

METU Department of Economics

Spring 2025

ECON 206

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Problem Set VI

Content: Simple Linear Regression I

Question 1

Consider the simple linear regression model as follows $Y_t = \beta_0 + \beta_1 (X_t - \bar{X}) + u_t$, $t=1, 2, \dots, n$ where the error terms are independently distributed with zero mean and constant variance, σ^2 . Find the least square estimates of β_0 and β_1 .

Question 2

Consider the following regression model: $Y_t = \beta_0 + \beta_1 X_t + u_t$. Show that $R^2 = 1 - \frac{\sum \tilde{u}_t}{\sum y_t}$.

Question 3

Show that $r_{xy} = \sqrt{R^2}$ in simple linear regression model.

Question 4

Assume the population regression function is as follows and satisfies the assumptions of CLRM:

$$Y_i = \beta_0 + \beta_1 X_i + u_i$$

By stating the problem explain the properties of the OLS estimators if:

- a. $cov(X_i, u_i) \neq 0$
- b. $var(u_i) = \sigma_i^2$
- c. $cov(u_i, u_j) \neq 0$ where $i \neq j$

Question 5

Assume the population follows the regression function:

$$Y_i = \beta_0 + \beta_1 X_i + u_i$$

where $\hat{u}_i$ denotes the residuals and $\hat{Y}_i$ is the fitted values show that $\sum y_i = \sum \hat{y}_i$ and $\sum \hat{y}_i \hat{u}_i = 0$

Question 6

Given the following simple linear regression model without intercept

$$Y_i = \beta_1 X_i + u_i$$

- Derive the OLS estimator of the slope.
- Does $\sum \hat{u}_i = 0$ hold?

Question 7

Consider the following regression equations:

$$Y_t = \beta_0 + \beta_1 X_t + u_t \quad \text{and} \quad X_t = \alpha_0 + \alpha_1 Y_t + v_t$$

Find the relation between $\hat{\beta}_1$ and $\hat{\alpha}_1$.

Question 8

A researcher is using a sample of 274 male employees to investigate the relationship between hourly wage rates Y_i (measured in dollars per hour) and firm tenure X_i (measured in years). Here are the sample information when $N = 274$.

$$\begin{array}{lll} \sum_{i=1}^N Y_i = 1945.26 & \sum_{i=1}^N X_i^2 = 30608.00 & \sum_{i=1}^N y_i^2 = 4726.377 \\ \sum_{i=1}^N X_i = 1774.00 & \sum_{i=1}^N X_i Y_i = 16040.72 & \sum_{i=1}^N x_i^2 = 19122.32 \\ \sum_{i=1}^N Y_i^2 = 18536.73 & \sum_{i=1}^N x_i y_i = 3446.226 & \sum_{i=1}^N \hat{u}_i^2 = 4105.297 \end{array}$$

- Compute OLS estimates of the intercept coefficient β_1 and the slope coefficient β_2 .
- Interpret the slope coefficient estimate you find in part a.
- Calculate an estimate of σ^2 , the error variance.

- d. Calculate an estimate of $Var(\hat{\beta}_2)$, variance of $\hat{\beta}_2$; $Var(\hat{\beta}_1)$ and $Cov(\hat{\beta}_1, \hat{\beta}_2)$.
- e. Compute the value of R^2 , the coefficient of determination for estimated OLS sample regression equation. Explain what R^2 means.
- f. Calculate a sample value of t- statistic for testing the null hypothesis $H_0 : \beta_2 = 0$ against the alternative hypothesis $H_1 : \beta_2 \neq 0$.