



$$e = \frac{\partial Q}{\partial P} \cdot \frac{P}{Q} = -\frac{1}{b} \cdot \frac{a/2}{a/2b} = (-1) \text{ unit elasticity at the midpoint}$$

Q₈) $P = 53 - Q$

a) $TR = (53 - Q)Q$

$MR = MC$

$$53 - 2Q = 5 \Rightarrow Q^* = 24$$

$$P^* = 29$$

$$\pi^* = P^* Q^* - AC \cdot Q^*$$

$$= Q^* (P^* - AC)$$

$$= 24^2 = 576$$

$$b) p = 53 - q_1 - q_2$$

$$\max_{q_1} \pi_1 = (53 - q_1 - q_2) q_1 - TC$$

$$\Rightarrow 53 - 2q_1 - q_2 = 5$$

$$\left. \begin{array}{l} q_1 = \frac{48 - q_2}{2} \\ \text{symmetric firms: } q_2 = \frac{48 - q_1}{2} \end{array} \right\} q = \frac{48 - q}{2}$$

$$q = 16$$

$$q_1 = q_2 = q$$

$$Q_C = 32, P_C = 21$$

$$\pi_i = (P_C - 5)q = (21 - 5)16 = 256$$

$$\text{Industry prof.: } 2 \cdot \pi_i = 512$$

$$c) P_B = MC = 5$$

$$Q_B = 48$$

$$\pi_1 = \pi_2 = 0$$

$$d) Q = \sum_{j=1}^n q_j, p = 53 - Q$$

$$\pi_i = (P - 5)q_i = (48 - Q)q_i$$

$$Q = q_i + Q_{-i}$$

$$\Rightarrow \pi_i = (48 - q_i - Q_{-i})q_i$$

$$\frac{\partial \pi_i}{\partial q_i} = 48 - 2q_i - Q_{-i} = 0 \Rightarrow q_i = \frac{48 - Q_{-i}}{2}$$

$$\text{Symmetric firms: } Q_{-i} = (n-1)q, q_i = q$$

$$q = \frac{48 - (n-1)q}{2} \Rightarrow q^* = \frac{48}{n+1}$$

$$Q^* = nq^* = \frac{48n}{n+1}$$

$$\pi_i^* = (P^* - 5)q^* = \left(\frac{48}{n+1}\right)\left(\frac{48}{n+1}\right) = \frac{2304}{(n+1)^2}$$

$$\text{Industry prof.: } n\pi_i^* = \frac{2304n}{(n+1)^2}$$