

Problem Set IV

Content: Method of Moments, MLE

Question 1

X_i are normally distributed random variables with mean μ and variance σ^2 .

a. Show that

$$\hat{\mu} = \frac{\sum X_i}{n} = \bar{X} \quad \text{and} \quad \hat{\sigma}^2 = \frac{\sum (X_i - \bar{X})^2}{n}$$

are both maximum likelihood and method of moments estimators of μ and σ^2 , respectively.

b. Are the estimators unbiased for their respective parameters?

c. If not, find unbiased estimators for the population parameters.

Question 2

Suppose that $Y_1, Y_2, \dots, Y_n$ denote a random sample from the Weibull density function, given by

$$f(y | \theta) = \begin{cases} \left(\frac{2y}{\theta}\right) e^{-y^2/\theta} & y > 0 \\ 0 & \text{elsewhere} \end{cases}$$

a. Find the sufficient statistics of θ

b. Is maximum likelihood estimator for θ sufficient?

Question 3

Let $X_1, X_2, \dots, X_n$ be gamma random variables with parameters α and θ , so that the probability density function is:

$$f(x) = \frac{1}{\Gamma(\alpha)\theta^\alpha} x^{\alpha-1} e^{-x/\theta}$$

for $x > 0$. What are the method of moments estimators of α and θ ?

Question 4

$$f(x; \beta) = \begin{cases} \frac{x^2}{2\beta^3} e^{-\frac{x}{\beta}} & , \quad x \geq 0 \\ 0 & , elsewhere. \end{cases}$$

- a. Find the MLE estimator $\hat{\beta}$ of β .
- b. Is $\hat{\beta}$ an unbiased estimator for β .
- c. Is $\hat{\beta}$ MVUE?
- d. Is $\hat{\beta}$ a consistent estimator?

Question 5

$$f(x; \theta) = \begin{cases} \theta e^{-\theta x} & , \quad x \geq 0 \\ 0 & , elsewhere. \end{cases}$$

- a. Find the method of moment estimator of θ .
- b. Is it an unbiased estimator for θ . If not find an unbiased estimator for θ .
- c. Calculate the bias of the MM estimator.
- d. Calculate the MSE of the MM estimator.
- e. Is MM consistent estimator?

Question 6

Suppose δ is known. Let $X_1, X_2, \dots, X_n$ be a random sample from the distribution with the probability density function

$$f(x; \beta, \delta) = \beta \delta x^{\delta-1} e^{-\beta x^\delta}, \quad x > 0$$

zero otherwise. Obtain the maximum likelihood estimator for $\beta, \hat{\beta}$.