

ECON 203.01 - Problem Set 1

Spring 2025

1. In an exchange economy with two consumers A and B, the initial endowments of goods X and Y are: $\omega_A = (1, 2)$ and $\omega_B = (2, 1)$ respectively. Their utility functions are

$$U_A = X_A^\alpha Y_A^{1-\alpha} \quad \text{and} \quad U_B = X_B^\beta Y_B^{1-\beta} \quad \alpha, \beta \in (0, 1)$$

Find the competitive equilibrium price ratio and equilibrium allocation of goods.

2. Consider a two agent and two good exchange economy where agent 1 has preferences represented by $u_1(x_{11}, x_{12}) = \min\{2x_{11}, x_{12}\}$, and agent 2 has preferences represented by $u_2(x_{21}, x_{22}) = \min\{2x_{21}, x_{22}\}$. Both agents have endowments (1,1).

(a) What are the Pareto efficient allocations for this economy?

(b) What are the Walrasian equilibrium allocations and prices?

3. Find the competitive (Walrasian) equilibrium(s) for the following three-person two-good exchange economy where the utility function and endowment of each agent is:

$$U_A = x_1^{\frac{1}{2}} x_2^{\frac{1}{2}} \quad \text{and} \quad W_A = (0, 1)$$

$$U_B = \min\{x_1, x_2\} \quad \text{and} \quad W_B = (1, 1)$$

$$U_C = x_1^{\frac{3}{4}} x_2^{\frac{1}{4}} \quad \text{and} \quad W_C = (1, 0)$$

4. Consider the following Robinson Crusoe economy. Robinson the consumer is endowed with zero units of coconuts, x , and 24 hours of time, h , so that $e = (0, 24)$. His preferences are defined over R_+^2 and represented by $u(x, h) = xh$. Robinson the producer uses the consumer's labor services, l , to produce 1 coconuts, y , according to the production function $y = \sqrt{l}$. The producer sells the coconuts to the consumer. All profits from the production and sale of coconuts are distributed to the consumer. Find the Walrasian equilibrium prices and allocation of this economy.

5. Consider a one-consumer, one-producer economy. Compute the equilibrium prices, consumptions, and profits when the consumer's utility function is given by $U(x, y) = \ln(x) + \ln(y)$ where y denotes leisure, the production function is $f(z) = \sqrt{z}$ where z denotes labor and the total endowment of time is 1.

6. Consider an economy with two goods. Good 1 is the numeraire and its price is normalized to 1. Good 2 is candy, which is the economy's main consumption good. There are two consumers. Alice has the utility function $u_1(x_{11}, x_{21}) = x_{11} + 4\sqrt{x_{21}}$. Her endowment is $\omega_1 = (100, 0)$. Bob has the utility function $u_1(x_{12}, x_{22}) = x_{12} + \sqrt{92x_{22}}$. His endowment is $\omega_2 = (100, 0)$. There is one firm producing candy. It uses the numeraire commodity as its sole input. The firm's cost function is $c_1(y_{21}) = \frac{1}{2}y_{21}^2$. Alice and Bob each own $\frac{1}{2}$ of the firm's shares. Find a competitive equilibrium in this economy.

7. Consider an economy with two firms and two consumers. Firm 1 is entirely owned by consumer 1. It produces guns from oil via the production function $g = 2x$. Firm 2 is entirely owned by consumer 2; it produces butter from oil via the production function $b = 3x$. Each consumer owns 10 units of oil. Utility functions of the consumers are given below:

$$U_1(g, b) = g^{0.4}b^{0.6}$$

$$U_2(g, b) = 10 + 0.5 \ln(g) + 0.5 \ln(b)$$

- a) Find the market clearing prices for guns, butter, and oil.
- b) How many guns and how much butter does each consumer consume?
- c) How much oil does each firm use?

8. Consider the following economy. There are two firms, firm 1 and firm 2. Firm 1 produces commodity 1 out of labor, l , according to the production function $y_1 = \sqrt{l}$. Firm 2 produces commodity 2 out of l according to the production function $y_2 = l$. There are two agents, A and B, with identical utility function $u(x_1, x_2, h) = x_1x_2$, where, x_k ; $k = 1, 2$, denotes commodity k , and h denotes the leisure time. Each consumer is endowed with 6 units of time. There is no initial endowment of any of the two commodities. Finally, both consumers own half of each firm. Compute the Walrasian equilibrium prices and allocation.

9. There are two consumers, three goods, and two firms. The utility functions of the consumers are:

$$u_1(x_{11}, x_{21}, x_{32}) = 4 \log(x_{11}) + 6 \log(x_{21})$$

$$u_2(x_{12}, x_{22}, x_{32}) = 6 \log(x_{12}) + 4 \log(x_{22})$$

Thus, neither consumer derives any utility from consuming good 3. As luck would have it, however, the agents' endowments are $\omega_1 = (0, 0, 10)$ and $\omega_2 = (0, 0, 10)$. Firm 1 produces only good 1 using good 3 as an input. Its production function is

$$y_{11} = f_1(y_{31}) = 4y_{31}$$

Firm 2 produces only good 2 using good 3 as an input. Its production function is

$$y_{22} = f_2(y_{32}) = 2y_{32}$$

Suppose firm 1 is entirely owned by person 1. Firm 2 is entirely owned by person 2. Find the Walrasian equilibrium in the economy described above.