

ECON 201 – Problem Set 3

Department of Economics

1. Find the Marshallian demand functions for the given utility functions.

- (a) $U(x_1, x_2) = x_1^{1/4} x_2^{3/4}$
- (b) $U(x_1, x_2) = 3x_1 + x_2$
- (c) $U(x_1, x_2) = \min\{2x_1, 3x_2\}$
- (d) $U(x_1, x_2) = x_1 + x_2 + \sqrt{x_2}$
- (e) $U(x_1, x_2, x_3, x_4) = \sqrt{x_1 x_2} + \sqrt{x_3 x_4}$
- (f) $U(x_1, x_2) = e^{(x_1 + \ln(x_2))^{1/3}}$

2. A consumer has the utility function over goods x and y :

$$u(x, y) = x^{1/2} + y^{1/2}.$$

- (a) Derive the consumer's demand function for good y .
 - (b) What is λ ? What does it mean?
 - (c) Is good y normal or inferior?
 - (d) If we have $p_x = 2$, $p_y = 1$, and $M = 12$, compute the utility maximizing consumption bundle of goods x and y .
3. Lily is a teenager who likes video games G and music M . Each week, she receives \$60 to buy games and music CDs. Her utility function for these goods is $U(G, M) = G^{1/2} + M^{1/2}$. The price of a game is \$20 and the price of a CD is \$10.
 - (a) Solve for Lily's demand using the Lagrangian Method.
 - (b) Lily is a bit disappointed with these calculations. She thinks that she can buy more CDs if her utility is logarithmic instead. Take the natural log of $U(G, M)$ and solve the optimization problem again. Compare the optimal levels with those found in (a).
 4. Leo consumes only nuts and berries. Fortunately, he likes both goods. The consumption bundle where Leo consumes x_1 units of nuts per week and x_2 units of berries per week is written as (x_1, x_2) .
 - (a) The set of consumption bundles (x_1, x_2) such that Leo is indifferent between (x_1, x_2) and $(16, 4)$ is the set of bundles such that $x_1 \geq 0$, $x_2 \geq 0$, and $x_2 = 20 - 4\sqrt{x_1}$. Plot several points that lie on the indifference curve that passes through the point $(16, 4)$ and sketch this curve.
 - (b) In fact, Leo's preferences can be represented by the utility function $u(x_1, x_2) = 4\sqrt{x_1} + x_2$. The price for nuts is \$1 per unit and the price for berries is \$2 per unit. Leo has \$24 to spend on the two goods. Write down Leo's budget constraint and solve for his optimal consumption bundle.

- (c) Now Leo has only \$34 to spend. Is he still able to consume the same amount of units as in part (b)? Explain.
 - (d) Now Leo has only \$9 in total to spend. Is he still able to consume the same amount of nuts as in part (b)? Explain.
5. Suppose that a consumer consumes only food (f) and entertainment (e) where $p_f = 5$, $p_e = 10$ and $m = 600$. Suppose that the consumer has to have a minimum of 50 units of f , and a maximum of 20 units of e .
- (a) Draw the budget set.
 - (b) Suppose the utility function is $u(f, e) = f \cdot e$. Find the optimal consumption bundle.
6. Suppose that a consumer only consumes good 1 and good 2 under the following prices and income: $p_1 = 2$, $p_2 = 1$ and $m = 100$. Consumer's preferences can be represented by the utility function $u(x_1, x_2) = 3x_1 + x_2$.
- (a) Find the optimal bundle.
 - (b) Now suppose that the consumer receives a coupon of \$20 which can be spent only on good 2. Draw the new budget constraint and find the new optimal consumption bundle.