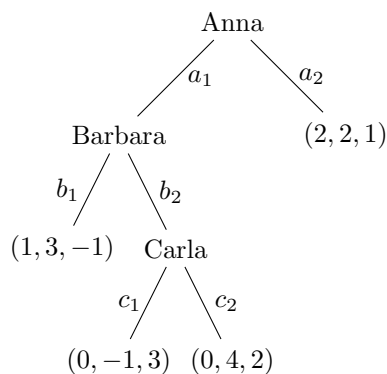
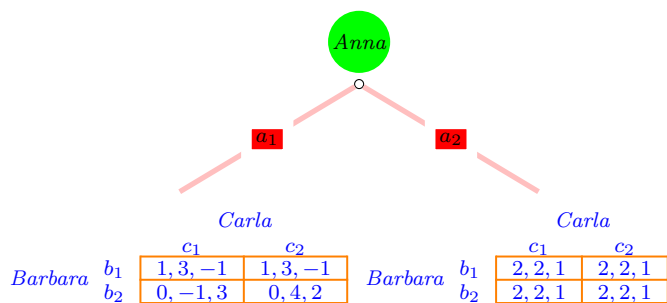


Problem 1: Anna, Barbara and Clara are playing the following extensive form game,



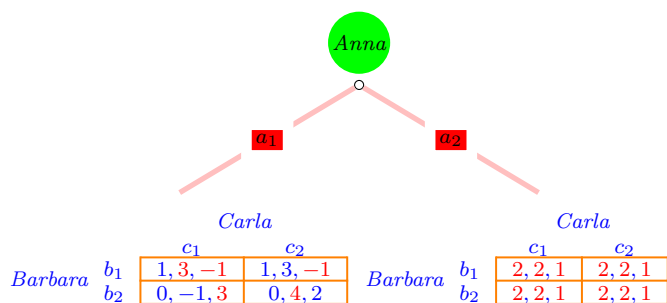
(a) Write the game in its normal form.

Solution:



(b) Find the pure strategy Nash equilibria of the game.

Solution: The best replies of the players are



The NE are the following.

- (a_2, b_1, c_1) .
- (a_2, b_1, c_2) .
- (a_2, b_2, c_1) .
- (a_2, b_2, c_2) .

(c) Find the proper subgames of the game and write them in their normal forms.

Solution:

- The game that starts with Barbara,

		Carla	
		c_1	c_2
Barbara	b_1	1, 3, -1	1, 3, -1
	b_2	0, -1, 3	0, 4, 2

- The game that starts with Carla,

		Carla	
		c_1	c_2
		0, -1, 3	0, 4, 2

- (d) Find the Nash equilibria of such subgames.

Solution:

- The NE of the game that starts with Barbara is (b_1, c_1) .
- The NE of the game that starts with Barbara is (c_1) .

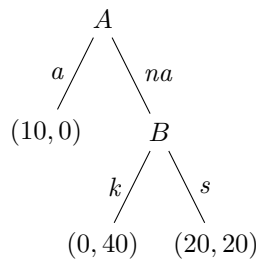
- (e) Find the subgame perfect Nash equilibria of the game.

Solution: (a_2, b_1, c_1) .

Problem 2: A firm's foundation has to choose whether to contribute to University A or to University B . Each University is interested only in the contributions it receives. The foundation announces that the contribution will be decided by the following mechanism. University A is proposed a contribution of \$10,000 for itself and \$0 for University B . If University A accepts, the contribution is assigned. Otherwise the Foundation increases the total contribution to \$40,000 and asks University B whether it wants to keep all the \$40,000 for itself or whether it prefers that the contribution is equally shared between the two Universities (\$20,000 to A and \$20,000 to B).

- (a) Draw the extensive form of the game (i.e., the tree whose nodes represent when players play and whose branches represent their decisions).

Solution:



- (b) Find the subgame perfect equilibrium of the game.

Solution: (a, k) .

- (c) Represent the game in its normal form (i.e., the matrix whose rows represent University A 's strategies and whose columns represent University B 's strategies, and the cells containing players' payoffs for any possible combination of strategies).

Solution: $S_A = \{a, na\}$, $S_B = \{k, s\}$. The normal game for is the following.

		B	
		k	s
A	a	10, 0	10, 0
	na	0, 40	20, 20

- (d) Find the pure-strategy Nash equilibria. Has one of the players a dominant or a dominated strategy?

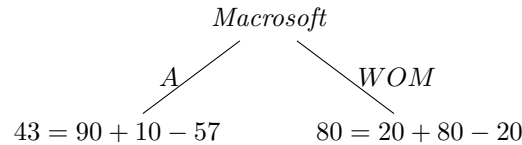
Solution: The NE is (a, k) . Strategy a is weakly dominated for player 2, but it is not strictly dominated.

Problem 3: Macrosoft has developed a new video game and it wants to launch it on the market. In particular the choice is between using an impacting advertising campaign or leaving the promotion to the communication between buyers and potential buyers ('word of mouth'). Macrosoft knows that the video game will last only 2 years on the market (as in 2 years newer and better video games will be introduced). It also knows that the sales do not depend on marketing strategy, but the temporal distribution of the sales do. The advertising campaign produces high sales in the first year but lower in the second due to market saturation. Using the word of mouth strategy, sales will be lower in the first year and higher in the second year. The following table describes Macrosoft's income depending on the strategy it adopts.

	Advertising	Word of mouth
Macrosoft first year income	900,000	200,000
Macrosoft second year income	100,000	800,000
Marketing cost	570,000	200,000

- (a) Draw the decision tree that represents Macrosoft problem and derive Macrosoft optimal decision.

Solution:

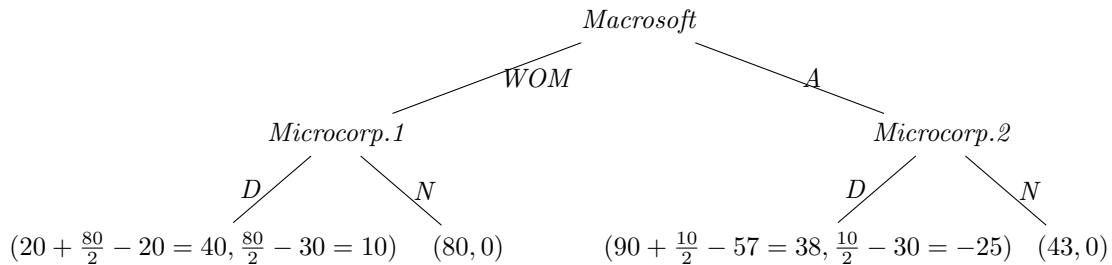


The optimal decision is WOM.

Assume now that a competitor, Microcorp, observes Macrosoft's strategy and can decide to develop a copy of the videogame in a year starting from the beginning of the promotion campaign. If Microcorp develops the copy, both firms share the market equally. The cost for Microcorp of developing the copy is 300.000.

- (b) Draw the extensive form of the game(the 'game tree').

Solution:



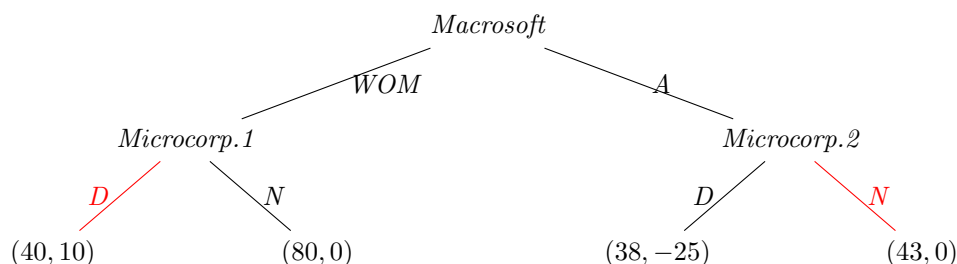
where

- WOM = Word of mouth.
- A = Advertise.
- DN = Do not develop.

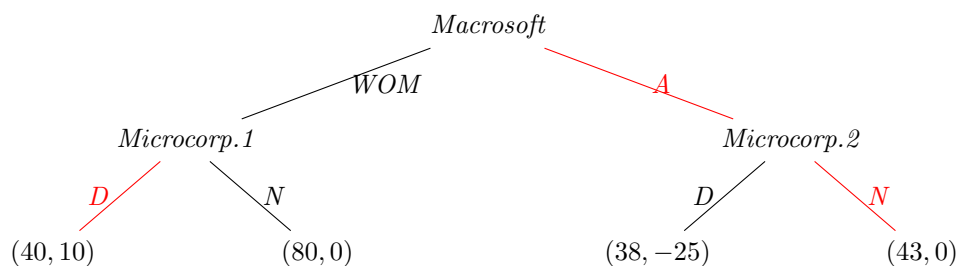
- $N = \text{Develop}$.

(c) Find the subgame perfect Nash equilibria of the game.

Solution: We will use the backwards induction procedure. We draw in red the best responses of the players. In the first step, Microcorp decides D at the node Microcorp.1 and N at the node Microcorp.2.



Given this, Macrosoft's best option is to choose A .



The SPNE is (A, DN) .

(d) What is Macrosoft's strategy space?

Solution: $S_{\text{Macrosoft}} = \{WOM, A\}$.

(e) What is Microcorp's strategy space?

Solution: $S_{\text{Microcorp}} = \{DD, DN, ND, NN\}$.

(f) What is the normal form of the game?

Solution:

		Microcorp			
		DD	DN	ND	NN
Macrosoft	WOM	40, 10	40, 10	80, 0	80, 0
	A	38, -25	43, 0	38, -25	43, 0

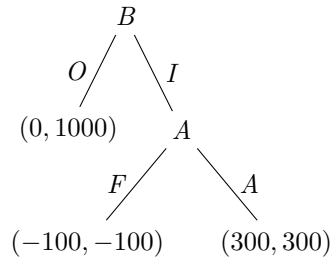
(g) Find the pure strategy Nash equilibria.

Solution: The set of NE is (A, DN) and (WOM, DD) . The NE (WOM, DD) is based on a non-credible threat.

Problem 4: Companies A and B are considering developing a new commercial aircraft. Suppose that A is ahead in the development process and that B is considering whether to enter the competition. If B stays out, it earns zero profit and A , being the monopolist, earns a profit of \$1 billion. If B decides to enter and develop the rival airplane, then A has to decide whether to accommodate B peaceably, or to wage a price war. In the event of peaceful competition, each firm will make a profit of \$300 million. If there is a price war, each will lose \$100 million.

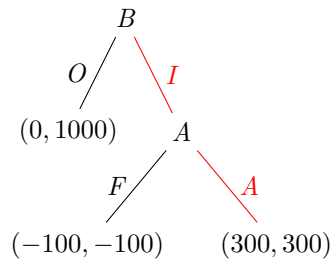
- (a) Draw the game tree ('extensive form') for this game.

Solution:



- (b) What is the subgame perfect Nash equilibrium?

Solution:



The SPNE is (I, A) . The payoffs are $u_A = u_B = 300$.

- (c) How many strategies does each player have?

Solution: $S_A = \{F, A\}$, $S_B = \{I, O\}$.

- (d) Write the normal form of the game.

Solution:

		A	
		A	F
B	I	300, 300	-100, -100
	O	0, 1000	0, 1000

- (e) Find all Nash equilibria of this game.

Solution: Let us look for a NE of the form

$$\begin{aligned}\sigma_1 &= xI + (1-x)O \\ \sigma_2 &= yA + (1-y)F\end{aligned}$$

We compute the expected utilities of the players

$$\begin{aligned}u_1(I, \sigma_2) &= 300y - 100(1-y) = 400y - 100 \\ u_1(O, \sigma_2) &= 0y + 0(1-y) = 0 \\ u_2(\sigma_1, A) &= 300x + 1000(1-x) = 1000 - 700x \\ u_2(\sigma_1, F) &= -100x + 1000(1-x) = 1000 - 1100x\end{aligned}$$

Note that

(a) $0 > 400y - 100$ for $0 < y < \frac{1}{4}$.

(b) $0 = 400y - 100$ for $y = \frac{1}{4}$.

(c) $0 < 400y - 100$ for $\frac{1}{4} < y \leq 1$.

Thus, we have that best reply of player 1 is

$$BR_1(\sigma_2) = \begin{cases} x = 0 & \text{if } 0 \leq y < \frac{1}{4} \\ [0, 1] & \text{if } y = \frac{1}{4} \\ x = 1 & \text{if } \frac{1}{4} < y \leq 1 \end{cases}$$

Note now that

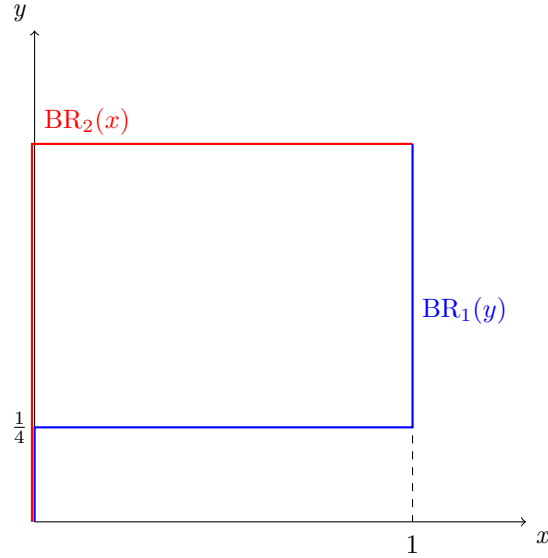
(a) $1000 - 700x > 1000 - 1100x$ for $x > 0$.

(b) $1000 - 700x = 1000 - 1100x$ for $x = 0$.

Thus, we have that best reply of player 2 is

$$BR_2(\sigma_1) = \begin{cases} y = 1 & \text{if } 0 < x \leq 1 \\ [0, 1] & \text{if } x = 0 \end{cases}$$

Graphically,



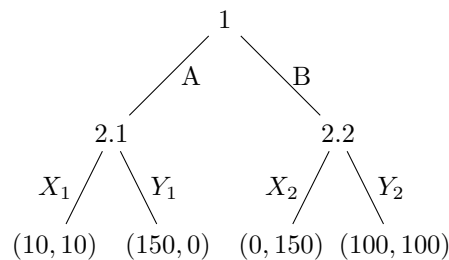
We see that there are the following NE,

- (O, F) with payoffs $u_A = 1000$, $u_B = 0$.
- (I, A) with payoffs $u_A = 300$, $u_B = 300$.
- $(O, yA + (1 - y)F)$ $0 \leq y \leq \frac{1}{4}$, with payoffs $u_A = 1000$, $u_B = 0$.

Only the NE (I, A) is a SPNE. All the other NE are based on non-credible strategies.

Problem 5: Find the subgame perfect Nash equilibria of the following games.

(a)



For this utility function and a lottery that is normally distributed (which is this case for large n), the agent's willingness to pay is always $\mu - \sigma^2/2$

In part b , we found that the agent would never buy into the company that only insures people like him. This is the equivalent of paying for a company to sell himself insurance. However, when the company insures many different people, the company's splits the risk of all of the people. Thus, a $1/N$ share of the company holds less risk as N increases. The agent that buys a share holds a part of everyone's risk, which is preferable to holding only his own risk.

2. Consider the game in which the following are commonly known. First, Ann chooses between actions a and b . Then, with probability $1/3$, Bob observes which action Ann has chosen and with probability $2/3$ he does not observe the action she has chosen. In all cases (regardless of whether he has observed Ann chose a , or he has observed Ann chose b , or he has not observed any action), Bob chooses between actions α and β . The payoff of each player is 1 after (a, α) and (b, β) and 0 otherwise.

(a) Write the above game in extensive form.

(b) Write the above game in normal form.

Strategy of Ann is simple: $\{a, b\}$. Strategy of Bob is $s^B = (s^1, s^2, s^3)$, where $s^1 \in \{\alpha, \beta\}$, $s^2 \in \{\alpha, \beta\}$, $s^3 \in \{\alpha, \beta\}$. s^1 denotes the choice of the information set (numbered as 1 in the picture) and s^2 is for node 2, and s^3 is for node 3. Utility of outcomes is in the following table.

	a	b
$\alpha\alpha\alpha$	1,1	0,0
$\alpha\alpha\beta$	1,1	$\frac{1}{3}, \frac{1}{3}$
$\alpha\beta\alpha$	$\frac{2}{3}, \frac{2}{3}$	0,0
$\alpha\beta\beta$	$\frac{2}{3}, \frac{2}{3}$	$\frac{1}{3}, \frac{1}{3}$
$\beta\alpha\alpha$	$\frac{1}{3}, \frac{1}{3}$	$\frac{2}{3}, \frac{2}{3}$
$\beta\alpha\beta$	$\frac{1}{3}, \frac{1}{3}$	1,1
$\beta\beta\alpha$	0,0	$\frac{2}{3}, \frac{2}{3}$
$\beta\beta\beta$	0,0	1,1

3. Consider the following variation of the above game. First, Ann chooses between actions a and b . Then, Bob decides whether to observe the chosen action of Ann or not, by choosing between the actions Open and Shut, respectively. In all cases, Bob then chooses between actions α and β . The payoff of Ann is 1 after (a, α) and (b, β) and 0 otherwise, regardless of whether Bob chooses Open or Shut. The payoff of Bob is equal to the payoff of Ann if he has chosen Shut, and his payoff is equal to the payoff of Ann minus $1/2$ if he has chosen Open.

(a) Write the above game in extensive form.

(b) Write the above game in normal form.

Strategy of Ann is simple: $\{a, b\}$. Strategy of Bob is $s^B = (s^1, s^2, s^3, s^4)$, where $s^1 \in$

$\{O, S\}$, $s^2 \in \{\alpha, \beta\}$, $s^3 \in \{\alpha, \beta\}$, $s^4 \in \{\alpha, \beta\}$. s^2 denotes the choice of the node 1 (the left one) and s^3 is for node 2, s^4 is for the information set on the right side (numbered as 3 in the picture). Utility of outcomes is in the following table.

	a	b
$O\alpha\alpha\alpha$	1, 0.5	0,-0.5
$O\alpha\alpha\beta$	1, 0.5	0,-0.5
$O\alpha\beta\alpha$	1, 0.5	1, 0.5
$O\alpha\beta\beta$	1, 0.5	1, 0.5
$O\beta\alpha\alpha$	0,-0.5	0,-0.5
$O\beta\alpha\beta$	0,-0.5	0,-0.5
$O\beta\beta\alpha$	0,-0.5	1, 0.5
$O\beta\beta\beta$	0,-0.5	1, 0.5
$S\alpha\alpha\alpha$	1,1	0,0
$S\alpha\alpha\beta$	0,0	1,1
$S\alpha\beta\alpha$	1,1	0,0
$S\alpha\beta\beta$	0,0	1,1
$S\beta\alpha\alpha$	1,1	0,0
$S\beta\alpha\beta$	0,0	1,1
$S\beta\beta\alpha$	1,1	0,0
$S\beta\beta\beta$	0,0	1,1

4. Federal government is planning to build an interstate highway between two states, named A and B . The highway costs $C > 0$ to the government, and the value of the highway to the states A and B are $v_A \geq 0$ and $v_B \geq 0$, respectively. Simultaneously, each state $i \in \{A, B\}$ is to bid $b_i \in [0, \infty)$. If $b_A + b_B \geq C$ the highway is constructed. For any distinct $i, j \in \{A, B\}$, state i pays $C - b_j$ to the federal government if $b_j < C \leq b_A + b_B$. (There is no payment otherwise.) The payoff of a state is the value of the highway to the state minus its own payment to the government if the highway is built, and 0 otherwise. (You can focus on the case $v_A + v_B < C$.)

- (a) Write this in the normal form.

Strategy of player i is choice of $b_i \in [0, \infty)$. Utility from strategy profile x of player i is

$$u_i(b_A, b_B) = v_i - C + b_j \text{ if } b_A + b_B \geq C \\ = 0 \text{ if } b_A + b_B < C$$

- (b) Check if there is a dominant strategy equilibrium, and compute it if there is one.

There is a unique dominant strategy equilibrium, (v_A, v_B) . In other words, bidding own value is the dominant strategy equilibrium.

From player A 's point of view, there are three cases. If $b_B \geq C$, then $u_A = v_A$, regardless of what b_A is. If $C - v_A \leq b_B < C$, then A wants to build the highway as $u_A = v_A - C + b_B \geq 0$. Thus, $b_A \geq C - b_B$ is the best response. Lastly, if $b_B < C - v_A$, then A does not want to build the highway as $u_A = v_A - C + b_B < 0$. The best response for this case is $b_A < C - b_B$.

Figure 1: Question 2

HW1-12, Problem 2

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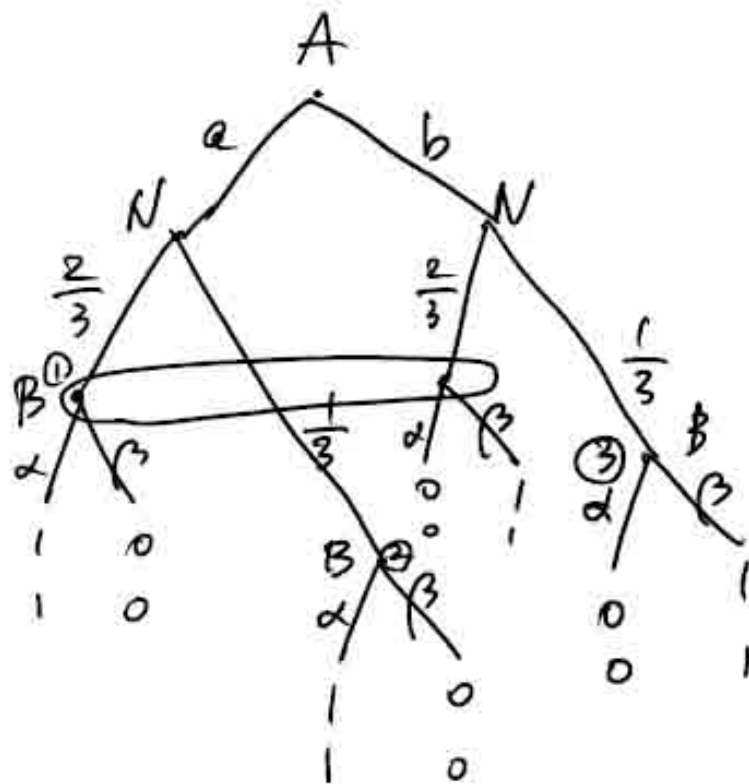


Figure 2: Question 3

HW1-12, Problem 3

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