

PS6 - Solutions

1) $f(x, y) = x + \min\{x, y\}$ and for $\lambda > 0$

$$f(\lambda x, \lambda y) = \lambda x + \min\{\lambda x, \lambda y\} = \lambda x + \lambda \min\{x, y\} \\ = \lambda f(x, y)$$

$\Rightarrow f$ is a h.o.d 1 $\Leftrightarrow f$ is constant returns to scale.

True

$$2) TRS = -\frac{MP_1}{MP_2} = -\frac{\frac{1}{2}(2x_1 + 4x_2)^{-\frac{1}{2}} \cdot 2}{\frac{1}{2}(2x_1 + 4x_2)^{-\frac{1}{2}} \cdot 4} = -\frac{1}{2} = -\frac{\Delta x_2}{\Delta x_1}$$

3) $f(x, y) = x^{3/4} y^{1/4}$ $TRS = -\frac{MP_1}{MP_2}$ is the slope of the isoquant

$$TRS = -\frac{\frac{3}{4} x^{-1/4} y^{1/4}}{\frac{1}{4} x^{3/4} y^{-3/4}} = -3 \frac{y}{x}$$

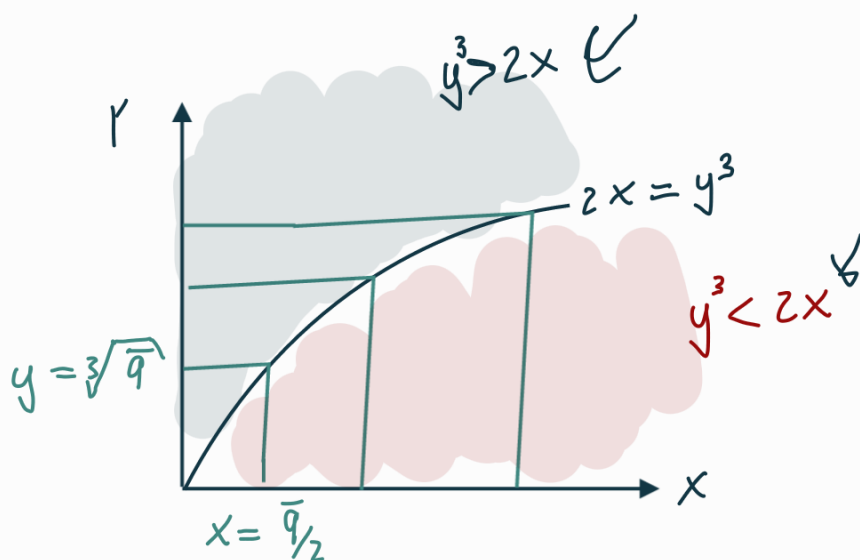
if $-9 = -3 \frac{y}{x} \Rightarrow y = 3x$

5)

a) $f(x, y) = \max(2x, y^3)$

$$2x = y^3$$

$$f(x, y) = y^3 = 1$$

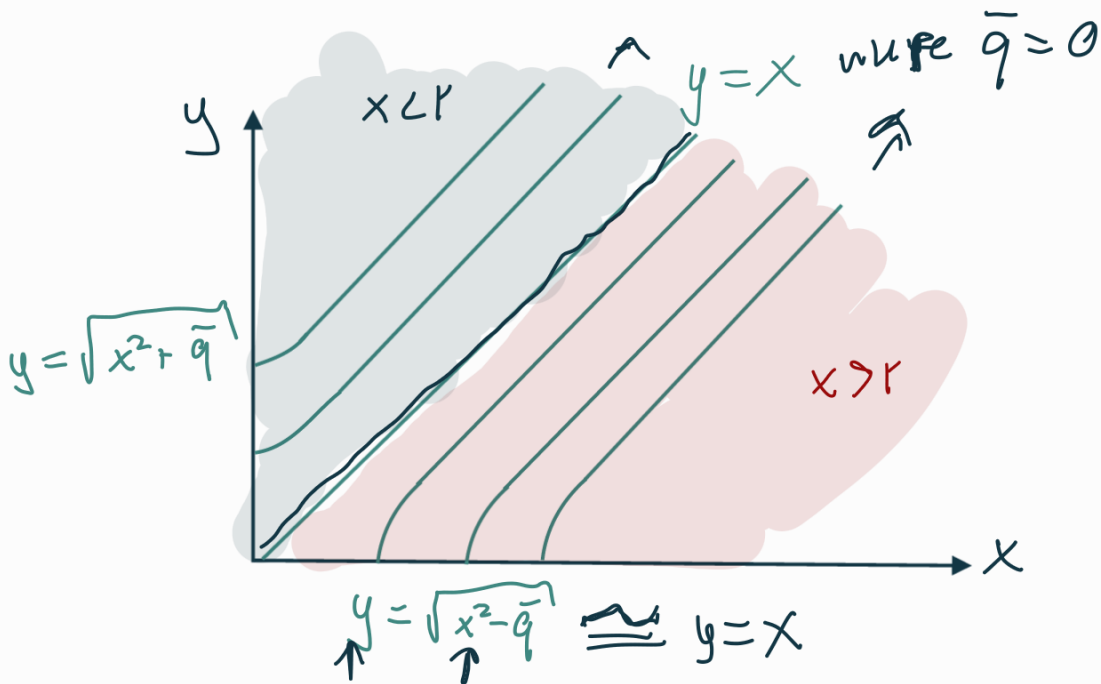


$$b) f(x, y) = (\max\{x, y\})^2 - (\min\{x, y\})^2$$

$$\text{if } x \geq y \Rightarrow f(x, y) = x^2 - y^2 \Rightarrow y = \sqrt{x^2 - \bar{q}}$$

$$\text{if } x < y \Rightarrow f(x, y) = y^2 - x^2 \Rightarrow y = \sqrt{x^2 + \bar{q}}$$

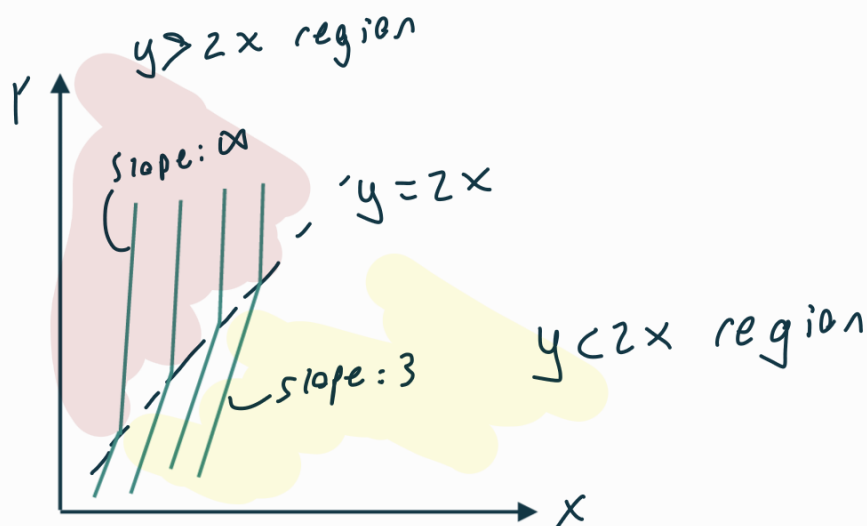
$$\rightarrow \text{if } x = y \Rightarrow f(x, y) = x^2 - x^2 = 0$$



$$c) f(x, y) = 3x - \min\{2x, y\}$$

$$\Rightarrow \text{if } 2x \geq y \Rightarrow f(x, y) = 3x - y \Rightarrow \underline{y = 3x - \bar{q}}$$

$$\Rightarrow \text{if } 2x \leq y \Rightarrow f(x, y) = x \Rightarrow \underline{\bar{q} = x}$$



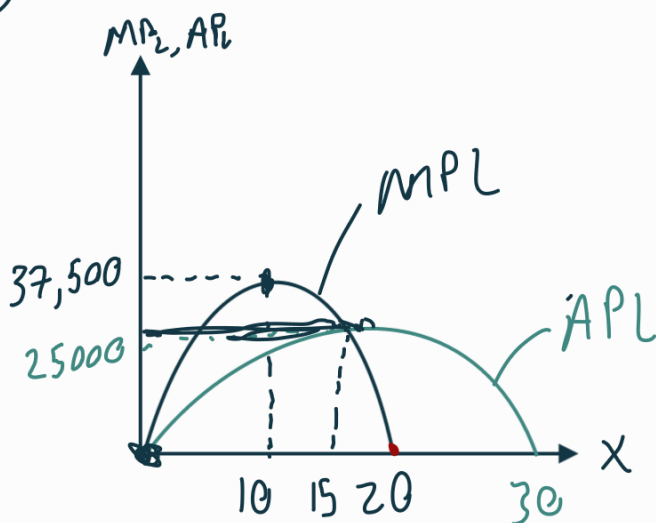
6)

$$a) Q(L) = 150 \cdot 25 \cdot L^2 - 125 L^3 = 3750 L^2 - 125 L^3 //$$

$$MP_L = \frac{\partial Q(L)}{\partial L} = 2 \cdot 3750 L - 3 \cdot 125 L^2 = \underline{\underline{7500 L - 375 L^2}}$$

$$AP_L = \frac{Q(L)}{L} = \underline{\underline{3750 L - 125 L^2}}$$

b, c)



$$\frac{\partial MP_L}{\partial L} = 7500 - 750 L = 0$$

$$L^{max} = 10$$

$$MP_L(L^{max} = 10) = 37500$$

$$\frac{\partial AP_L}{\partial L} = 3750 - 250 L = 0$$

$$L^{max} = 15$$

$$AP_L(L^{max} = 15) = 37500 - 12500 = 25000$$

$$d) AP_L = MP_L \Rightarrow L = 0 \text{ or } L = 15$$

$$e) TP \text{ is max at } MP_L = 0 \Rightarrow L = 20 \Rightarrow$$

$$Q(20) = TP_{max}$$

$$TP_{max} = 3750 \cdot 20^2 - 125 \cdot 20^3$$

4) For a constant $k > 1$, $f(kx_1, kx_2) =$

$$= (kx_1)^b + (kx_2)^b)^c = (k^b(x_1^b + x_2^b))^c = k^{bc}(x_1^b + x_2^b)^c = k^{bc} f(x_1, x_2)$$

if $bc > 1 \Rightarrow f$ is increasing returns to scale.

$$= k^{bc} f(x_1, x_2) > k f(x_1, x_2)$$

$$\begin{aligned} 7) \quad \frac{TP(L)}{L} = AP_L &\Rightarrow \frac{\partial AP_L}{\partial L} = \frac{\partial \left(\frac{TP(L)}{L} \right)}{\partial L} = \frac{\frac{\partial TP(L)}{\partial L} L - 1 \cdot TP(L)}{L^2} \\ &= \frac{MP_L L - \frac{TP(L)}{L}}{L^2} = \frac{MP_L}{L} - \frac{AP_L}{L} \end{aligned}$$

a) Hence if $MP_L > AP_L \Rightarrow \frac{\partial AP_L}{\partial L} > 0$ implies

APL is increasing.

b) Similarly if $MP_L < AP_L \Rightarrow \frac{\partial AP_L}{\partial L} < 0$ implies

APL is decreasing with L

c)

Additionally, APL is at max when $MP_L = AP_L$ ($\frac{\partial AP_L}{\partial L} = 0$)

$$8) MP_L = \frac{\partial Q}{\partial L} = \frac{20L(K+L) - 20KL}{(K+L)^2} = \frac{20L^2}{(K+L)^2} > 0$$

$$9) \frac{\partial MP_L}{\partial K} = \frac{\partial^2 Q}{\partial K^2} = \frac{-40L^2}{(K+L)^3} < 0 \Rightarrow \text{diminishing } MP_L$$

$$MP_L = \frac{\partial Q}{\partial L} = \frac{20L^2}{(K+L)^2}, \quad \frac{\partial MP_L}{\partial L} = \frac{-40L^2}{(K+L)^3} < 0 \Rightarrow \text{diminishing returns for } L.$$

$$b) f(cK, cL) = \frac{20cKcL}{(cK+cL)} = \frac{c^2 20KL}{c(K+L)} = c f(K, L)$$

constant.

$$9) \pi = TR - TC = P \cdot Q - X_1 - 4X_2 = 8(6X_1^{1/2} + 8X_2^{1/2}) - X_1 - 4X_2$$

$$a) \left. \begin{aligned} \frac{\partial \pi}{\partial X_1} = 0 &\Rightarrow 24X_1^{-1/2} - 1 \Rightarrow X_1^* = 576 \\ \frac{\partial \pi}{\partial X_2} = 0 &\Rightarrow 32X_2^{-1/2} - 4 \Rightarrow X_2^* = 64 \end{aligned} \right\} f(576, 64) = 208$$

$$b) \pi = P \ln(xy) - P_x X - P_y Y \Rightarrow \frac{\partial \pi}{\partial X} = 0 \Rightarrow \frac{P}{xy} - P_x \Rightarrow X^* = \frac{P}{P_x}$$

$$\frac{\partial \pi}{\partial Y} = 0 \Rightarrow Y^* = \frac{P}{P_y}$$

$$f(x^*, y^*) = \ln\left(\frac{P^2}{P_y P_x}\right) = 2\ln(P) - \ln(P_y) - \ln(P_x)$$

$$10) \quad Y(x) = \begin{cases} x^{3/2} & \text{if } 0 \leq x \leq 4 \\ x+4 & \text{if } x > 4 \end{cases} \quad \begin{matrix} P_y = 1 \\ P_x = 3 \end{matrix}$$

$$\pi(x) = TR - TC = P_y \cdot Y - P_x \cdot X = Y(x) - 3X$$

$$\pi(x) = \begin{cases} x^{3/2} - 3x & \text{if } 0 \leq x \leq 4 \\ \underbrace{x+4-3x}_{=4-2x} & \text{if } x > 4 \end{cases}$$

$$\frac{\partial \pi(x)}{\partial x} = \begin{cases} \frac{3}{2} x^{1/2} - 3 & \text{if } 0 \leq x \leq 4 \\ -2 & \text{if } x > 4 \end{cases}$$

$$\text{For } x > 4 \quad MP_x < 0 \Rightarrow \pi_i^{\max}(4.01) = -4.02 < 0$$

$$\text{For } 0 \leq x \leq 4 \quad \frac{\partial MP_x}{\partial x} = \frac{3}{6} x^{-1/2} > 0 \Rightarrow MP_x = 0 \text{ gives the minimum}$$

$\Rightarrow \pi^{\max}(x=0) = 0$ is the maximum possible output.

11)

$$a) \quad Y = f(K, L) = \sqrt{\min\{K, L\}} \quad \text{in S.R.} \quad \bar{K} = 16$$

$$Y(L) = \begin{cases} \sqrt{L} & \text{if } L < 16 \\ 4 & \text{if } L \geq 16 \end{cases} \Rightarrow$$

$$\Rightarrow MP_L = \begin{cases} \frac{1}{2} L^{-\frac{1}{2}} & \text{if } L \leq 16 \\ 0 & \text{if } L > 16 \end{cases}$$

$$b) \pi = 4 \cdot r(L) - L$$

$$\text{if } L \geq 16 \Rightarrow \pi(L) = 16 - L \Rightarrow \pi(L=16) = 0 \text{ is max.}$$

$$\text{if } L < 16 \Rightarrow \pi(L) = 4\sqrt{L} - L \Rightarrow \frac{\partial \pi}{\partial L} = 0 \Rightarrow L = 4 //$$

$$c) \text{ if } L \geq 16 \Rightarrow \pi(L) = 40 - L \Rightarrow \text{max at } L = 16 \Rightarrow \pi = 24 //$$

$$\text{if } L < 16 \Rightarrow \pi(L) = 10\sqrt{L} - L \Rightarrow$$

$$\Rightarrow \frac{\partial \pi}{\partial L} = \frac{5}{\sqrt{L}} - 1 = 0 \Rightarrow L = 25 \text{ (contradiction)}$$

$$\Rightarrow \pi^{\max} = 24$$

$$d) \pi = \begin{cases} p\sqrt{L} - wL & \text{if } L \leq 16 \\ 4p - wL & \text{if } L > 16 \end{cases}$$

$$\text{if } L \leq 16 \Rightarrow \frac{\partial (p\sqrt{L} - wL)}{\partial L} = \frac{p}{2\sqrt{L}} - w = 0$$

$$\Rightarrow L = (p/2w)^2$$

if $L \geq 16 \Rightarrow 4P - wL$ is linear in L

$\Rightarrow \pi^{\max}$ is $L = 16$

$$\Rightarrow L = \begin{cases} \left(\frac{P}{2w}\right)^2 & \text{if } \left(\frac{P}{2w}\right)^2 \leq 16 \\ 16 & \text{o.w.} \end{cases}$$

12) $MP_x = 10 - \frac{x}{100}$, $P_r = 5$, $P_x = \frac{3}{2}$

$\Rightarrow V(x) = A + 10x - \frac{x^2}{200}$ (A is scalar)

$$\pi(x) = 5\left(A + 10x - \frac{x^2}{200}\right) - \frac{3}{2}x$$

$$\frac{\partial \pi}{\partial x} = 0 \Rightarrow x = 970 //$$

13) $f(x, y, z) = (x+y)^{\frac{1}{2}} z^{\frac{1}{2}}$ $w_x=1, w_y=2, w_z=3$ and $w'_y=4$

There'll be no change in the cost of production as x and y are perfect substitutes, and since $w_x < w_y$ in both cases $y=0$.