

Econ 206: Statistics for Economists-II

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Recitation Questions

Question 1

Consider a population with the following information: Population size: $N = 4$. X : age of individuals; Population values: 18, 20, 22, 24 (years).

1. What is the distribution for the population?
 2. Find the mean and variance for this population.
 3. Assume that we select a sample from this population with $n = 2$. Construct a table for all possible samples of size $n = 2$ with replacement. What is the mean and variance for the sample?
 4. Construct a table for all possible sample means based on $n = 2$ and plot a histogram.
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Question 2

Let X_1 and X_2 be two random observations from a population with mean μ and variance σ^2 . Consider the following point estimators of μ :

$$\hat{\mu}_1 = \frac{1}{2}X_1 + \frac{1}{2}X_2,$$
$$\hat{\mu}_2 = \frac{1}{4}X_1 + \frac{3}{4}X_2; \quad \hat{\mu}_3 = \frac{1}{3}X_1 + \frac{2}{3}X_2$$

1. Show that each estimator is an unbiased estimator of μ .
 2. Which one is the most efficient estimator?
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Question 3

The reading on a voltage meter connected to a test circuit is uniformly distributed over the interval $(\theta, \theta + 1)$, where θ is the true but unknown voltage of the circuit. Suppose that $Y_1, Y_2, \dots, Y_n$ denote a random sample of such readings.

1. Show that $\bar{Y}$ is a biased estimator of θ and compute the bias.
 2. Find a function of $\bar{Y}$ that is an unbiased estimator of θ .
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Question 4

Consider a population that is uniformly distributed over an interval $[0, \beta]$ where β is an unknown positive number that we would like to estimate. We take a sample of size 3 from this population and use the sample maximum as an estimate for the population maximum.

1. What is the PDF of this estimator, i.e., what is the sampling distribution of the sample maximum?
 2. What is the expected value of this estimator?
 3. Is this estimator an unbiased estimator for β ?
 4. Suggest one more estimator for β which is unbiased.
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Question 5

Let $X_1, \dots, X_n$ be i.i.d. random variables with common pdf:

$$f(x) = \begin{cases} e^{-(x-\theta)} & x > \theta, -\infty < \theta < \infty \\ 0 & \text{elsewhere} \end{cases}$$

This is the shifted exponential distribution. Let $Y_n = \min\{X_1, \dots, X_n\}$. We will prove that $Y_n \xrightarrow{P} \theta$.

Question 6

Let $Y_1, Y_2, \dots, Y_n$ constitute a random sample from the probability density function given by

$$f(y|\theta) = \begin{cases} \frac{2}{\theta^2}(\theta - y), & 0 \leq y \leq \theta, \\ 0, & \text{elsewhere.} \end{cases}$$

Find an estimator for θ by using the method of moments.

Question 7

Let $Y_1, Y_2, \dots, Y_n$ denote independent and identically distributed random variables from a power family distribution with parameters $\alpha > 0$ with probability density function:

$$f(y|\alpha) = \begin{cases} \frac{\alpha y^{\alpha-1}}{3^\alpha} & 0 \leq y \leq 3 \\ 0, & \text{elsewhere.} \end{cases}$$

Given that

$$E(Y_1) = \frac{3\alpha}{\alpha + 1}$$

derive the method-of-moments estimator for α and determine whether it is consistent.