

Problem Set VII

Content: Simple Linear Regression II

Question 1

Discuss which of the following population regression functions are linear in parameters.

- a. $Y_i = \beta_0 + \beta_1 X_i + u_i$
- b. $Y_i = \beta_0 + \beta_1 X_i + \beta_2 X_i^2 + u_i$
- c. $Y_i = \beta_0 X_{1i}^{\beta_1} X_{2i}^{\beta_2} e^{u_i}$
- d. $Y_i = \beta_0 + \beta_1 10^{X_i} + u_i$
- e. $Y_i = \beta_0 + \ln(\beta_1 + X_i) + u_i$
- f. $Y_i = \beta_0 X_i^{\beta_1} + u_i$

Question 2

Consider a simple linear model,

$$Y_t = \beta_0 + \beta_1 X_t + u_t$$

where $t = 1, 2, \dots, T$ and the model satisfies the assumptions. Show that $\widehat{\beta}_1$ is the best linear unbiased estimator for β_1 .

Question 3

Auditors are often required to compare the audited (or current) value of an inventory item with the book (or listed) value. If a company is keeping its inventory and books up to date, there should be a strong linear relationship between the audited and book values. A company sampled ten inventory items and obtained the audited and book values given in the accompanying table.

Item	Audit Value (y_i)	Book Value (x_i)
1	9	10
2	14	12
3	7	9
4	29	27
5	45	47
6	109	112
7	40	36
8	238	241
9	60	59
10	170	167

- Fit the model $Y = \beta_0 + \beta_1 x + \varepsilon$ to these data.
- If the book value is $x = 100$, what would you use to estimate the audited value?
- Write 95% confidence interval for β_1 .

Question 4

Assume that simple linear regression model $Y_i = \beta_1 X_i + u_i$ is appropriate for the data consisted of driving a car at constant speed 50 km/h and observing with 5 minute intervals the distance you have gone. Assume that the errors are iid normal with 0-mean and constant variance σ^2 .

Time	Distance
5	4.4
10	14.5
15	14.1
20	18
25	19.6
30	26.7
35	28.3
40	32.4
45	38
50	43.2
55	45.5
60	49.7

- Write the normal equation(s).

- b. Fit a simple linear regression model relating Y to X.
- c. Estimate σ^2 .
- d. Write 95% confidence interval for β_1 .

Question 5

In a study of credit unions, the following regression was fitted to eighty-seven quarterly observations:

$$\log y = 0.47 + 0.31 \log x \quad \bar{R}^2 = 0.86 \quad (0.13)$$

where y = output of loans, x = expenditure on capital equipment

The figure in the parenthesis under the least square parameter estimate is the estimated standard error.

- a. Interpret the estimated regression coefficients.
- b. Construct and interpret a 95% confidence interval for the true coefficient on log x.
- c. Interpret the coefficient of determination.

Question 6

Assume the population regression function is as follows:

$$\begin{aligned} Y_i &= \beta \ln(X_t) + u_t \\ E(u_t) &= 0 \quad \text{where } t = 1, \dots, T \\ E(u_t^2) &= \sigma^2 \\ E(u_t u_s) &= 0 \quad \text{for all } t \neq s \end{aligned}$$

- a. Derive the OLS estimator of β .
- b. Derive the variance of β .