

Problem Set I - Answers

Q1) A chef can make starters, main courses, and desserts. Four different starters, seven different main course, and three different desserts are under consideration.

i) If she wants to make a menu with a single starter, main course, and dessert, how many experimental menus are possible?

$n_1=4, n_2=7, n_3=3$ So, there are $n_1*n_2*n_3=4*7*3=84$ different menus

ii) How many experimental menus consists of only tomato soup as a starter?

$n_1=1$ (since she makes only tomato soup as a starter), $n_2=7, n_3=3$ So, there are $1*7*3=21$ different menus

Q2)

a) Two cards are drawn from a standard 52-card playing deck. What is the probability that the draw will yield an ace and a face card?

There are $\binom{52}{2}=1326$ ways to draw two cards from the deck. The probability is $4*12/1326=0.0362$.

b) Five cards are dealt from a standard 52-card deck. What is the probability that we draw 3 aces and 2 kings?

There are $\binom{52}{5}=2598960$ ways to draw five cards from the deck. There are $\binom{4}{3}*\binom{4}{2}=24$ ways to draw three Aces and two kings. The probability is $24/2598960$.

c) Five cards are dealt from a standard 52-card deck. What is the probability that we draw 1 ace, 1 two, 1 three, 1 four, and 1 five?

$\binom{4}{1}*\binom{4}{1}*\binom{4}{1}*\binom{4}{1}*\binom{4}{1}=4^5=1024$. The probability is $1024/2598960$.

Q3) Cards are dealt, one at a time, from a standard 52-card deck. If the first 2 cards are both spades, what is the probability that the next three cards are also spades?

There are 13 spades in a deck. Given the 2 cards are spades, there are 11 spades left in the deck. The probability is $\frac{\binom{11}{3}}{\binom{50}{3}}=0.0084$.

Q4) Same product is produced by machines A, B and C. %5 of the products produced by Machine A, %4 of the product produced by Machine B and %3 of the products produced by Machine C is broken. In a single day %25 of the products are produced by Machine A, %30 of the products are produced by Machine B and %45 of the products are produced by Machine C. A product produced in a single day is randomly selected and known to be broken then what is the probability that that product is produced by Machine C.

SOLUTION

Apply Bayes' rule:

$H = \{ \text{Selected product is broken} \}$

$A = \{ \text{Product is produced by Machine A} \}$

$B = \{ \text{Product is produced by Machine B} \}$

$C = \{ \text{Product is produced by Machine C} \}$

$P(A) = 0.25, P(H|A) = 0.05, P(B) = 0.30, P(H|B) = 0.04, P(C) = 0.45, P(H|C) = 0.03$

$$P(H) = P(A)P(H|A) + P(B)P(H|B) + P(C)P(H|C) = \frac{1}{4} \frac{1}{20} + \frac{3}{10} \frac{1}{25} + \frac{9}{20} \frac{3}{100} = \frac{19}{500}$$

$$P(C|H) = \frac{P(C \cap H)}{P(H)} = \frac{P(C)P(H|C)}{P(H)} = \frac{(9/20)(3/100)}{(19/500)} = \frac{27}{76}$$

Q5) One needs to distribute 20 labors to four different construction jobs. The first job required 6 labors, the second, third and fourth utilized 4, 5 and 5 labors, respectively. Find the probability of the observed events assuming that the labors are randomly assigned to jobs.

a) All 4 members of a particular ethnic group are placed on job 1.

The number of ways to assign 20 labors to 4 jobs is $\binom{20}{6 \ 4 \ 5 \ 5} = \frac{20!}{6!4!5!5!} = N$

All 4 members are assigned to job 1. We need to place the remaining 16. $\binom{16}{2 \ 4 \ 5 \ 5} = \frac{16!}{2!4!5!5!} = n_a$

The probability is $\frac{n_a}{N} = 0.0031$.

b) An ethnic group member is assigned to each type of job.

$$n_b = \binom{4}{1\ 1\ 1\ 1} \binom{16}{5\ 3\ 4\ 4}$$

The probability is $\frac{n_b}{N} = 0.1238$.

Q6) A local fraternity is conducting a raffle where 50 tickets are to be sold- one per customer. There are three prizes to be rewarded. If the four organizers of the raffle each buy one ticket, what is the probability that the four organizers win

- All the prizes?
- Exactly two of the prizes?
- Exactly one of the prizes?
- None of the prizes?

SOLUTION

There are $\binom{50}{3} = 19,600$ ways to choose the 3 winners. Each of these is equally likely.

a. There are $\binom{4}{3} = 4$ ways for the organizers to win all of the prizes. The probability is $4/19600$.

b. There are $\binom{4}{2} \binom{46}{1} = 276$ ways the organizers can win two prizes and one of the other 46 people to win the third prize. So, the probability is $276/19600$.

c. $\binom{4}{1} \binom{46}{2} = 4140$. The probability is $4140/19600$.

d. $\binom{46}{3} = 15,180$. The probability is $15180/19600$.

Q7) A monkey is to demonstrate that she recognize colors by tossing one red, one black and one white ball into boxes of the same respective colors, one ball to a box. If the monkey has not learned the colors and merely tosses one ball into each box at random, find the probability of the following results:

- There are no colors matches.
- There is exactly one color match.

SOLUTION

Solution This problem can be solved by listing sample points because only three balls are involved, but a more general method will be illustrated. Define the following events:

- A_1 : A color match occurs in the red box.
- A_2 : A color match occurs in the black box.
- A_3 : A color match occurs in the white box.

There are $3! = 6$ equally likely ways of randomly tossing the balls into the boxes with one ball in each box. Also, there are only $2! = 2$ ways of tossing the balls into the boxes if one particular box is required to have a color match. Hence,

$$P(A_1) = P(A_2) = P(A_3) = 2/6 = 1/3.$$

Similarly, it follows that

$$P(A_1 \cap A_2) = P(A_1 \cap A_3) = P(A_2 \cap A_3) = P(A_1 \cap A_2 \cap A_3) = 1/6.$$

We can now answer parts (a) and (b) by using the event-composition method.

a Notice that

$$\begin{aligned} P(\text{no color matches}) &= 1 - P(\text{at least one color match}) \\ &= 1 - P(A_1 \cup A_2 \cup A_3) \\ &= 1 - [P(A_1) + P(A_2) + P(A_3) - P(A_1 \cap A_2) \\ &\quad - P(A_1 \cap A_3) - P(A_2 \cap A_3) + P(A_1 \cap A_2 \cap A_3)] \\ &= 1 - [3(1/3) - 3(1/6) + (1/6)] = 2/6 = 1/3. \end{aligned}$$

b We leave it to you to show that

$$\begin{aligned} P(\text{exactly one match}) &= P(A_1) + P(A_2) + P(A_3) \\ &\quad - 2[P(A_1 \cap A_2) + P(A_1 \cap A_3) + P(A_2 \cap A_3)] \\ &\quad + 3[P(A_1 \cap A_2 \cap A_3)] \\ &= (3)(1/3) - (2)(3)(1/6) + (3)(1/6) = 1/2. \quad \blacksquare \end{aligned}$$