

Problem Set IV Solutions

Section I: Chapter 9

Question 1

Consider the following effects of an increase in taxes for a consumer.

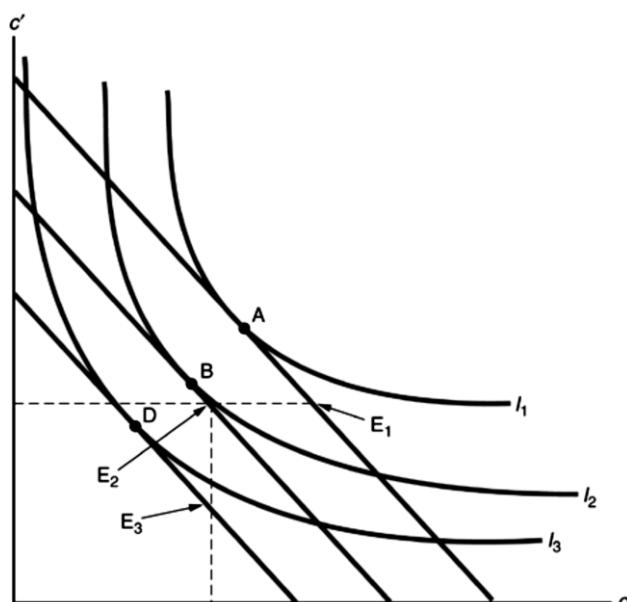
- The consumer's taxes increase by Δt in the current period. How does this affect current consumption, future consumption, and current saving?
- The consumer's taxes increase permanently, increasing by Δt in the current and future periods. Using a diagram, determine how this affects current consumption, future consumption, and current saving.

Explain the differences between your results here and in part (a).

Solution:

Temporary and Permanent Tax Increases.

(a) The increase in first-period taxes induces a parallel leftward shift in the budget line. The original budget line passes through the initial endowment, E_1 . The new budget line passes through E_2 . The consumer reduces both current and future consumption. In the figure below the consumer's optimum point moves from point A to point B. First-period consumption falls by less than the increase in taxes and so savings fall.



(b) Next consider a permanent increase in taxes. A permanent tax increase adds a second tax increase to the first tax increase, the current-period tax increase. The increase in second-period taxes induces a parallel downward shift in the budget line. The new budget line passes through E_2 in the figure above. The second part of the tax increase also reduces both first-period and second-period consumption. The consumer moves from point B to point D. Because the second tax increase reduces first-period

consumption holding first-period disposable income fixed, savings must rise. Since the permanent tax increase is the sum of the two individual tax increases, the permanent tax increase reduces both first-period and second-period consumption, but on net, savings may either rise, fall, or remain unchanged.

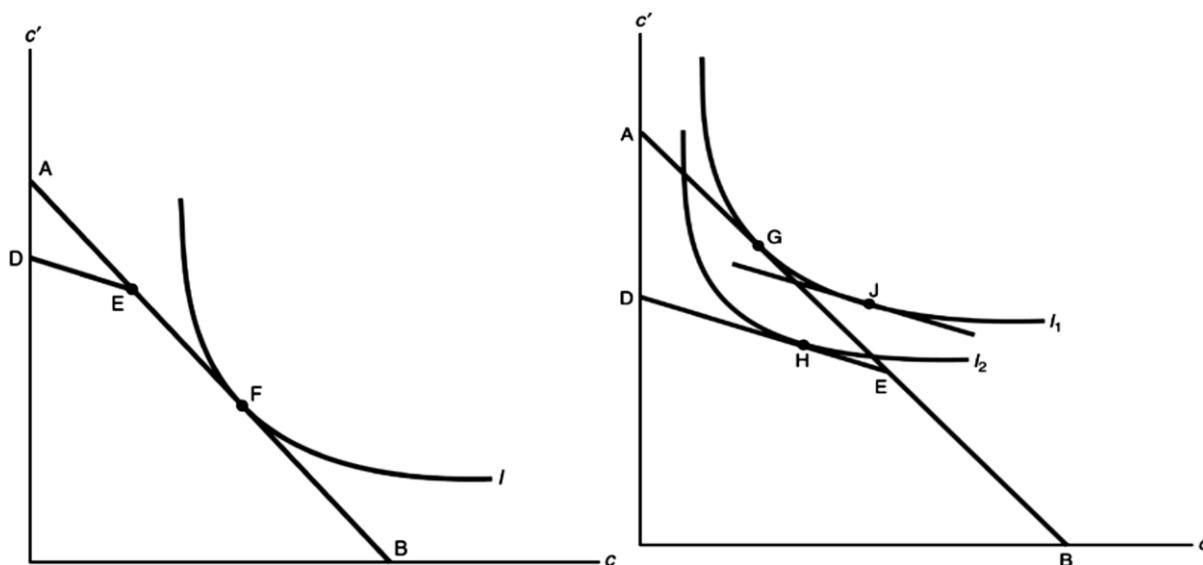
Question 2

Suppose that the government introduces a tax on interest earnings. That is, borrowers face a real interest rate of r before and after the tax is introduced, but lenders receive an interest rate of $(1 - x)r$ on their savings, where x is the tax rate. Therefore, we are looking at the effects of having x increase from zero to some value greater than zero, with r assumed to remain constant.

- Show the effects of the increase in the tax rate on a consumer's lifetime budget constraint.
- How does the increase in the tax rate affect the optimal choice of consumption (in the current and future periods) and savings for the consumer? Show how income and substitution effects matter for your answer and show how it matters whether the consumer is initially a borrower or a lender.

Solution: A tax on interest income.

- Initially, AB in the first figure below depicts the consumer's budget constraint. The introduction of the tax results in a kink in the budget constraint, since the interest rate at which the consumer can lend, $r(1 - t)$ is now smaller than the interest rate at which the consumer borrows, r . The kink occurs at the endowment, E.



- The first figure above shows the case of a consumer who was a borrower before the imposition of the tax. This consumer is unaffected by the introduction of the tax. The second figure above shows the case of a consumer who was a lender before the imposition of the tax. Initially the consumer chooses point G, and then chooses point H after the imposition of the tax. There is a substitution effect that results in an increase in first-period consumption and a reduction in second-period consumption, and moves the consumer from point G to point J. Savings also fall from point G to point J. The income effect is the movement from point D to point B, and the income effect reduces both first-period and second-period consumption, and increases savings. On net, consumption must fall in period 2, but in period 1, consumption may rise or fall. The figures above show the case in which first-period consumption increases, which is a case where the substitution effect dominates.

Question 3

An employer offers his or her employee the option of shifting x units of income from next

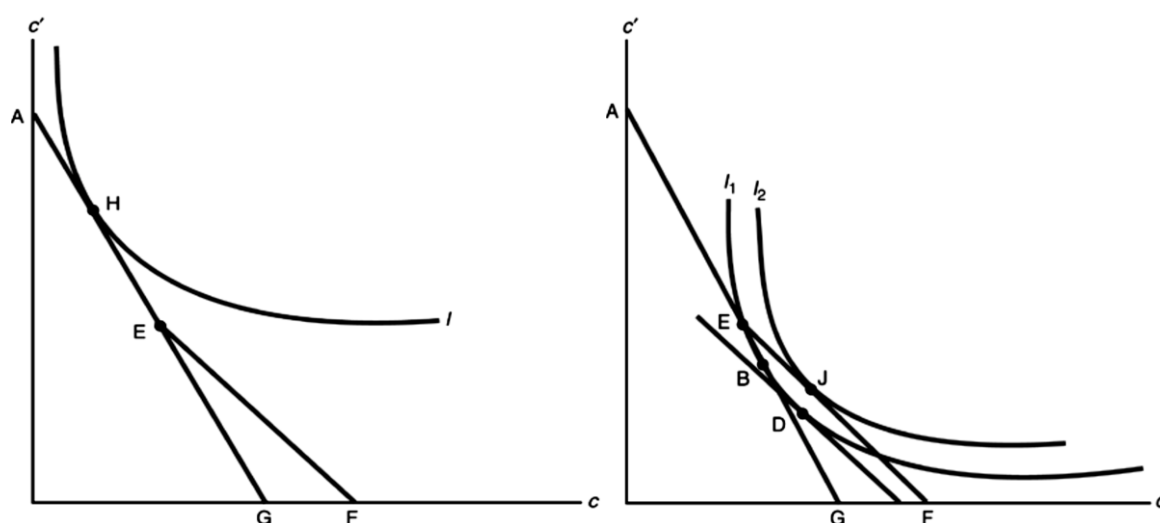
year to this year. That is, the option is to reduce income next year by x units and increase income this year by x units.

- Would the employee take this option (use a diagram)?
- Determine, using a diagram, how this shift in income will affect consumption this year and next year and saving this year. Explain your results.

Solution: This problem involves a firm's offer to provide an interest-free advance on the consumer's income. If the consumer takes the advance, then his lifetime wealth is given by:

$$we = y + \frac{y'}{1+r} + x \left(1 - \frac{1}{1+r}\right)$$

Therefore, provided that $r > 0$, the consumer should take the advance, as any increase in his lifetime wealth makes him better off.



Question 4

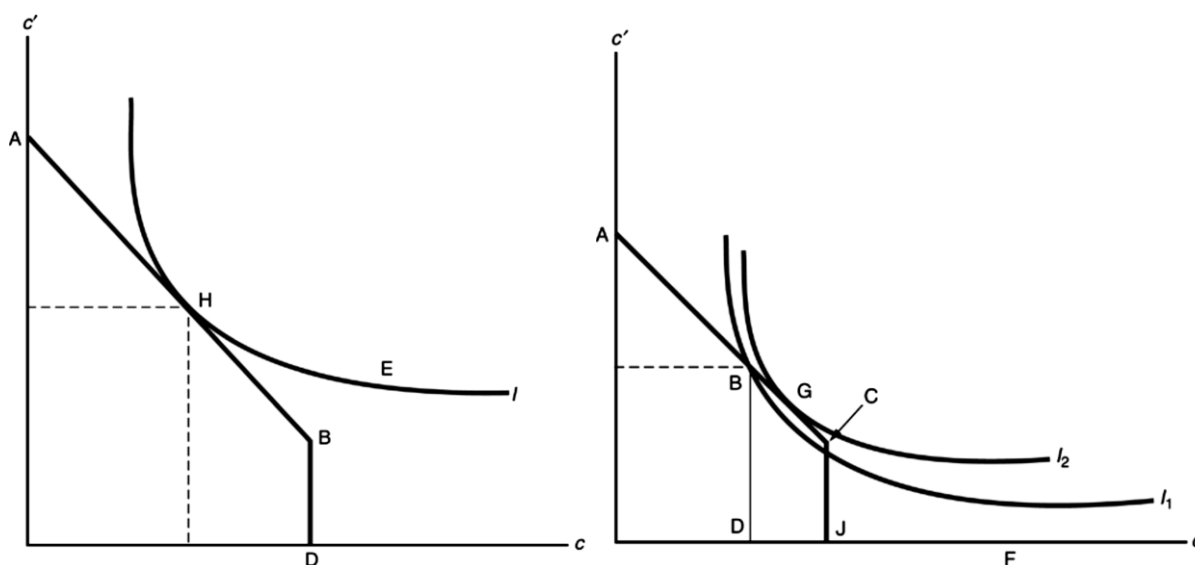
A consumer receives income y in the current period, income y' in the future period, and pays taxes of t and t' in the current and future periods, respectively. The consumer can borrow and lend at the real interest rate r . This consumer faces a constraint on how much he or she can borrow, much like the credit limit typically placed on a credit card account. That is, the consumer cannot borrow more than x , where $x < we - y + t$, with we denoting lifetime wealth. Use diagrams to determine the effects on the consumer's current consumption, future consumption, and savings of a change in x , and explain your results.

Solution:

The consumer faces a borrowing constraint that places a ceiling on the level of current consumption. The consumer may consume more than the current endowment, $y - t$ but less than the amount of the lifetime endowment, we . The consumer's budget line is as in the first figure below. The budget line becomes vertical at $c = x$. An example of such a budget line is depicted in the two panels of the figure as ABD. As one possibility, the constraint is nonbinding as in the figure below. The consumer chooses point H. A change in the level of x has no effect on such a consumer.

Alternatively, the consumer depicted in the second figure below originally chooses the corner solution, point B. The consumer achieves the level of utility corresponding to indifference curve, I_1 . An increase in x produces the new budget line, ACJ. This consumer now chooses point G. She increases current

consumption and decreases both current saving and future consumption. This consumer is able to improve her level of utility to that corresponding to indifference curve, I_2 .



Question 5

In the example laid out in the subsection titled “Ricardian Equivalence: An Example,” suppose That

$$b < \frac{Ny' - G'}{Ny - G}$$

- Solve for the equilibrium real interest rate, and the consumption of lenders and borrowers in the current and future periods.
- Does Ricardian equivalence hold in this case? Explain why or why not.

Solution:

Out of coursebook content.

Question 6

Suppose that all consumers are identical and also assume that the real interest rate r is fixed. Suppose that the government wants to collect a given amount of tax revenue R , in present value terms. Assume that the government has two options: (i) a proportional tax of s per unit of savings, in that the tax collected per consumer is $s(y - c)$; (ii) a proportional tax u on consumption in the current and future periods, so that the present value of the total tax collected per consumer is

$$uc + \frac{uc'}{1+r}.$$

Note that the tax rate s could be positive or negative. For example, if consumers borrow, then s would need to be less than zero for the government to collect tax revenue. Show that option (ii) is preferable to option (i) if the government wishes to make consumers as well off as possible and explain why this is so. [Hint: Show that the consumption bundle that consumers choose under option (i) could have been chosen under option (ii), but was not.]

Solution:

Let (c_1, c'_1) denote the consumption bundle chosen by each consumer under the first tax scheme with tax rate s on savings, and (c_2, c'_2) the consumption bundle chosen if the consumer faces the tax scheme with tax rate u on current-period and future-period consumption. Then, since the consumer's budget constraint is satisfied in each case, then

$$c_1 + \frac{c'_1}{1+r} = y + \frac{y'}{1+r} - (y - c_1)s$$

and

$$c_2(1+u) + \frac{c'_2(1+u)}{1+r} = y + \frac{y'}{1+r}$$

As well, since tax revenue generated in each case is R , we have

$$c_i + \frac{c'_i}{1+r} = y + \frac{y'}{1+r} - R$$

for $i = 1, 2$. Then, equations (2) and (3) give

$$u = \frac{R}{y + \frac{y'}{1+r} - R}$$

As a result,

$$c_1(1+u) + \frac{c'_1(1+u)}{1+r} = \left(c_1 + \frac{c'_1}{1+r} \right) \left(\frac{y + \frac{y'}{1+r}}{y + \frac{y'}{1+r} - R} \right) = y + \frac{y'}{1+r}$$

In the above equation, the first equality follows from equation (4), and the second from (3). Therefore, the consumption bundle (c_1, c'_1) is affordable under the second tax system. But that was not the consumption bundle the consumer chose. The consumer chose the consumption bundle (c_2, c'_2) which is not the same as (c_1, c'_1) , as the consumer's marginal rate of substitution will be different under the two tax systems. As a result, consumers are better off under the second tax system than the first, as the second tax system does not distort the choice the consumer makes over current consumption vs. future consumption – under the second system current and future consumption are taxed at the same rate.

Section II: Chapter 10

Question 7

Suppose that there is a credit market imperfection due to asymmetric information. In the economy, a fraction b of consumers consists of lenders, who each receive an endowment of y units of the consumption good in the current period, and 0 units in the future period. A fraction $(1-b)a$ consumers are good borrowers who each receive an endowment of 0 units in the current period and y units in the future period. Finally, a fraction $((1-b)(1-a))$ of consumers are bad borrowers who receive 0 units of endowment in the current and future periods. Banks cannot distinguish between good and bad borrowers. The government sets $G = G' = 0$, and each consumer is asked to pay a lump-sum tax of t in the current period and t' in the future period. The government also cannot distinguish between good and bad borrowers, but as with banks can observe endowments.

(a) Write down the government's budget constraint, making sure to take account of who is able to pay their taxes and who does not.

(b) Suppose that the government decreases t and increases t' in such a way that the government budget constraint holds. Does this have any effect on each consumer's decisions about how much to consume in each period and how much to save? Show with the aid of diagrams.

(c) Does Ricardian equivalence hold in this economy? Explain why or why not.

Solutions:

Question 8

Suppose that there is limited commitment in the credit market, but lenders are uncertain about the value of collateral. Each consumer has a quantity of collateral H , but from the point of view of lender, there is a probability a that the collateral will be worth p in the future period, and probability $1 - a$ that the collateral will be worthless in the future period. Suppose that all consumers are identical.

- Determine the collateral constraint for the consumer and show the consumer's lifetime budget constraint in a diagram.
- How will a decrease in a affect the consumer's consumption and savings in the current period and consumption in the future period? Explain your results.

Solution:

(a) The bank will be lending so that it will be able to get the loan back in expectation. Thus, the new collateral constraint is

$$-s(1 + r) \leq a p H$$

which leads to

$$c \leq y - t + \frac{a p H}{1 + r}$$

This is much like Figure 10.5 in the textbook, simply with a lower collateral value.

(b) If the collateral is more likely to be of no value, banks will lend less. Thus, some household will not be able to borrow as much as they could before, leading for them and in aggregate to a reduction of current consumption. For a constrained borrower, this causes no change in future consumption, but current consumption drops by the same amount as the decrease in the value of collateral if the borrower is collateral-constrained. For a nonconstrained borrower both current consumption and future consumption will decrease due to income effect. Thus we have exactly the same impact as if we had a reduction in p , as in Figure 10.5 of the textbook.

Question 9

Suppose a credit market with a good borrowers and $1 - a$ bad borrowers. The good borrowers are all identical and always repay their loans. Bad borrowers never repay their loans. Banks issue deposits that pay a real interest rate r_1 , and make loans to borrowers. Banks cannot tell the difference between a good borrower and a bad one. Each borrower has collateral, which is an asset that is worth A units of future consumption goods in the future period.

- Determine the interest rate on loans made by banks.
- How will the interest rate change if each borrower has more collateral?
- Explain your results, and discuss.

Solution:

If each good borrower chooses to borrow L , then the cost of deposits for the bank is $(1 + r_1)L$. The payoff on each loan to a good borrower is $(1 + r_2)L$, and bad borrowers always default, but the bank can

have each borrower post A as collateral, so the payoff to the bank on a loan to a bad borrower is the smaller of A or $(1 + r_2)L$. Suppose first that $A < (1 + r_2)L$. Then, the expected profit for the bank to making a loan is zero in equilibrium, or

$$-(1 + r_1)L + a(1 + r_2)L + (1 - a)A = 0$$

Then, if we solve the above equation for r_2 , we obtain

$$r_2 = \frac{1 + r_1 - (1 - a)\frac{A}{L}}{a} - 1$$

Therefore, the larger is A , the smaller is r_2 , the loan interest rate. The larger the quantity of collateral available, the larger the payoff the bank receives when a bad borrower defaults. This increases bank profits, but in equilibrium expected profits are zero for the bank, borrowers benefit from having a higher quantity of collateral with which to secure loans. If A is sufficiently large, i.e. $A \geq (1 + r_2)L$, then the bank's payoff on a loan to a bad borrower is $(1 + r_2)L$, and so $r_1 = r_2$. Thus, if A is large enough then there is sufficient collateral to eliminate the credit market friction.