer is relaxed. Therefore, for tax cuts that leave lifetime wealth unchanged, lenders will not change their current consumption, but credit-constrained borrowers will increase current consumption.