

METU, Department of Economics

ECON 201 – Problem Set 1

- 1- Zach and Edward are talented eighteenth-century painters. The world's stock of Zach's paintings is 100 and the world's stock of Edward's paintings is 70. The demand for each painter's work depends on its own price and the price of the other painter's work. If P_z is the price of Zach's paintings (assume all paintings of the same painter is identical in quality) and P_e is the price of Edward's paintings, the demand function for Zach's paintings is $101 - 3P_z + 2P_e$ and the demand function for Edward's paintings is $72 + P_z - P_e$. What are the equilibrium prices for Zach's and Edward's paintings?

Solution:

Firstly, define demand and supply functions for Zach.

$$Q_z^s = 100, Q_z^d = 101 - 3P_z + 2P_e$$

Demand and supply are equal at the equilibrium.

$$Q_z^s = 100 = Q_z^d = 101 - 3P_z + 2P_e \Rightarrow 3P_z - 2P_e = 1$$

Secondly, define demand and supply functions for Edward.

$$Q_e^s = 70, Q_e^d = 72 + P_z - P_e \Rightarrow P_z - P_e = 2$$

Rest is solving two equations with two unknowns to find market prices and then market quantities.

- 2- Jean, Emma, and Kurt are all buyers of widgets. Jean's demand function is $Q_j = 520 - 13P$, Emma's demand function is $Q_e = 40 - P$, and Kurt's demand function is $Q_k = 200 - 5P$. Together, these three constitute the entire demand for widgets. At what price will the price elasticity of market demand be -1?

Solution:

If the price is less than 50, all the buyers have a positive market demand meaning the market demand function is $Q_d = Q_j + Q_e + Q_k = 760 - 19P$

$$\text{Then } e_{d,p} = \frac{\partial Q_d}{\partial P} \frac{P}{Q_d} = 19 \frac{P}{760 - 19P} = -1 \Rightarrow P = 20 (< 50)$$

- 3- Consider the following demand function:

$$Q = \beta P^\alpha$$

- (a) Find the conditions on α and β for which the demand curve makes sense, i.e. Q is decreasing in P .
- (b) Find the price elasticity of demand. Does it depend on Q or P ? Explain.

Solution:

- a) There are two conditions to satisfy to get a reasonable demand func.

i. $Q \geq 0$. This is satisfied with $\beta \geq 0$ (assuming $P > 0$)

ii. $\frac{\partial Q}{\partial P} = \frac{\beta \alpha P^\alpha}{P} < 0$. This is satisfied with assuming $\beta \alpha < 0$

This implies a positive β and a negative α .

- b) $e_{d,p}$ depends neither quantity demanded nor the price level.

$$\frac{\partial Q}{\partial P} \frac{P}{Q} = \frac{\beta \alpha P^\alpha}{P} \frac{P}{\beta P^\alpha} = \alpha$$

- 4- Consider the following market demand and supply functions:

$$Q_D = D(P, \alpha), \quad Q_S = S(P, \beta)$$

where α and β denote variables which shifts demand and supply curves, respectively (e.g. $\alpha = \text{income}$, $\beta = \text{input costs}$).

Assuming that equilibrium is preserved and β is kept constant, derive the following equation:

$$e_{P,\alpha} = \frac{e_{D,\alpha}}{e_{S,P} - e_{D,P}}$$

where $e_{P,\alpha}$ gives the percentage change in equilibrium price in response to percentage change in α and $e_{D,\alpha}$ is the elasticity of demand with respect to α .

Solution:

$$a) \quad dQ_D = \frac{\partial D}{\partial P} dP + \frac{\partial D}{\partial \alpha} d\alpha = D_P dP + D_\alpha d\alpha$$

$$dQ_S = \frac{\partial S}{\partial P} dP + \frac{\partial S}{\partial \beta} d\beta = S_P dP + S_\beta d\beta$$

β is constant meaning $d\beta = 0$

At eqb. $Q_D = Q_S$

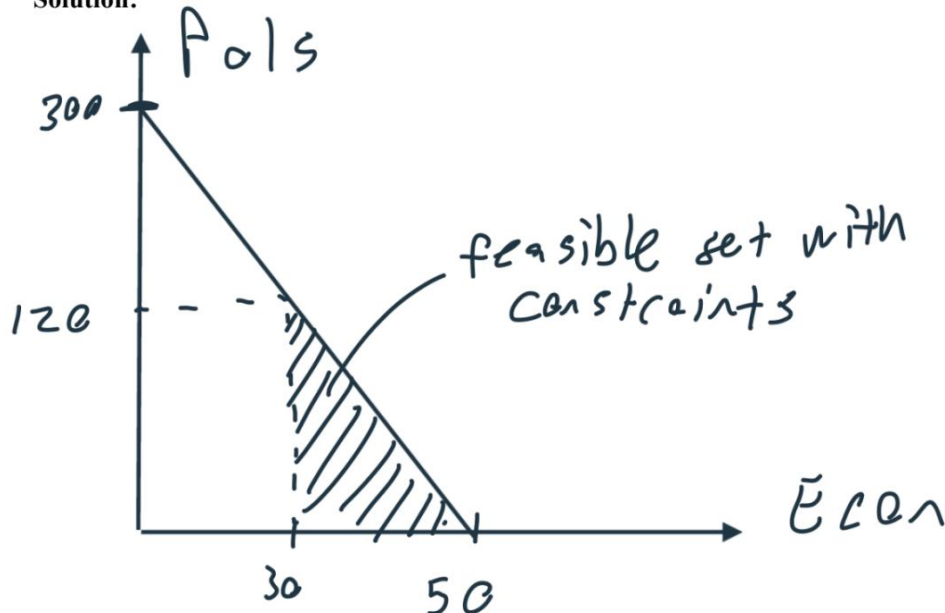
$$dQ_D = dQ_S \rightarrow D_p dP + D_\alpha d\alpha = S_p dP \rightarrow \frac{D_\alpha}{S_p - D_p} = \frac{dP}{d\alpha} \rightarrow \frac{D_\alpha}{S_p - D_p} \frac{\alpha}{P} \frac{Q}{Q} = \frac{dP}{d\alpha} \frac{\alpha}{P} = e_{p,\alpha}$$

$$\rightarrow e_{p,\alpha} = \frac{e_{D,\alpha}}{e_{S,P} - e_{D,P}}$$

5- Answer the following questions by drawing the associated budget sets.

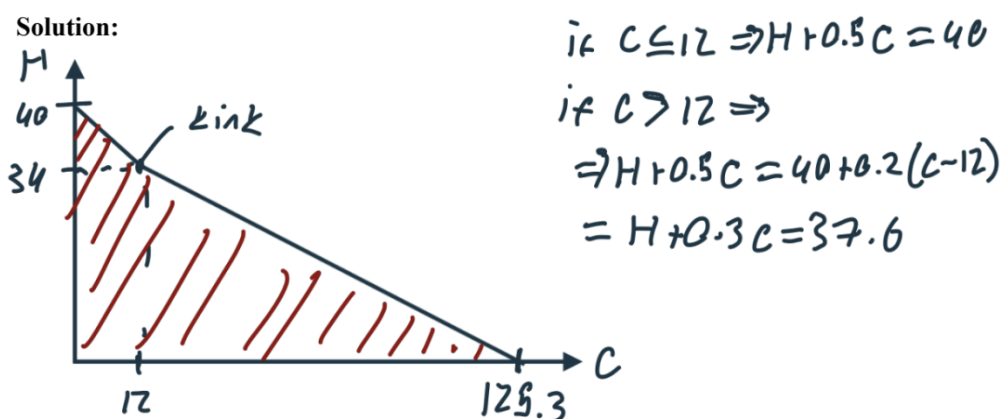
- a) Felicity is studying economics and political science. She can read 30 pages of political science per hour but only 5 pages of economics per hour. This week she has a 50-page assignment in economics and a 150-page assignment in political science. Because of sorority rush, she cannot devote more than 10 hours to studying these subjects this week. She realizes she cannot complete all of her assignments but is determined to complete at least 30 pages of her economics reading. Draw a graph with pages of economics on the horizontal axis and pages of political science on the vertical axis. On this graph, show the possibilities that are consistent with the constraints that Felicity has imposed on herself. (She is allowed to read ahead in either subject.) Label key points on your graph with their numerical values.

Solution:



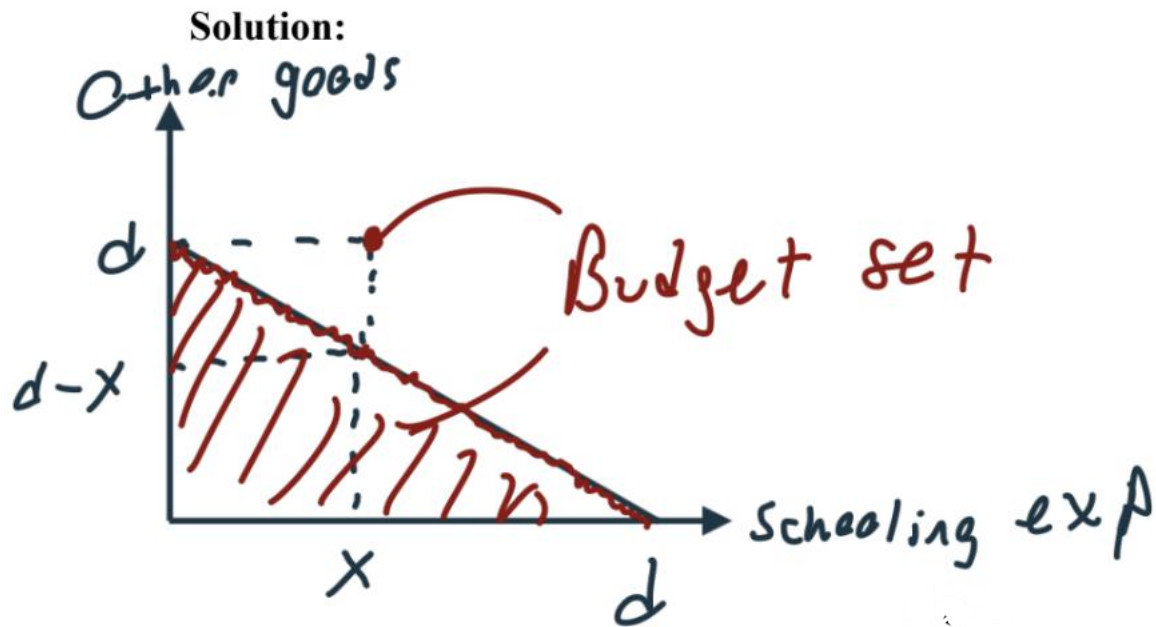
- b) Brenda likes hot dogs and Coca-Cola. Hot dogs cost \$1 each and Cokes cost \$.50 per bottle. There is a special promotion for Coke that will last for one month. If Brenda sends in the bottle tops from the Cokes she drinks during the next month, she will get a refund of \$.20 for every bottlecap beyond the first 12 that she

returns. For example, if she returns 25 bottle caps she will get back $\$2.60 = \$0.20(25 - 12)$. Brenda has \$40 to spend on hot dogs and Coke during the next month. Draw her budget line with Coke on the horizontal axis and hot dogs on the vertical axis. Find the points where the budget line hits the axes and the point where it has a kink. At each of these three points write down the quantities of each good consumed.



- c) Ed Moore and his family live in a city with many private schools and one public school. The Moores are thinking of sending their only child to private school because they would like a school that has more teachers and other resources per student than the local public school. The Moores must pay taxes to support local public schools whether or not their child goes to private school. There is such a variety of private schools that the Moores can get just about any level of inputs per student by choosing the appropriate private school. Tuition in the private schools equals expenditure per student. Draw a diagram to show the Moores' budget constraint. Put expenditures per student in the child's school on the horizontal axis and other goods on the vertical.

Solution:



6- True/False

- If there are two goods with positive prices and the price of one good is reduced, while income and other prices remain constant, then the size of the budget set is reduced.
- If good 1 is measured on the horizontal axis and good 2 is measured on the vertical axis and if the price of good 1 is p_1 and the price of good 2 is p_2 , then the slope of the budget line is $-p_2/p_1$.
- There are two goods. You know how much of good 1 a consumer can afford if she spends all of her income on good 1. If you knew the ratio of the prices of the two goods, then you could draw the consumer's budget line without any more information.
- In year 1, the price of good x_1 was \$4, the price of good x_2 was \$1, and income was \$70. In year 2, the price of x_1 was \$9, the price of good x_2 was \$2, and income was \$80. On a graph with x_1 on the horizontal axis and x_2 on the vertical, the new budget line is steeper than the old one and lies below it.
- Suppose there are two goods. If you could exactly afford either 2 units of x_1 and 7 units of x_2 , or 4 units of x_1 and 3 units of x_2 , then if you spent all of your income on x_2 , the maximum unit of y you can buy is

- f) Normal goods have negative income elasticities of demand, while inferior goods have positive income elasticities of demand.

Solution:

- a) False – If the price of one good decreases while income and the price of the other good remain constant, the consumer can afford more of that good (and potentially more of both goods), which means the budget set expands, not shrinks.

$$(1) p_1x_1 + p_2x_2 = m$$

Algebraic operations yield the following equations from (1):

$$(2) x_1 = m/p_1 - p_2x_2/p_1$$

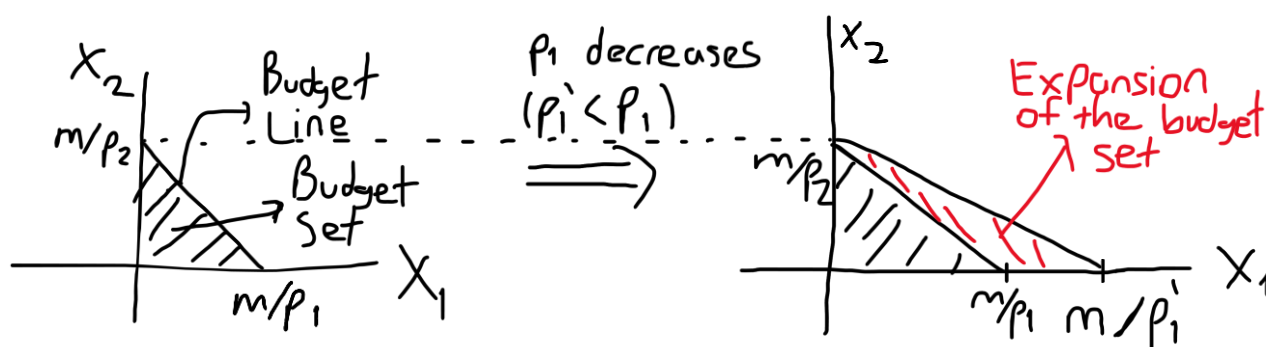
$$(3) x_2 = m/p_2 - p_1x_1/p_2$$

And thus,

$$0 \leq x_1 \leq m/p_1 \text{ from equation (2),}$$

$$0 \leq x_2 \leq m/p_2 \text{ from equation (3).}$$

Say p_1 decreases while m and p_2 are constant. Then the set of values that (x_1, x_2) can take expands. Or, graphically,



- b) False

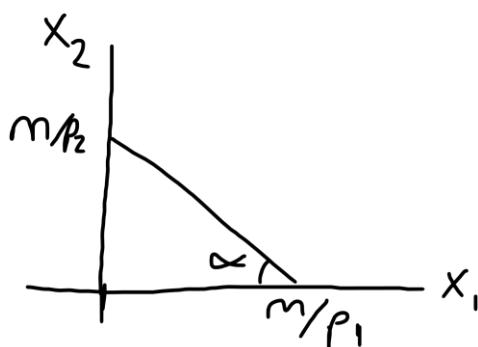
$$p_1x_1 + p_2x_2 = m$$

$$x_1 = m/p_1 - p_2x_2/p_1$$

$$x_2 = m/p_2 - p_1x_1/p_2$$

$$\text{Slope of the budget line: } dx_2/dx_1 = -p_1/p_2$$

Or, graphically:



$$\tan \alpha = - \frac{m/p_2}{m/p_1} = - \frac{p_1}{p_2}$$

c) True

$$(1) p_1x_1 + p_2x_2 = m$$

$$(2) x_1 = m/p_1 - p_2x_2/p_1$$

$$(3) x_2 = m/p_2 - p_1x_1/p_2$$

You know m/p_1 and p_1/p_2 . All you need is m/p_2 , which can be obtained by multiplying m/p_1 by p_1/p_2 .

d) True

In year 1,

$$-p_1/p_2 = -4$$

$$m/p_1 = 17.5$$

$$m/p_2 = 70$$

In year 2,

$$-p_1/p_2 = -4.5$$

$$m/p_1 = 8.6$$

$$m/p_2 = 40$$

The budget line is steeper because the slope has increased (in absolute terms).

The new budget line lies below the old one because both m/p_1 and m/p_2 decreased.

e) $p_1x_1 + p_2x_2 = m$

$$(1) 2p_1 + 7p_2 = m$$

$$(2) 4p_1 + 3p_2 = m$$

$$\Rightarrow 2p_1 + 7p_2 = 4p_1 + 3p_2$$

$$2p_1 = 4p_2$$

$$p_1 = 2p_2$$

Plugging p_1 into (1),

$$11p_2 = m$$

Therefore, you can afford a maximum of 11 units of x_2 .