

ECON 203.01 - Problem Set 6

Spring 2026

1. Consider the market for education. The marginal social cost of education (MSC) and the marginal private benefit of education (MPB) are given by the following equations where Q is the number of units of education provided per year.

$$MSC = 10 + Q$$

$$MPB = 100 - Q$$

You are also told that each unit of education provides an external benefit to society of \$10 per unit. This external benefit is currently not being internalized in the market.

- (a) Given the MSC and MPB curves, what is the current number of education units being produced by the market?
- (b) Is the current level of market production for education the socially optimal amount of education? Explain your answer.
- (c) What is the value of consumer surplus (CS), the value of producer surplus (PS), and the value of the external benefits of the current level of production. Sum together (CS + PS + external benefits). Draw a diagram illustrating each of these concepts in the market for education.
- (d) Given the market level of production, what is the deadweight loss in this market?
- (e) Suppose that the external benefit is internalized in this market when the government provides a subsidy of \$10 per educational unit to consumers. What will be the socially optimal amount of education to provide given this subsidy?
- (f) Given the subsidy in (e), calculate the value of consumer surplus with the subsidy (CS'), producer surplus with the subsidy (PS'). With the subsidy there are no longer any external benefits that the market fails to account for. Sum together CS' + PS' : does this total equal the sum of CS + PS + external benefits + DWL from parts (c) and (d)?

2. Consider a steel firm by a river which has a cost function $C_s(s, x) = s^2 + (x - 4)^2$ where s is the steel production and x is the pollution level produced by the firm. There is a fishery by the same river which is negatively effected by the pollution produced by the steel firm. Fishery's cost function is $C_f(f, x) = f^2 + x^2$. What is the Pigouvian tax that the government should impose on the steel firm to achieve Pareto efficiency?

3. There is an apiary and a farm. The apiary makes honey and does not need the farm, while the farm needs the bees to pollinate its flowers. Profits for the apiary are:

$$\pi_a(b) = b - \frac{1}{2}b^2$$

while the profits for the farm are

$$\pi_f(q, b) = bq - q^2$$

- (a) What is the socially optimal quantity of bees b and flowers q ?
- (b) What quantities of bees and flowers do the apiary and farm pick on their own?
- (c) Suppose the government used a subsidy to provide the bee farmer with incentives to produce the socially optimal level of bees. What is the optimal subsidy?

4. Consider a plant that manufactures dynamite d and a nearby farm producing tomatoes t . The dynamite's production cost is:

$$TC_d(d, x) = \left(\frac{1}{2}\right)d^2 + (x - 2)^2$$

where d is the amount of dynamite produced and x is the intensity of use of a nitrogen in the production process. The side product associated with use of the nitrogen is ammonia - a fertilizer that is released to the air. Such fertilizer promotes growth of tomatoes making the production on the farm cheaper. In particular the higher the intensity x the lower is the farmer's cost:

$$TC_t(t, x) = \left(\frac{1}{2}\right)t^2 + 2t - xt$$

The price of tomatoes and dynamite are: $p_d = p_t = \$1$

- (a) Is the market interaction associated with a positive or negative externality?
- (b) Find the level of production of dynamite d and intensity x that maximizes the profit of the dynamite manufacturer. What is the maximal level of profit?
- (c) What is the marginal benefit (negative marginal cost) from using x in optimum. Give one number and show it on the graph with x is on the horizontal axis. Explain why this is a reasonable number.
- (d) Given the intensity x from (a), find the optimal level of production of tomatoes t , and the profit of the farmer.
- (e) Find the joint profit of the dynamite manufacturer and the farmer.
- (f) Find the pareto efficient level of production of d , t , and use of nitrogen x . Compare these values to the ones obtained in points b and d .
- (g) Is the marginal benefit from using x in (f) positive, zero, or negative? Why?
- (h) Economists say that the positive externality is associated with too little activity, compared to the efficient outcome. Are your findings in this problem confirming this statement?

5. Consider an exchange economy with two agents, A and B, who consume two goods, x and y.

$$U_A = Zx_A - \frac{x_A^2}{2} + y_A$$

$$U_B = (Z + \gamma x_A)x_B - \frac{x_B^2}{2} + y_B$$

where Z is a positive parameter, while γ is either positive or negative. Assume that the initial endowments of the two agents are $(\bar{x}_A, \bar{y}_A)$ and $(\bar{x}_B, \bar{y}_B)$.

- (a) Find the Walrasian equilibrium and comment on how does the consumptions change when the externality increases.
- (b) Find the set of Pareto efficient allocations.

6. There are 150 fisherman in a town. They each choose whether to fish in the lake or in the ocean. Since the ocean is large, each fisherman can catch 0.05 tons of fish no matter how many other fisherman are there. The lake however can get crowded. If there are x fisherman in the lake, each of them catches $\frac{1}{2}x^{-\frac{1}{2}}$ tons. Fishing in the ocean and lake are perfect substitutes in consumption and there are no costs associated with fishing. The demand for fish is perfectly elastic at price \$5 per ton.

- (a) If each fisherman is free to choose whether to fish in the lake or the ocean, how many fisherman will go to the lake, how many to the ocean, and what will be the average yield for the 150 fisherman?
- (b) If the government restricts access to the lake, how many fisherman should it allow in the lake to maximize the total fish yield in the town? What is the average yield from the lake and what are total profits? Is this an improvement over the allocation in part (a)?
- (c) The government decides to sell permits for fishing in the lake. It chooses a price p and sells as many permits as fisherman wish to buy at this price. What price p should the government charge per fishing permit to achieve the optimal allocation determined in part (b)?