

ECON 203.02 - Problem Set 10

Spring 2023

1. Consider the market for new cars. Suppose there are two types of new cars available at dealerships -good cars and bad cars (i.e. lemons), which break down often. The fraction of lemons at any dealership is x . The seller does not differentiate between good cars and lemons -meaning there is no big sign stating whether or not a car is a lemon; the seller sells any car on the lot for the sticker price on each car. Buyers cannot tell good cars from lemons, but they know that a proportion x of cars at any dealership are lemons. Once they are bought the car and owned the car for a while, buyer can tell whether or not they have bought a lemon. The good cars are valued at \$2000 and lemons are worth \$1000.
 - (a) What does asymmetric information mean in this context?
 - (b) What is the equilibrium price for new cars? (Hint: This price will be in terms of x)
 - (c) Now, consider the used car market. Assume that used car sellers are willing to sell their cars at 20% below their new value but buyers do not know this. Given that good cars are valued at \$2000 and lemons are worth \$1000, what are the price ranges at which used car sellers are willing to sell their cars?
 - (d) Assuming that cars do not deteriorate, could there be any gains from trade from each sale?
 - (e) Let $x = 0.4$. What is the average value of a used car to a buyer? Will both good used cars and lemons be sold in the market? If so, does either the buyer or the seller get to keep any surplus?
2. A monopoly insurance company provides accident insurance to two types of customers: low risk customers, for whom the probability of an accident is 0.25, and high risk customers, for whom the probability of an accident is 0.5. There is an equal number of both types of customers. Without insurance, each customer's wealth is 16 if there is no accident, but 0 if there is an accident. Customers' utility of wealth is $u(w) = \sqrt{w}$. The insurance company cannot identify the type of a customer when the customer applies for an insurance contract. Suppose the insurance company offers the following two contracts. The first contract offers a payout of 8 in case there is an accident, and requires customers to pay a premium of 7. The second contract offers a payout of 16 in case there is an accident, but requires customers to pay a premium of 10.
 - (a) Determine for low and high risk customers which if any of these contracts they will buy.
 - (b) Does the insurance company manage to screen its customers with these contracts?
 - (c) Calculate the insurance company's expected profit if it offers these contracts.

3. There are two types of workers: high and low ability. Education (e) is continuous and observed, but individual ability (and output) is not. The labor productivity for the low type is $y_l(e) = \alpha_1$ and the cost of education is $c_l(e) = \frac{3}{2}e^2$. For the high type, labour productivity is $y_h(e) = \alpha_1 + \alpha_2 e$ and the cost of education is $c_h(e) = e^2$.
 - (a) Solve for the most efficient equilibrium of the signaling game.
 - (b) Show that the high type does not have an incentive to deviate from your proposed equilibrium strategy.
 - (c) Does the high type's investment in education differ from what would have obtained in the perfect-information case? Why or why not?

4. Consider an economy with three types of workers that differ in their abilities $a_1 = 1$, $a_2 = 2$ and $a_3 = 3$ and their reservation utilities 0.5, 1 and 2.2. The probability of each type is $\frac{1}{3}$. A perfectly competitive firm considers hiring the workers, but the productivity is private information only known to each worker. Once the workers are hired, the production function of the firm is $y = a_1 L_1 + a_2 L_2 + a_3 L_3$, where L_i is the amount of work done by a worker of type a_i . The product y is sold at the price of 1. Suppose that workers can educate themselves. The utility function of the workers is $u_i = w - \frac{e}{a_i^2}$. The company observes the education level e , of each worker.
 - (a) Compute, if possible, the education levels e_1 , e_2 and e_3 such that a worker of type i prefers the education level e_i and the company believes them when he sees the level of education. Note: start by arguing that $e_1 = 0$. Of all the possible levels of education, find those that are better for the consumers. Explain explicitly the beliefs of the company.
 - (b) Find, if possible, an equilibrium in which $e_1 = e_2 = 0$, and $e_3 > 0$. In this way, the company does not discriminate among the first two types, but distinguish among the first two types and the third one. Compute the equilibrium with the most convenient education level for the workers of type 3. Write explicitly the beliefs of the company. Compare the utilities with those of part (a).

5. Suppose there are two companies which compete in quantities (Cournot's competition). The inverse demand is $P(q) = a - q$ where $q = q_1 + q_2$ and q_1, q_2 are the amounts produced by the companies. The cost function of Firm 1 is $c_1(q_1) = cq_1$ and the cost function of Firm 2 is:

$$c_2(q_2) = \begin{cases} c_a q_2 & \text{with probability } \theta \\ c_b q_2 & \text{with probability } 1 - \theta \end{cases} \quad (1)$$

with $c_b < c_a$. There is asymmetric information: Firm 2 knows its cost cost function (c_a or c_b) and knows the cost of Firm 1 but, Firm 1 only knows its cost function. It also knows that the marginal cost of Firm 2 is c_a with probability θ and c_b with probability $1 - \theta$. Find the possible levels of output.