

Problem Set V Solutions

Chapter 11

Question 1

Tom lives on an island and has 20 coconut trees in the current period, which currently produce 180 coconuts. Tom detests coconuts, but he can trade them with people on other neighboring islands for things that he wants. Further, Tom can borrow and lend coconuts with neighboring islands. In the coconut credit market, a loan of 1 coconut in the current period is repaid with 2 coconuts in the future period. Each period, Tom's trees produce, and then 10% of them die. If Tom plants a coconut in the ground in the current period, it will grow into a productive coconut tree in the future period. At the end of the future period, Tom can sell any remaining coconut trees for 1 coconut each. When Tom plants coconuts in the current period, he plants them in successively less fertile ground, and the less fertile the ground, the less productive is the coconut tree. For convenience, we assume here that fractions of coconuts can be produced by trees. Output in the future period, for given numbers of trees in production in the future period, is given in the following table:

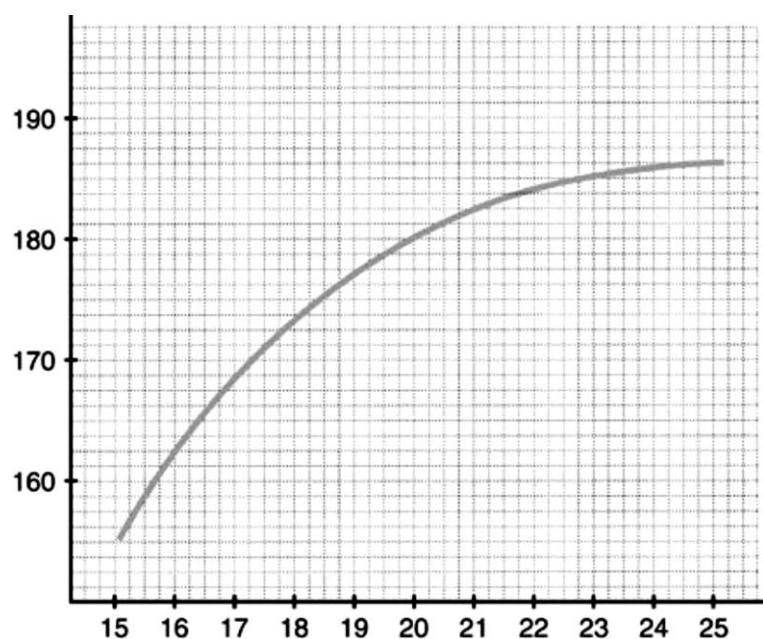
Trees in Production in the Future	Future Output of Coconuts
15	155
16	162
17	168
18	173
19	177
20	180
21	182
22	183.8
23	184.8
24	185.2
25	185.4

- (a) Plot the level of output against the quantity of capital for the future period.
- (b) Plot the marginal product of capital against the quantity of capital for the future period.
- (c) Calculate Tom's present value of profits given each quantity of future trees.
- (d) Calculate the net marginal product of capital for each quantity of future trees.
- (e) Determine Tom's optimal quantity of investment, and explain your results.

Solution:

The problem supplies the following production function, where future output only depends on the level of second-period capital, in this case the number of trees.

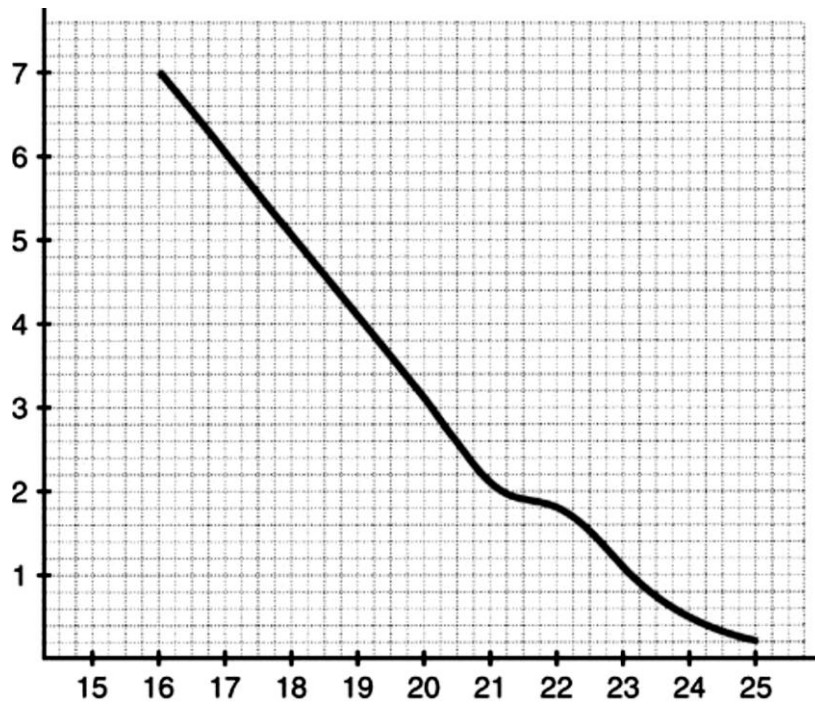
(a) The production function is depicted below.



(b) The marginal product of capital schedule is computed from the previous table. In table form:

Future Trees	Future Output	MP'_K
15	155.0	—
16	162.0	7.0
17	168.0	6.0
18	173.0	5.0
19	177.0	4.0
20	180.0	3.0
21	182.0	2.0
22	183.8	1.8
23	184.8	1.0
24	185.2	0.4
25	185.4	0.2

These data are plotted in the figure below.



- (c) Tom's first-year profits are equal to $\pi = Y - I$. The present value of second-year profits is equal to $\pi' = \frac{Y' - (1-d)K'}{1+r} = \frac{Y' - (1-d)K'}{2}$. These calculations are given in column V , below.
- (d) The net marginal product of capital is equal to $MPK' - d = MPK' - 0.1$. These calculations are also included in the table below.

Future Trees	Future Output	Required I	V	$MPK' - d$
15	155.0	-3	267.25	—
16	162.0	-2	270.20	6.9
17	168.0	-1	279.65	5.9
18	173.0	0	274.60	4.9
19	177.0	1	276.05	3.9
20	180.0	2	277.00	2.9
21	182.0	3	277.45	1.9
22	183.8	4	277.80	1.7
23	184.8	5	277.50	0.9
24	185.2	6	277.50	0.3
25	185.4	7	276.95	0.1

- (e) Tom's optimal level of V is equal to 277.80. To earn this amount of profit, Tom needs to plant 4 new trees. Note that at $I = 4$, $MPK' - d = 1.7 > r = 1.0$. Planting the 4th tree is therefore profitable. However, at $I = 5$, $MPK' - d = 0.9 < r = 1.0$. Planting the 5th tree is not profitable. The maximum V is therefore attained at the last tree for which $MPK' - d > r$.

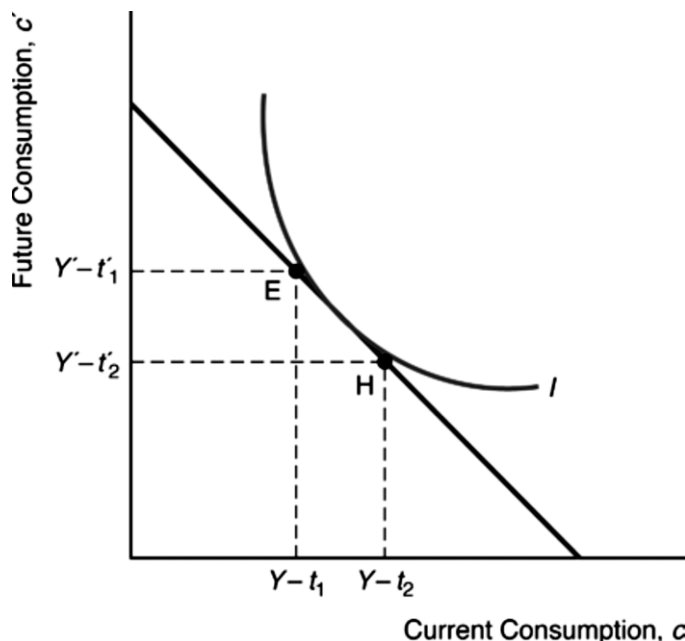
Question 2

The government decreases current taxes, while holding government spending in the present and the future constant.

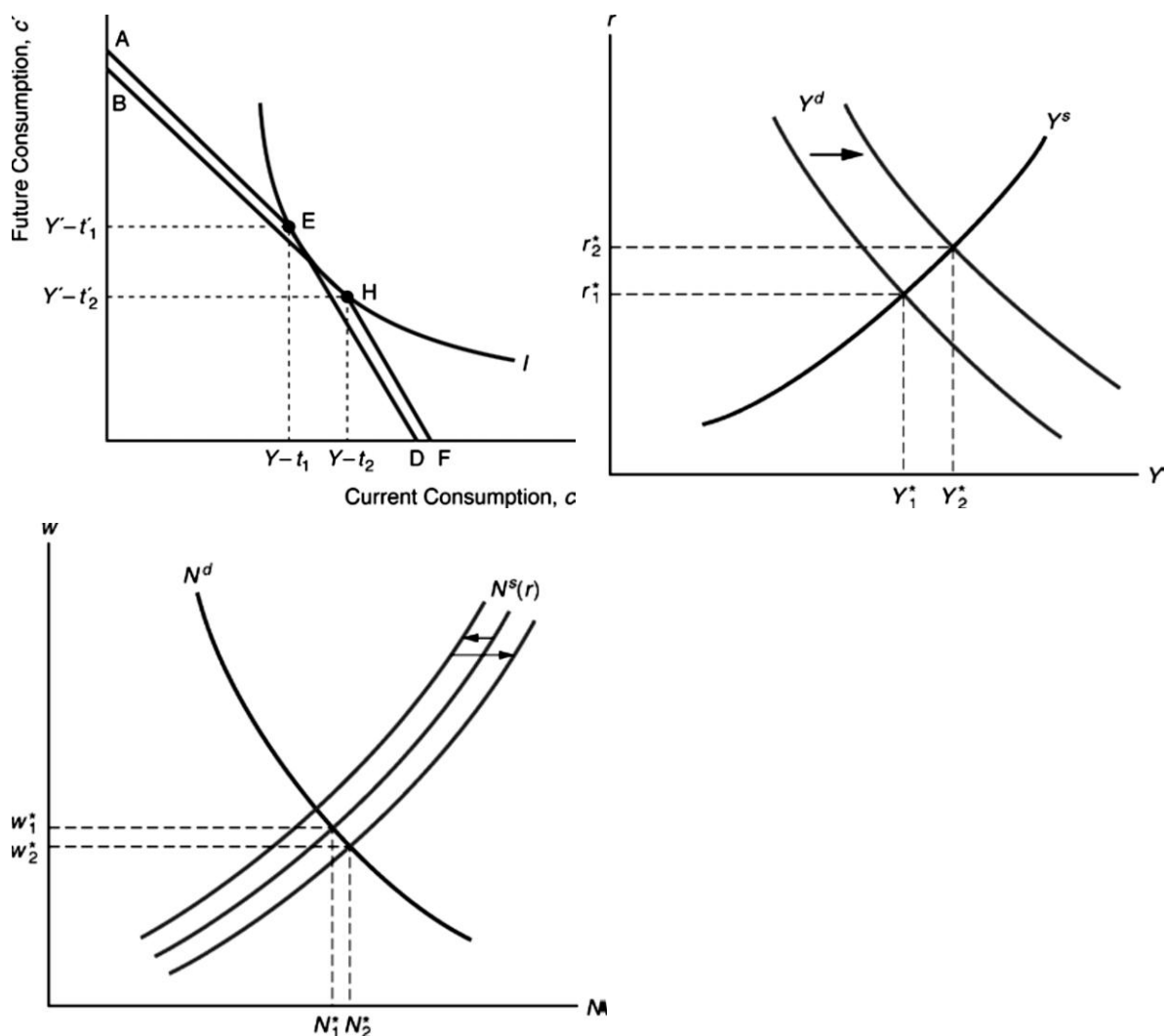
- (a) Using diagrams, determine the equilibrium effects on consumption, investment, the real interest rate, aggregate output, employment, and the real wage. What is the multiplier, and how does it differ from the government expenditure multiplier?
- (b) Now suppose that there are credit market imperfections in the market for consumer credit, for example due to asymmetric information in the credit market. Repeat part (a), and explain any differences in your answers in parts (a) and (b).

Solution:

- (a) As government expenses are unchanged, future taxes need to increase to satisfy the intertemporal budget constraint of the government. We are therefore in the context of the Ricardian equivalence. Thus, neither of the real interest rate, aggregate output, employment or the real wage is affected.



- (b) We are now violating the conditions of the Ricardian equivalence. There is potentially an impact. Indeed, the endowment point is moving towards the right, and some borrowers now become lenders, and thus face a lower interest rate. This leads to a positive income effect (and thus an increase in current consumption demand and a decrease in labor supply) and a substitution effect that increases demand for current goods. In then end, consumption demand increases and labor supply decreases.



The implications are that the interest rate increases, thus leading to a second shift in the labor supply, an increase that has to be larger than the initial decrease, as equilibrium output is up. In the end, aggregate output, consumption, labor and the real interest rate are all up, while the real wage is down.

Question 3

Determine how the following affect the slope of the output supply curve, and explain your results:

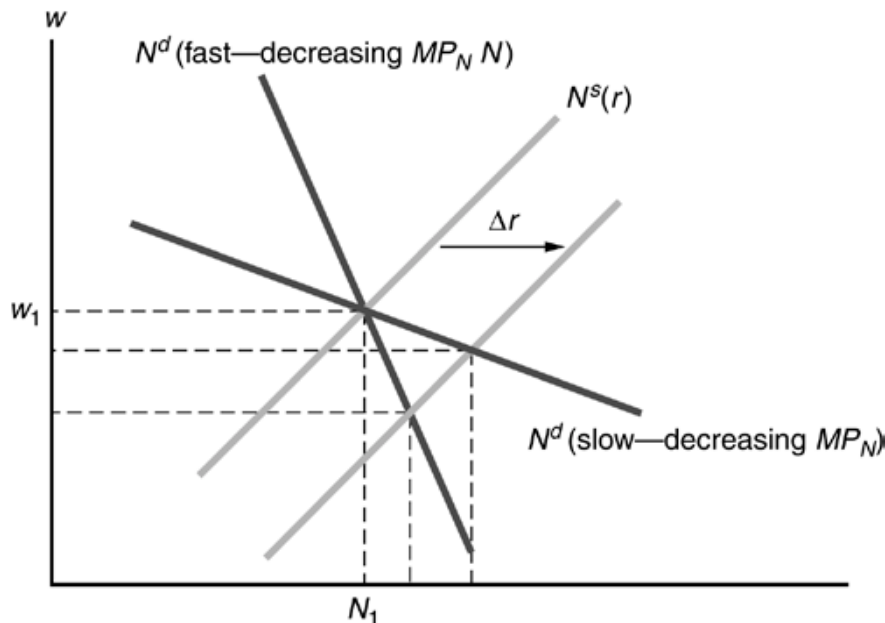
- (a) The marginal product of labor decreases at a faster rate as the quantity of labor used in production increases.
- (c) The intertemporal substitution effect of the real interest rate on current leisure decreases.

Solution:

Slope of the output supply curve.

- (a) The figure below depicts the effect of an increase in labor supply, due to an increase in the real interest rate, on the equilibrium level of employment. The diagram shows two

alternate labor demand curves with differing slopes. Note that the equilibrium level of employment increases more when the marginal product of labor declines at a slower rate with increases in the level of employment. Therefore, when the marginal product of labor declines at a faster rate as the quantity of labor used in production increases, there is a smaller increase in employment and therefore a smaller increase in output supply. The output supply curve is steeper in this case.



- (b) When the substitution effect of an increase in the real rate of interest decreases, there is a smaller effect on equilibrium employment of an increase in the real interest rate. Therefore, there is a smaller increase in output supply. The output supply curve is steeper in this case.

Question 4

Suppose that there is a shift in the representative consumer's preferences. Namely, the consumer prefers, given the market real interest rate, to consume less current leisure and more current consumption goods.

- (a) Determine the effects of this on current aggregate output, current employment, the current real wage, current consumption, and current investment.
- (b) Explain your results. What might cause such a change in the preferences of consumers?

Solution:

Labor supply shifts to the right, so output supply also shifts to the right. Consumption demand also increases, so the output demand curve must also shift to the right. Output must increase although the real rate of interest may rise or fall. In light of the increase in output, equilibrium employment must increase. A higher level of employment, in the absence of a shift in the labor demand curve, assures us that the real wage rate must also fall. Investment rises if the real rate of interest declines, and investment falls if the real rate of interest increases. Because output has increased, consumption will rise as long as investment remains the same or

declines. Consumption falls only in the case of a decline in the rate of interest of sufficient size to increase investment by more than the increase in output.

(a) To summarize: $Y \uparrow, N \uparrow, w \downarrow, r?, I?, C?$, but most likely increases.

(b) As one possibility, at low levels of nutrition, it may be infeasible for the consumer to work very much (a very high $MRS_{l,c}$). In this case, an increase in nutrition would make the consumer more willing (and able) to work more and consume more. One could also imagine some change in the technology of using leisure that is more goods intensive. In this case the value of leisure is low without a lot of consumption goods.

Question 5

Suppose that we modify the model of the firm's investment behavior by assuming that any capital the firm has remaining at the end of the period can be sold at the price p'_K (in our model we assumed the capital could be sold at a price of one, in terms of consumption goods).

(a) Determine how this change affects the optimal investment rule for the firm.

(b) Suppose that we interpret p'_K as the firm's stock price. If p'_K increases, what effect does this have on the firm's optimal investment schedule? What does this imply about the relationship between investment expenditures and stock prices?

Solution:

The new second-period profits of the firm are now $\pi' = Y' - w'N' + (1 - d)p'_K K$.

(a) The new marginal benefit from investment is now

$$MB(I) = (MP'_K + (1 - d)p'_K) / (1 + r)$$

As the marginal cost from investment remains at one, the new investment rule is then

$$MP'_K = (1 + r) - (1 - d)p'_K$$

(b) With an increase in p'_K , the marginal product of future capital needs to be reduced, thus more future capital is needed and investment rises. Indeed, as the liquidation value of capital goes up, you want to invest more in capital. Thus, investment is positively correlated with stock prices.