

Middle East Technical University
Department of Economics
Econ 206: Statistics for Economists-II

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Recitation 4

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Question 1

A company requires the lifespan of the batteries it purchases to be at least 50 hours. For this purpose, they tested 9 batteries before purchasing. The average lifespan of these batteries was found to be 48.2 hours. Based on past experience, the company knows that these lifespans follow a normal distribution with a standard deviation of 3 hours.

Test the hypothesis that the average life of all batteries is at least 50 hours against the alternative hypothesis at a **10% significance level** ($\alpha = 0.10$), and find the **p-value** of the test.

Solution: $n = 9$, $\bar{x} = 48.2$, $\sigma = 3$, $\alpha = 0.10$

- $H_0 : \mu \geq 50$
- $H_1 : \mu < 50$

Test statistic:

$$Z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \frac{48.2 - 50}{3/3} = -1.8$$

Critical value: $z_\alpha = z_{0.1} = -1.28$

Since $Z_{\text{calculated}} < z_\alpha \rightarrow$ reject the null hypothesis.

p-value:

$$p = P(Z < -1.8) = 0.0359$$

Since $p = 0.0359 < \alpha = 0.10 \rightarrow$ reject the null hypothesis.

Question 2

In a study of 21 individuals who made changes to their individual pension plans, the average percentage change was found to be 0.078 of their monthly payments. The sample standard deviation was 0.142.

Considering the population of those who change their pension plans, what is the **p-value** when the hypothesis that the average monthly change is 0% is tested against a **two-sided alternative hypothesis**?

Solution: $\bar{x} = 0.078$, $\sigma = 0.142$, $n = 21$

- $H_0 : \mu = 0$
- $H_1 : \mu \neq 0$

Test stat:

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{0.078 - 0}{0.142/\sqrt{21}} = 2.51$$

p-value:

$$p = 2 \cdot P(T_{20} > 2.51) \approx 0.2$$

Question 3

The proportion of undesirable substances in 64 drugs produced by a pharmaceutical manufacturer was found to be 3.7%.

- a) Test the null hypothesis that the population proportion of undesirable substances is 3% or less ($p \leq 0.03$) against the alternative hypothesis that it is greater than 3%, at a **5% significance level** ($\alpha = 0.05$).
- b) If the true value of the proportion of undesirable substances in the population is actually 3.10%, find the probability of rejecting the null hypothesis in part (a) (i.e., find the **power of the test**).

Solution: $n = 64$, $\hat{p} = 0.037$, $\alpha = 0.05$

- $H_0 : \hat{p} = 0.03$
- $H_1 : \hat{p} > 0.03$

a) The null hypothesis is rejected if $\frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} > z_{1-\alpha}$

$$\Rightarrow \frac{0.037 - 0.03}{\sqrt{\frac{0.03(1-0.03)}{64}}} = \frac{0.007}{0.021} = 0.33$$

Critical value: $z_{0.95} = 1.64$

Since $0.33 < 1.64$, we fail to reject the null hypothesis.

b) $P(\text{fail to reject } H_0 \mid H_0 \text{ is false}) = \beta \rightarrow$ Type II error.

$1 - \beta \rightarrow$ power of a test: the probability of rejecting the null when it is false.

$$P \left(\frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} < z_{1-\alpha} \mid H_0 \text{ is false} \right)$$

$$\hat{p} < 1.64 \times \sqrt{\frac{0.03(1-0.03)}{64}} + 0.03 \Rightarrow \hat{p} < 0.064$$

$$\begin{aligned}\text{Normalize } \hat{p} \Rightarrow z &= \frac{\hat{p}-0.031}{\sqrt{\frac{0.031(1-0.031)}{64}}} < \frac{0.064-0.031}{\sqrt{\frac{0.031(1-0.031)}{64}}} \\ &\Rightarrow P(Z < 1.571) = 0.9418 = \beta\end{aligned}$$

The power of the test = $1 - \beta = 1 - 0.9418 = 0.0582$

Question 4

A professor teaches various sections of the same course. In the final exam, the marks of all students showed a variance of 300. During the semester, additional problem-solving sessions were held, and in one of the sections, these sessions were conducted by a new teaching assistant.

If the variance of the final exams in this 30-person section was 480, test the hypothesis that the population variance of the student profile in this section is 300 ($\sigma^2 = 300$) against a **two-sided alternative hypothesis** at both **5%** and **2% significance levels**.

Solution: $n = 30$, $s^2 = 480$

- $H_0 : \sigma^2 = 300$
- $H_1 : \sigma^2 \neq 300$

Test statistic:

$$\chi^2 = \frac{(n-1)s^2}{\sigma_0^2} = \frac{29 \cdot 480}{300} = 46.4$$

a) $\alpha = 0.05$

Critical values: $\chi_{29,0.975}^2 = 16.047$, $\chi_{29,0.025}^2 = 45.722$

Since $\chi_{calc}^2 > 45.722$, reject H_0 .

b) $\alpha = 0.02$

Critical values: $\chi_{29,0.99}^2 = 14.256$, $\chi_{29,0.01}^2 = 49.588$

Since χ_{calc}^2 is in the non-rejection region, fail to reject H_0 .

Question 5

According to a study, it takes 60 minutes to prepare a CV for a job position. A sample of 100 applicants who applied for a job position last year showed that they took on average 62.6 minutes to prepare the CV. The standard deviation for the sample was 11 minutes.

- a) Find the **p-value** for the test that the mean time taken to prepare the CV is different from 60 minutes ($H_1 : \mu \neq 60$).
- b) Using the p-value calculated in part (a), will you reject the Null Hypothesis if $\alpha = 0.01$? What if $\alpha = 0.05$?

Solution:

$$n = 100, \quad \bar{x} = 62.6, \quad s = 11$$

- $H_0 : \mu = 60$
- $H_1 : \mu \neq 60$

Test statistic: $t = \frac{62.6-60}{11/\sqrt{100}} = 2.36$

p-value: $2 \cdot P(T_{99} > 2.36) \approx 0.02$

b) For $\alpha = 0.01 \rightarrow p = 0.02 > 0.01 = \alpha$ we fail to reject the null hypothesis.

For $\alpha = 0.05 \rightarrow p = 0.02 < 0.05 = \alpha$ we reject the null hypothesis.

- Increasing the significance level (α) expands the rejection region within the probability distribution, thereby reducing the stringency of the evidence required to reject the null hypothesis. As the critical thresholds migrate toward the mean of the null distribution, the statistical test becomes more liberal, increasing both the Power of the test ($1 - \beta$) and the probability of committing a Type I error.

Question 6

Consider the following null and alternative hypothesis:

$$H_0 : \mu = 120 \quad \text{versus} \quad H_1 : \mu > 120$$

A random sample of 81 observations taken from this population produced a sample mean of 123.5 and a sample standard deviation of 15.

- If this test is made at the **2.5% significance level**, would you reject the null hypothesis?
- Suppose that the null hypothesis is false and the true population mean is 121. What is the probability of making a **Type II error** (β) in part (a)? What is the **power of the test** ($1 - \beta$)?

Solution:

Note: We normally use t-test when the population variance is unknown, but if we have enough sample size, we can use z-test. And since it is more convenient to use z-table when calculating the power of a test, in this question we will use z-test.

a) $n = 81, \quad \bar{x} = 123.5, \quad s = 15, \quad \alpha = 0.025$

- $H_0 : \mu = 120$
- $H_1 : \mu > 120$

Test stat: $Z = \frac{123.5-120}{15/\sqrt{81}} = 2.1$

Critical value: $z_{0.025} = 1.96$. Since $2.1 > 1.96$, reject H_0 .

b) Power ($1 - \beta$):

Reject if $\bar{x} > 1.96 \cdot \frac{15}{9} + 120 \Rightarrow \bar{x} > 123.26$

$$\beta = P(\bar{x} < 123.26 \mid \mu = 121) = P(Z < \frac{123.26-121}{15/9}) = P(Z < 1.356) = 0.9131$$

$$\text{Power} = 1 - 0.9131 = 0.0869$$