

① $u(x, y) = x + 10y - \frac{y^2}{2}$; $I = 200$; $P_x = 1$ & $P_y = 5$, $P_y' = 1$

$\max_{x, y} u(x, y) \text{ s.t. } I = P_x \cdot x + P_y \cdot y$

$\mathcal{L} = x + 10y - \frac{y^2}{2} + \lambda(I - P_x \cdot x - P_y \cdot y)$

FOC: $\left. \begin{aligned} (1) \partial \mathcal{L} / \partial x &= 1 - \lambda P_x = 0 \\ (2) \partial \mathcal{L} / \partial y &= 10 - y - \lambda P_y = 0 \\ (3) \partial \mathcal{L} / \partial \lambda &= I - P_x x - P_y y = 0 \end{aligned} \right\} \frac{1}{10-y} = \frac{P_x}{P_y} \Rightarrow y = 10 - P_y/P_x$
 where $y \geq 0$
 so $10 > P_y/P_x$

$y^* = \begin{cases} 10 - \frac{P_y}{P_x} & ; \frac{P_y}{P_x} < 10 \\ 0 & ; \frac{P_y}{P_x} \geq 10 \end{cases}$

$\frac{P_y}{P_x} = 5 \Rightarrow y^* = 5 \wedge x^* = 175$

$u_0 = 175 + 5 \cdot 10 - \frac{5^2}{2} = 212.5 \quad \wedge \quad u_c = 212.5 = x + 10 \cdot 9 - \frac{9^2}{2}$

$\frac{P_y}{P_x} = 1 \Rightarrow y^* = 9 \text{ and } x^* = 163 \Rightarrow \text{choices that keep utility level constant (at } u_0) \text{ with new prices.}$

$I' = 163 \cdot 1 + 9 \cdot 1 = 172$

$CV = 172 - 200 = -28$

• **Note:** CV - adjustment in income that returns the consumer to the original utility after an economic change has occurred.

In case of a positive economic change (such as a fall in price of a good), CV is often referred to as the maximum a consumer is willing to pay in order to have the economic change happen. When there is a negative economic change, CV is the minimum the consumer needs in order to accept the economic change.

EV - adjustment in income that changes the consumer's utility equal to the level that would occur if the event had happened

In the case of a positive economic change, such as a fall in price, EV would be the increase in income that would give the consumer the same additional utility that would happen if a price did fall. In the case of a negative economic change, EV would be the amount of income that would be taken away to lower the consumer's utility to the level that would happen if the change had occurred.

② $u(x_1, x_2) = \sqrt{x_1 x_2}$; $p_1=1, p_2=1, I=100$
 $p'_1=2$

• Use Lagrangian method to find $x_1^* = \frac{I}{2p_1}, x_2^* = \frac{I}{2p_2}$

so, initial demand $(50, 50) \Rightarrow \bar{u}_0 = \sqrt{50 \cdot 50} = 50$

new demand $(25, 50) \Rightarrow \bar{u}_1 = \sqrt{25 \cdot 50} = 25\sqrt{2} \approx 35$

• In order to have the same utility level ($\bar{u}_0 = 50$) with new prices, income should rise (compensating variation):

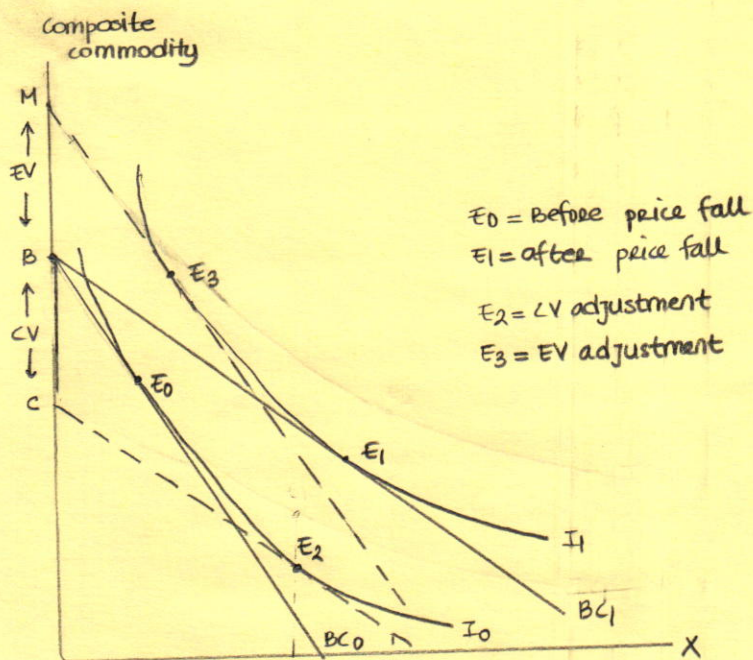
$$50 = \sqrt{x'y'} = \sqrt{\frac{I'}{2p_x'}} \sqrt{\frac{I'}{2p_y'}} = \frac{I'}{2\sqrt{2}} \Rightarrow I' = 10\sqrt{2}$$

$$CV = 100\sqrt{2} - 100 \approx 41$$

• Equivalent variation, to bring consumer to the new utility level with old prices, what is the change in income?

$$\bar{u}_1 = 25\sqrt{2} = \frac{I'}{2\sqrt{p_x p_y}} \Rightarrow I' = 50\sqrt{2}$$

$$EV = I - I' = 100 - 50\sqrt{2} \approx 29.3$$



③ $u(x,y) = \min \{x,y\}$, $I=150$, $P_x=P_y=1$ + $P_y'=2$

$$x=y \Rightarrow x^*=y^* = \frac{I}{P_x+P_y}$$

Initially: $\bar{u}_0 = 75$

with new prices: $\bar{u}_1 = 50$

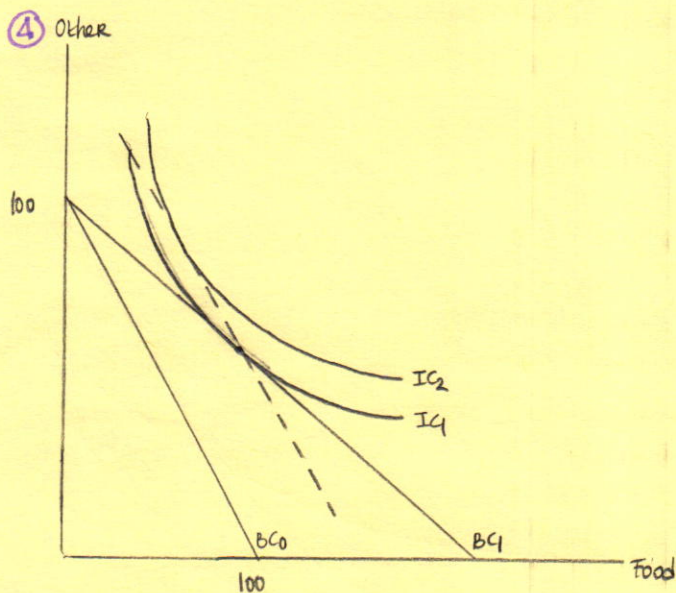
In order to get $x=y=75$ with new prices, income must be

$$\frac{I_{\text{new}}}{P_x+P_y} = 75 \Rightarrow I_{\text{new}} = (1+2) \cdot 75 = 225.$$

$$CV = 225 - 150 = 75 \text{ (B)}$$

$$\bar{u}_1 = 50 \Rightarrow 50 = x = y = \frac{I_{\text{new}}}{\underbrace{P_x}_{=1} + \underbrace{P_y}_{=2}} \Rightarrow I_{\text{new}} = 100$$

$$EV = 100 - 150 = -50 \text{ (A)}$$

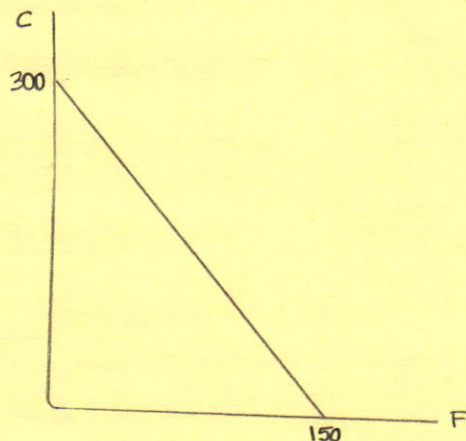


• without food stamps, the budget constraint is BC_0 . Food stamps enable the consumer to effectively purchase food for \$0.10 (rather than \$1).

Resulting budget constraint is BC_1 and optimal bundle occurs where IC_1 is tangent to BC_1 .

Alternatively, if govt eliminates food stamps but provide \$450, the budget constraint is parallel to BC_0 . Consumer is still able to purchase the same bundle (with food stamp policy) but now consumer reoptimize to reach a higher indifference curve.

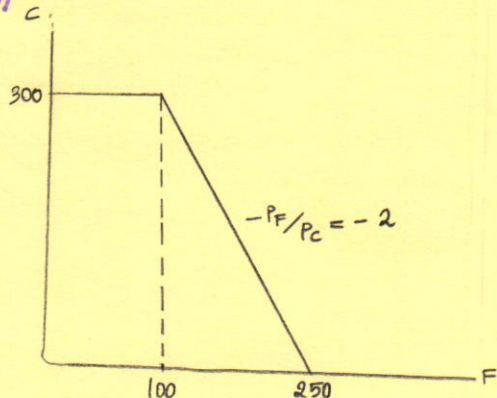
⑤ (a)



- Let F denote the amount of food, and C denote the amount of clothing.

The budget line is: $2F + C = 300$

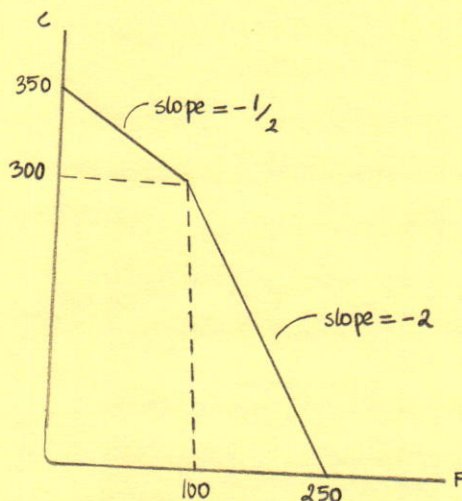
(b)



- For the first 100 units of food, William can use coupons, and resale is prohibited.

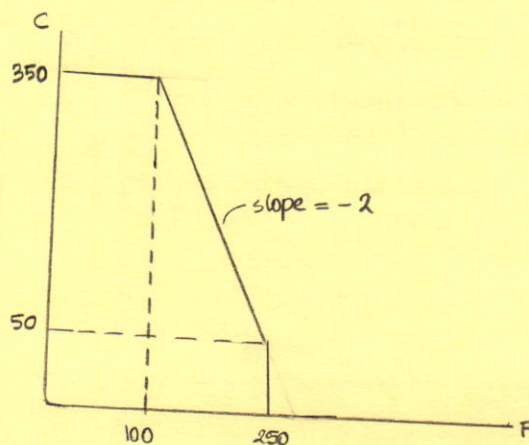
(c)

- Now, he has 100 coupons for food, but he can sell it in the black mkt at \$0.5 per coupon.



(d)

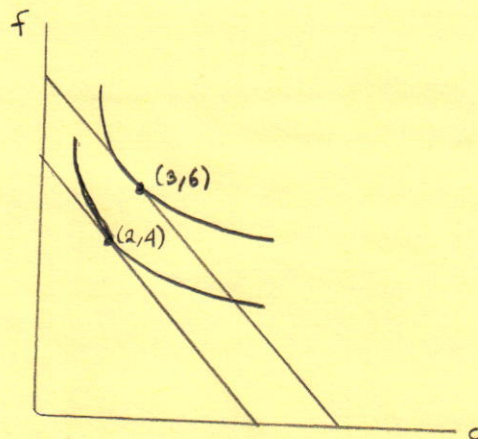
- He has coupon for both C and F (trading in black mkt is prohibited)



⑥ $u(c, f) = c^{1/5} f^{4/5}$; $P_c = 2$, $P_f = 4$, $I = 20$

(a) $c^* = \frac{1}{5} \cdot \frac{20}{5} = 2$, $f^* = \frac{4}{5} \cdot \frac{20}{4} = 4$

(b) $c^* = \frac{1}{5} \cdot \frac{30}{2} = 3$, $f^* = \frac{4}{5} \cdot \frac{30}{4} = 6$



(c) Proper solution should come from the following problem:

$$\max c^{1/5} f^{4/5} \quad \text{s.t.} \quad 2c + 4f \leq 30, \quad c \geq 4, \quad f \geq 0.5$$

However, in the cash transfers case, optimum was (3, 6). Thus, transfer forces the consumer to consume more coal than he would have under the cash transfer, thus the optimal choice is to consume as much coal as he has to

$$c^* = 4, \quad f^* = 5.5$$

$$u(4, 5.5) = 5.16$$

(d) we need to find the minimum expenditure required to get $u = 5.16$.

$$\mathcal{L}(c, f, \lambda) = 2c + 4f + \lambda (5.16 - c^{1/5} f^{4/5})$$

$$\text{FOCs: } \partial \mathcal{L} / \partial c = 2 - \frac{1}{5} \lambda c^{-4/5} f^{4/5} = 0$$

$$\partial \mathcal{L} / \partial f = 4 - \frac{4}{5} \lambda c^{1/5} f^{-1/5} = 0$$

$$\partial \mathcal{L} / \partial \lambda = c^{1/5} f^{4/5} = 5.16 \Rightarrow c^{1/5} (2c)^{4/5} = 5.16$$

$$c^* = \frac{5.16}{2^{4/5}}, \quad f^* = \frac{10.32}{2^{4/5}}$$

$$\text{Total expenditure: } 2c^* + 4f^* = 29.636$$

$$\text{Cash equivalent: } 29.636 - 20 = 9.636$$

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(a) we have $p_c \cdot c = w(24 - n)$, that is $c = 8(24 - n)$ or $c + 8n = 192$.

(b) $u(c, n) = c \cdot n$ is Cobb-Douglas utility function, and we can also use $u(c, n) = c^{1/2} n^{1/2}$.

Then, with the budget set $c + 8n = 192$, we get the optimal bundle as $n^* = \text{income} / 2p_n$

$= 192 / 16 = 12$ and $c^* = \text{income} / 2p_c = 192 / 2 = 96$. Thus, at these prices the labor supply

is $h^* = 24 - 12 = 12$ hours.

(c) Budget line will be $c + 10n = 240$. Then the optimal $n^* = I / 2w = 240 / 2 \cdot 10 = 12$

and the labor supply is $h^* = 24 - 12 = 12$ hours.

So, wage increase will not affect leisure-labor decision.

8 We need to find the labor supply as a function of the wage rate, w . To find this we solve for optimal leisure demand.

The budget line is $c + wn = \bar{n}w$, with a slope $-w$. The marginal rate of substitution's magnitude is:

$$\frac{m_{un}}{m_{uc}} = \frac{\frac{1}{2} n^{-1/2}}{\frac{1}{2} c^{-1/2}} = \frac{c^{1/2}}{n^{1/2}}$$

Now, we can use the tangency condition since this utility function is well-behaved (monotone, has diminishing MRS, continuous, and differentiable). Then, we have $\frac{c^{1/2}}{n^{1/2}} = w$,

which gives $c = nw^2$. Using this in the budget constraint, we get $nw^2 + wn = \bar{n}w$

which implies $n^* = \frac{\bar{n}}{w+1}$, which is the leisure demand. As w goes up leisure

always goes down, thus the labor supply goes up. Thus, labor supply, $h^* = \bar{n} - \frac{\bar{n}}{w+1}$

$= \frac{\bar{n}w}{w+1}$ is always increasing as w goes up. The labor supply is never bending

backwards.

⑨ This utility function is monotone, and $|MRS| = \frac{m_u n}{m_u c} = \frac{c}{1+n}$ is diminishing as n increases and c decreases. Thus, first look at the tangency condition: $\frac{c}{1+n} = \frac{w}{p_c}$, which implies $w(1+n) = p_c \cdot c$.

Now, the budget line is $p_c \cdot c + wn = \bar{n}w$. Plugging $w(1+n) = p_c \cdot c$ into the budget we get $w(1+n) + wn = \bar{n}w$, that is, $(1+n) + n = \bar{n}$. Thus we get $1+2n = \bar{n}$, which gives the optimal leisure as $n^* = \frac{\bar{n}-1}{2}$. Thus, $h^* = \bar{n} - n^* = \bar{n} - \frac{\bar{n}-1}{2} = \frac{\bar{n}+1}{2}$, which is independent of the wage rate.

⑩ (a) $|MRS| = \frac{m_u n}{m_u c} = \frac{2(12-n)}{1} = \frac{n}{1}$ implies that $2(12-n) = w$. Thus if $w=0$, she will get $n^*=12$ and will supply $h^*=16-12=4$ (even if labor hour does not pay)

(b) If $w=10$ then $2(12-n)=10$ implies that $n^*=7$ and labor supply will be $h^*=16-7=9$.

(c) Note that the leisure is determined by $2(12-n)=w$ and the unearned income m does not enter the calculation of optimal leisure hours, n^* , thus it also does not enter the calculation of optimal labor hours, h^* . so even if her unearned income is cut down by half, she will still work for $h^*=9$ hours when $w=10$.

(d) Total income is $(16-n)w + m$. If the tax rate is τ , then after tax total income is $[(16-n)w + m](1-\tau)$. Then the budget constraint is given by $p_c c = (1-\tau)[(16-n)w + m]$. At the given wage rate, unearned income, tax rate and $p_c=1$, we get $c = 0.8[10(16-n) + 20] = 0.8[180 - 10n] = 144 - 8n$. or we can write as $c + (1-\tau)wn = (1-\tau)[16w + m]$, that is, $c + 0.8wn = 0.8[16w + 20]$, or $c + 8n = 144$.

Now the tangency implies that $\frac{2(12-n)}{1} = \frac{(1-\tau)w}{1}$ that is $2(12-n) = 8$. Thus we have $n^*=8$ which gives the optimal labor hour as $h^*=16-8=8$. And the consumption level is $c^*=144-8 \cdot 8 = 144-64=80$.