

ECON 203.02 - Problem Set 6

Spring 2023

1. Suppose an industry has a duopoly structure. Total cost functions of the duopolists are given by:

$$c_1(y_1) = y_1^2 \text{ if } y_1 \leq 0$$

$$c_2(y_2) = 12y_2 \text{ if } y_2 \leq 0$$

Denote total output produced in the industry by $y = y_1 + y_2$, and the inverse demand function for the good produced in the industry is given by: $p = 100 - y$.

- (a) Find the reaction function of each duopolist.
 - (b) Obtain the output levels that will be produced in a Cournot-Nash equilibrium, and the price level in such an equilibrium.
 - (c) Illustrate your solution in a diagram.
2. Suppose that there are two firms producing video games. Firms have an average variable cost $c = \$50$ and compete in prices. Market demand for video games is given by: $Q(P) = 100 - P$ when $P \leq 100$ and zero otherwise. Write down the reaction functions of the firms and find the Bertrand equilibrium.
3. Suppose that there are n identical firms; and let q_i denote the quantity produced by firm i and let $Q = q_1 + \dots + q_n$ denote aggregate quantity. Let P be market-clearing price and assume that inverse demand is given as $P = \alpha - \beta Q$ (assuming that if $Q > \frac{\alpha}{\beta}$ then $P = 0$). Each firm has constant marginal cost c of producing output and assume that there is no other cost (fixed cost). Finally, suppose that firms choose their quantity simultaneously.
- (a) What is the Nash equilibrium of this game?
 - (b) What is the profit of each firm in equilibrium? What happens when the number of firms in the industry goes to infinity?
4. Suppose that the (inverse) market demand for paper is given by $P = 400 - 2Q$ where Q is total industry output. There are two firms that produce paper. Each firm has a constant marginal cost of production equal to \$40.
- (a) Suppose that the firms decide to form a cartel. Calculate the output for each firm.
 - (b) Calculate the optimal response (i.e. the cartel cheating response).

- (c) Set up the cartel formation problem as a prisoner's dilemma problem.
5. Two workers independently choose how much effort to put in their jobs. If worker 1 and worker 2 chooses $e_i \geq 0$ for $i = 1, 2$, their payoffs are given by

$$u_1(e_1, e_2, \delta_1, \delta_2) = \delta_1 e_1 + \beta e_1 e_2 - \frac{1}{2} e_1^2$$

$$u_2(e_1, e_2, \delta_1, \delta_2) = \delta_2 e_2 + \beta e_1 e_2 - \frac{1}{2} e_2^2$$

where $0 < \beta < 1$ and $0 \leq \delta_i \leq 1$ for $i = 1, 2$. You may think δ_i as worker i 's productivity parameter. Find the Nash equilibria of this game.

6. Consider a duopoly where firms compete in prices but their products are differentiated. If firm 1 chooses price p_1 and firm 2 chooses price p_2 , the demand functions for firm 1's product and firm 2's product are, respectively:

$$q_1 = 24 - 5p_1 + 2p_2$$

$$q_2 = 24 - 5p_2 + 2p_1$$

- Determine each firm's best response function and illustrate the functions in a diagram.
- Calculate for the Nash equilibrium in prices. What are the firm's profits?
- Suppose now the firms collude, i.e., they jointly determine prices to maximize their joint profit. Determine the optimal prices and total profit.
- Is there an incentive for either firm to deviate from the collusive agreement? If so, what would its price be?