

HOMEWORK - 1 Solutions

2.4.49

$\left. \begin{array}{l} 4 \text{ 40-W} \\ 5 \text{ 60-W} \\ 6 \text{ 75-W} \end{array} \right\} \text{total of 15 lightbulbs}$

Let A = At least one bulb (out of 2 bulbs chosen) is 75-W

B = Both bulbs are 75-W

a) $P(B|A) = ?$

$$P(B|A) = \frac{P(B \cap A)}{P(A)} = \frac{P(B)}{P(A)}$$

$$P(B) = \frac{\binom{6}{2}}{\binom{15}{2}} = \frac{\frac{6 \cdot 5}{2}}{\frac{15 \cdot 14}{2}} = \frac{1}{7}$$

$$\begin{aligned} P(A) &= 1 - P(\text{none of the bulbs are 75-W}) = 1 - \frac{\binom{9}{2}}{\binom{15}{2}} = 1 - \frac{\frac{9 \cdot 8}{2}}{\frac{15 \cdot 14}{2}} \\ &= 1 - \frac{12}{35} = \frac{23}{35} \end{aligned}$$

$$\text{So } P(B|A) = \frac{\frac{1}{7}}{\frac{23}{35}} = \boxed{\frac{5}{23}}$$

b) Let C = At least one (of the 2 selected) is NOT 75-W
 D = Both bulbs have the same rating.

$$P(D|C) = ? \quad P(D|C) = \frac{P(D \cap C)}{P(C)} = \frac{P(\text{both bulbs are 40-W or both bulbs are 60-W})}{P(C)}$$

$$P(C) = 1 - P(B) = 1 - \frac{1}{7} = \frac{6}{7}$$

$$P(D|C) = \frac{\frac{\binom{4}{2} + \binom{5}{2}}{\binom{15}{2}}}{\frac{6}{7}} = \frac{\frac{\frac{4 \cdot 3}{2} + \frac{5 \cdot 4}{2}}{\frac{15 \cdot 14}{2}}}{\frac{6}{7}} = \boxed{\frac{8}{45}}$$

2.5.75 Let U = The upper subsystem consisting of 1 and 2 works

L = The lower subsystem consisting of 3 and 4 works

S = The whole system works

$$P(S) = P(U \cup L) = P(U) + P(L) - P(U \cap L)$$

Since components work independently, $\rightarrow P(S) = P(U) + P(L) - P(U) \cdot P(L)$

$$P(U) = (0.9) + (0.9) - (0.9)^2 = 0.93$$

$$P(L) = (0.9)^2 = 0.81$$

$$\begin{aligned} \text{So } P(S) &= 0.93 + 0.81 - (0.93)(0.81) \\ &= 1.8 - 0.8013 = \boxed{0.9987} \end{aligned}$$

2.5.83 T = The number of tags lost

$$P(T=1 | T \leq 1) = ?$$

$$P(T=1 | T \leq 1) = \frac{P(T=1)}{P(T \leq 1)} = \frac{P(T=1)}{P(T=0) + P(T=1)}$$

$$= \frac{\pi(1-\pi) + (1-\pi) \cdot \pi}{(1-\pi) \cdot (1-\pi) + \pi \cdot (1-\pi) + (1-\pi) \cdot \pi}$$

$$= \frac{2 \cdot \pi (1-\pi)}{(1-\pi) [1-\pi + 2\pi]} = \boxed{\frac{2\pi}{1+\pi}}$$