

ECON 203.01/02 - Problem Set 2

Spring 2023

1. Consider following "Robinson Crusoe" economy with two-goods, one consumer and one firm. Compute all Pareto optimal allocations.

$$U(l, q) = \log(l) + \log(q) \quad \text{and} \quad \omega = (24, 0) = (\bar{L}, q)$$

$$q = \begin{cases} \frac{3L}{4} & \text{if } 0 \leq L \leq 20 \\ (L - 20)^2 + 15 & \text{if } 20 < L \end{cases}$$

where $\bar{L} = 24$ denotes the time units (time can be used as either "leisure time" or "working time") and the endowment of consumer good is zero. Note: l denotes leisure and q denotes consumption good.

2. Robinson and Friday both produce pizza (P) and cola (C). Both of them have 10,000 units of labor each that they can allocate to the production of pizza or the production of cola. How much pizza and cola they produce depends on the amount of labor that Robinson and Friday allocate to the production of each. Assume that Robinson's and Friday's production of pizza and cola are both subject to diminishing marginal returns and take the following form: The quantity of pizza that Robinson produces is equal to twice the labor he allocates to the production of pizza and the quantity of cola that Robinson produces is equal to the labor he allocates to the production of cola. The quantity of pizza that Friday produces is equal to the labor he allocates to the production of pizza and The quantity of cola that Friday produces is equal to twice the labor he allocates to the production of cola. Assume that their utility functions take the following form: $U = \sqrt{PC}$.
 - (a) Derive and draw the production possibility frontier for both Robinson and Friday.
 - (b) Find the opportunity cost of producing pizza for both Robinson and Friday.
 - (c) Find the equilibrium allocation and prices in the absence of trade.
 - (d) Suppose Robinson and Friday can trade. Find the equilibrium price and quantity in the combined market after the trade. Discuss whether Robinson and Friday gain from trade.
3. Suppose that Robinson Crusoe produces and consumes fish (F) and coconuts (C). Assume that during a certain period he has decided to work 200 hours and is indifferent as to whether he spends this time fishing or gathering coconuts. Robinson's production function for fish is $F = \sqrt{l_F}$ and for coconut

is: $C = \sqrt{l_C}$ where l are the number of hours spent on each activity. Robinson's utility function is: $U(F, C) = \sqrt{FC}$.

- (a) Assume that Robinson is the only human living in his island (no trade can take place). How will he choose to allocate his time? What are the optimal C and F levels? What is his utility?
- (b) Now assume that there are newcomers to the island and Robinson can trade fish and coconuts at a price ratio of $\frac{P_F}{P_C} = 2$. If Robinson continues to produce the quantities in part (a), how will his consumption change, given the opportunity to trade? What will be his new utility?
- (c) How would your answer to part (b) change if Robinson adjusts his production to take advantage of the world?

4. Consider the island economy with Friday and Robinson. They have agreed to share their resources and they have also agreed that the weight that Friday receives in the economy is $w_f \in (0, 1)$ and the weight that Robinson receives is $w_r = 1 - w_f$. Their preferences are different. Friday likes bananas more than coconuts, and Robinson likes coconuts more than bananas. The utility functions are given by:

$$u_f = \alpha \ln x_b^f + (1 - \alpha) \ln x_c^f$$

$$u_r = (1 - \alpha) \ln x_b^r + \alpha \ln x_c^r$$

The endowment of the island is $e_b = 2$ and $e_c = 2c \geq 2$.

- (a) For every weight $w_f \in (0, 1)$ find the allocation that maximized the social surplus.
 - (b) For every weight $w_f \in (0, 1)$, can you find an initial endowment of bananas and coconuts among Robinson and Friday and a pair of prices that the Pareto efficient allocation actually constitutes an equilibrium of the market. (Here we decentralize the Pareto efficient allocation via a market equilibrium.)
5. Suppose that the utility possibilities curve of a 2 person economy is given by the equation $u_1^2 + 4u_2^2 = 100$ and the social welfare function of the economy is $W(u_1, u_2) = \min\{u_1, u_2\}$
 - (a) Draw the utility possibilities set for this economy.
 - (b) Draw the social indifference curves for this economy.
 - (c) What is the maximum social welfare that can be achieved in this economy? What are the individual utilities in this case?
 6. There are two goods, with quantities denoted by x and y . The total amounts available to Ann and Barbara together are $\bar{x}$ and $\bar{y}$, and these quantities satisfy $\bar{x} = \bar{y}$. Ann's and Barbara's preferences are given by the following utility functions $u_A = \sqrt{x} + \sqrt{y}$ and $u_B = \min\{x, y\}$.
 - (a) Determine the set of Pareto allocations.
 - (b) Derive the utility frontier for both agents, and depict it in a diagram for $\bar{x} = \bar{y} = 16$.

(c) What allocation maximizes the social welfare function $W = u_A + u_B$.

7. Ann and Celine both have flower gardens. Their gardens are positioned in such a way that Ann can see Celine's garden as well as her own, and Ayse therefore derives utility both from Celine's garden and her own garden. But Celine can't see Ann's garden, so she derives utility only from her own garden. Their preferences are represented by the utility functions:

$$u_A = y_A + 12x_A - \frac{1}{2}x_A^2 + 6x_C - \frac{1}{2}x_C^2$$

$$u_C = y_C + 8x_C - \frac{1}{2}x_C^2$$

where y_i is i 's consumption of dollars and x_i is the size of i 's garden in square meters. The cost of a garden is four dollars per square meter, and each individual is endowed with 100 dollars.

- (a) Write down a maximization problem for which the solutions are the Pareto allocations.
- (b) Derive the first-order conditions and the Pareto optimal allocations.