

Spring 2024-
2025
ECON 205

Problem Set 6

- 1) Suppose you roll two fair, six-sided dice, one of which is red and the other of which is green. Define the following random variables:

X = The number shown on the red die

$$Y = \begin{cases} 0 & \text{if the two dice show the same number} \\ 1 & \text{if the number on the green die is bigger than the number on the red die} \\ 2 & \text{if the number on the red die is bigger than the number on the green die} \end{cases}$$

- a) Write down a table showing the joint probability mass function for X and Y .
b) Find the marginal probability mass function for Y , and compute its expected value.

2)

The random variable X has a range of $\{0,1,2\}$ and the random variable Y has a range of $\{1,2\}$. The joint distribution of X and Y is given by the following table.

		y	
		1	2
x	$p(x,y)$		
	0	0.2	0.1
	1	0.0	0.2
	2	0.3	0.2

- i) Write down tables for marginal distributions of X and Y .
ii) Compute $E(X)$ and $E(Y)$.
iii) Compute $E(XY)$

- 3) Of nine executives in a business firm, four are married, three have never been married, and two are divorced. Three of executives are to be selected for promotion. Let Y_1 denote the number of married executives and Y_2 denote the number of never-married executives among three selected for promotion.

i) Assuming that the three are randomly selected from nine available, find the joint probability of Y_1 and Y_2 .

ii) Find the $\text{Cov}(Y_1, Y_2)$.

- 4) Suppose that Y_1 and Y_2 are random variables and show that

$$V(Y_1 + Y_2) = V(Y_1) + V(Y_2) + 2\text{Cov}(Y_1, Y_2).$$

- 5) Let Y_1 and Y_2 have joint density function

$$f(y_1, y_2) = \begin{cases} e^{-(y_1 + y_2)}, & y_1 > 0, y_2 > 0, \\ 0, & \text{elsewhere.} \end{cases}$$

- a. What is $P(Y_1 < 1, Y_2 > 5)$?
b. What is $P(Y_1 + Y_2 < 3)$?

- 6) The joint density function of Y_1 and Y_2 is given by

$$f(y_1, y_2) = \begin{cases} 30y_1y_2^2, & y_1 - 1 \leq y_2 \leq 1 - y_1, 0 \leq y_1 \leq 1, \\ 0, & \text{elsewhere.} \end{cases}$$

- a. Find $F(1/2, 1/2)$.
b. Find $F(1/2, 2)$.
c. Find $P(Y_1 > Y_2)$.

7)

Let X and Y be jointly continuous random variables with joint pdf

$$f(x, y) = \begin{cases} cx + 1, & x, y \geq 0, x + y \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

- i) Show the range of (X, Y) , in the x - y plane.
- ii) Find the constant c .
- iii) Find the marginal pdf's $f_X(x)$ and $f_Y(y)$.
- iv) Find $P(Y < 2X^2)$.

8)

Assume we have the following form;

$$f(x, y) = \begin{cases} 6yx^2, & 0 < y < 1 \text{ and } 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}$$

Is $f(x, y)$ a joint probability density function? If so calculate the following:

- a) The marginal densities of X and Y .
- b) $P(X + Y > 1)$
- c) $P\left(\frac{1}{3} < Y < \frac{1}{2}\right)$.