

2025-2026

ECON 205

Problem Set 3 – Answers

- 1) A large stockpile of used pumps contains 20% that are in need of repair. A maintenance worker is sent to the stockpile with three repair kits. She selects pumps at random and tests them one at a time. If the pump works, she sets it aside for future use. However, if the pump does not work, she uses one of her repair kits on it. Suppose that it takes 10 minutes to test a pump that is in working condition and 30 minutes to test and repair a pump that does not work. Find the mean and variance of the total time it takes the maintenance worker to use her three repair kits.

SOLUTION

Let Y denote the number of the trial on which the third nonfunctioning pump is found. It follows that Y has a negative binomial distribution with $p = .2$. Thus, $E(Y) = 3/ (.2) = 15$ and $V(Y) = 3(.8)/ (.2)^2 = 60$. Because it takes an additional 20 minutes to repair each defective pump, the total time necessary to use the three kits is

$$T = 10Y + 3(20)$$

Using the result derived in Exercise 3.33, we see that

$$E(T) = 10E(Y) + 60 = 10(15) + 60 = 210$$

and

$$V(T) = 10^2 V(Y) = 100(60) = 6000.$$

- 2) In southern California, a growing number of individuals pursuing teaching credentials are choosing paid internships over traditional student teaching programs. A group of eight candidates for three local teaching positions consisted of five who had enrolled in paid internships and three who enrolled in traditional student teaching programs. All eight candidates appear to be equally qualified, so three are randomly selected to fill the open positions. Let Y be the number of internship trained candidates who are hired.
- Does Y have a binomial or hypergeometric distribution? Why?
 - Find the probability that two or more internship trained candidates are hired.

SOLUTION

- (i) The random variable Y follows a hypergeometric distribution. The probability of being chosen on a trail is dependent on the outcome of previous trials.

(ii)
$$P(Y \geq 2) = P(Y = 2) + P(Y = 3) = \frac{\binom{5}{2}\binom{3}{1}}{\binom{8}{3}} + \frac{\binom{5}{3}}{\binom{8}{3}} = 0.5357 + 0.1786 = 0.7143$$

- 3) A parking lot has two entrances. Cars arrive at entrance I according to a Poisson distribution at an average of three per hour and at entrance II according to a Poisson distribution at an average of four per hour. What is the probability that a total of three cars will arrive at the parking lot in a given hour? (Assume that the numbers of cars arriving at the two entrances are independent.)

SOLUTION

Define: $Y_1 = \#$ of cars through entrance I $Y_2 = \#$ cars through entrance II. Thus, Y_1 is Poisson with $\lambda = 3$ and Y_2 is Poisson with $\lambda = 4$.

Then,

$$\begin{aligned} P(3 \text{ cars arrive}) &= P(Y_1 = 0, Y_2 = 3) + P(Y_1 = 1, Y_2 = 2) \\ &\quad + P(Y_1 = 2, Y_2 = 1) + P(Y_1 = 3, Y_2 = 0) \end{aligned} \quad (1)$$

By independence,

$$\begin{aligned}
 P(3 \text{ cars arrive}) &= P(Y_1 = 0)P(Y_2 = 3) + P(Y_1 = 1)P(Y_2 = 2) \\
 &\quad + P(Y_1 = 2)P(Y_2 = 1) + P(Y_1 = 3)P(Y_2 = 0)
 \end{aligned} \tag{2}$$

Let's find the Poisson probabilities for all outcomes by using the formula

$$p(y) = \frac{\lambda^y e^{-\lambda}}{y!}$$

$$P(Y_1 = 0) = \frac{3^0 e^{-3}}{0!} = e^{-3} = 0.0497 \tag{3}$$

$$P(Y_1 = 1) = \frac{3^1 e^{-3}}{1!} = 3e^{-3} = 0.1493 \tag{4}$$

$$P(Y_1 = 2) = \frac{3^2 e^{-3}}{2!} = \frac{9}{2} e^{-3} = \frac{0.4480}{2} = 0.224 \tag{5}$$

$$P(Y_1 = 3) = \frac{3^3 e^{-3}}{3!} = \frac{27}{6} e^{-3} = \frac{1.3444}{6} = 0.2241 \tag{6}$$

$$P(Y_2 = 0) = \frac{4^0 e^{-4}}{0!} = e^{-4} = 0.0183 \tag{7}$$

$$P(Y_2 = 1) = \frac{4^1 e^{-4}}{1!} = 4e^{-4} = 0.0732 \tag{8}$$

$$P(Y_2 = 2) = \frac{4^2 e^{-4}}{2!} = \frac{16}{2} e^{-4} = \frac{0.293}{2} = 0.1465 \tag{9}$$

$$P(Y_2 = 3) = \frac{4^3 e^{-4}}{3!} = \frac{64}{6} e^{-4} = \frac{1.172}{6} = 0.1953 \tag{10}$$

Therefore, we can calculate the probability that three cars arrive.

$$\begin{aligned}
 P(3 \text{ cars arrive}) &= (0.0497)(0.1953) + (0.1493)(0.1465) \\
 &\quad + (0.224)(0.0732) + (0.2241)(0.0183)
 \end{aligned} \tag{11}$$

$$P(3 \text{ cars arrive}) = 0.0521 \tag{12}$$

4) A geological study indicates that an exploratory oil well should strike oil with probability 0.2.

i) What is the probability that the first strike comes on the third well drilled?

SOLUTION

Let X be the number of well drilled on which first strike comes. We need to find $P(X = 3)$. X is a geometric random variable, so X is just a special case of a negative binomial random variable. Here's the calculation:

$$P(X = 3) = (0.8)^2 * (0.2) = 0.128 \quad (1)$$

(Alternatively, X could be defined as the number of failures (unsuccessful drills) that precede the 1st success (strike). In that case we would need to find $P(X = 2)$ and that would give us the same answer:

$$P(X = 2) = nb(2; 1, 0.2) = \binom{2 + 1 - 1}{1 - 1} (0.2)(0.8)^2 = 0.128 \quad (2)$$

ii) What is the probability that the third strike comes on the seventh well drilled?

Assuming independent drillings and probability 0.2 of striking oil with any one well, let Y denote the number of the trial on which the third oil strike occurs. Then it is reasonable to assume that Y has a negative binomial distribution with $p = 0.2$. Because we are interested in $r = 3$ and $y = 7$,

$$P(Y = 7) = p(7) = \binom{6}{2} (0.2)^3 (0.8)^4 = 0.049 \quad (3)$$

iii) Find the probability that the third oil strike comes on the fifth well drilled.

The negative binomial probability distribution with $p = 0.2$, $r = 3$, and $y = 5$

$$P(Y = 5) = p(5) = \binom{4}{2} (0.2)^3 (0.8)^2 = 0.0307 \quad (4)$$

iv) What assumptions did you make to obtain the answers to parts (i) and (ii)?

- The trials are independent.
- The probability of success is the same from trial to trial.

v) Find the mean and variance of the number of wells that must be drilled if the company wants to set up three producing wells.

$$\mu = E(Y) = \frac{r}{p} = \frac{3}{0.2} = 15$$

$$\sigma^2 = V(Y) = \frac{r(1-p)}{p^2} = \frac{3 \times 0.8}{0.04} = 60$$

5) A city government wants to compare two transportation policies:

- the current fuel tax (Policy A)
- a new congestion-pricing policy (Policy B) that charges drivers for entering busy areas.

To see which policy people find more attractive, the city randomly selects four citizens. Each person is shown three policy summaries in random order - two about Policy A and one about Policy B.

Each person then chooses the one they prefer. Assume that people see no real difference and they pick randomly among the three summaries independently from each other.

- Write the probability function of Y .
- Find the probability that at least three out of four people choose the new policy.
- Find the expected value of Y .
- Find the variance of Y .

SOLUTION

Each person has a $1/3$ chance of choosing Policy B (because there is one B out of three choices). Choices are independent, so

$$Y \sim \text{Binomial} \left(n = 4, p = \frac{1}{3} \right).$$

a)

$$P(Y = y) = \binom{4}{y} \left(\frac{1}{3} \right)^y \left(\frac{2}{3} \right)^{(4-y)}, \quad y = 0, 1, 2, 3, 4.$$

b)

$$\begin{aligned}P(Y \geq 3) &= P(Y = 3) + P(Y = 4) = \binom{4}{3} (1/3)^3 (2/3)^{(4-3)} + \binom{4}{4} (1/3)^4 (2/3)^{(4-4)} \\&= 8/81 + 1/81 = 9/81 = 1/9.\end{aligned}$$

c)

$$E[Y] = n.p = 4 \cdot \frac{1}{3} = \frac{4}{3}.$$

d)

$$\text{Var}(Y) = np(1-p) = 4 \cdot \frac{1}{3} \cdot \frac{2}{3} = \frac{8}{9}.$$

6) If $M_X(t) = e^{5(e^t-1)}$, then find $E(X)$ and $V(X)$.

SOLUTION

We can get $E(X)$ by getting the first moment and set $t = 0$,

$$M'_X(0) = M_X^{(1)}(0) = E(X)$$

$$M'_X(t) = \frac{d}{dt} [e^{5(e^t-1)}]$$

$$M'_X(t) = e^{5(e^t-1)} * \frac{d}{dt} [5(e^t - 1)]$$

$$M'_X(t) = e^{5(e^t-1)} * 5e^t$$

$$M'_X(t) = 5e^{5(e^t-1)+t}$$

$$M'_X(0) = 5e^{5(e^0-1)+0} = 5 \rightarrow E(X) = 5$$

$$M_X''(0) = M_X^{(2)}(0) = E(X^2)$$

$$M_X''(t) = \frac{d}{dt}[5e^{5(e^t-1)+t}]$$

$$M_X''(t) = 5e^{5(e^t-1)+t} * \frac{d}{dt}[5(e^t - 1) + t]$$

$$M_X''(t) = 5e^{5(e^t-1)+t} * (5e^t + 1)$$

$$M_X''(t) = 5(5e^t + 1)e^{5(e^t-1)+t}$$

$$M_X''(0) = 5(5e^0 + 1)e^{5(e^0-1)+0} = 30 \rightarrow E(X^2) = 30$$

$$V(X) = E(X^2) - [E(X)]^2$$

$$V(X) = 30 - 5^2 = 5$$

- 7) The manager of an industrial plant is planning to buy a new machine of either Type A or Type B. If t denotes the number of hours of daily operation, the number of daily repairs Y_1 required to maintain a machine of type A is a random variable with mean and variance both equal to $.10t$. The number of daily repairs Y_2 for a machine of type B is a random variable with mean and variance both equal to $.12t$. The daily cost of operating A is $C_A(t) = 10t + 30Y_1^2$; for B it is $C_B(t) = 8t + 30Y_2^2$. Assume that the repairs take negligible time and that each night the machines are tuned so that they operate essentially like new machines at the start of the next day. Which machine minimizes the expected daily cost if a workday consists of (a) 10 hours and (b) 20 hours?

SOLUTION

The expected daily cost for A is

$$\begin{aligned} E[C_A(t)] &= E[10t + 30Y_1^2] = 10t + 30E(Y_1^2) \\ &= 10t + 30\{V(Y_1) + [E(Y_1)]^2\} = 10t + 30[.10t + (.10t)^2] \\ &= 13t + .3t^2 \end{aligned}$$

In this calculation, we used the known values for $V(Y_1)$ and $E(Y_1)$ and the fact that $V(Y_1) = E(Y_1^2) - [E(Y_1)]^2$ to obtain that $E(Y_1^2) = V(Y_1) + [E(Y_1)]^2 = .10t + (.10t)^2$. Similarly,

$$\begin{aligned} E[C_B(t)] &= E[8t + 30Y_2^2] = 8t + 30E(Y_2^2) \\ &= 8t + 30\{V(Y_2) + [E(Y_2)]^2\} = 8t + 30[.12t + (.12t)^2] \\ &= 11.6t + .432t^2 \end{aligned}$$

Thus, for scenario (a) where $t = 10$,

$$E[C_A(10)] = 160 \text{ and } E[C_B(10)] = 159.2$$

which results in the choice of machine B .

For scenario (b), $t = 20$ and

$$E[C_A(20)] = 380 \text{ and } E[C_B(20)] = 404.8$$

resulting in the choice of machine A .

In conclusion, machines of type B are more economical for short time periods because of their smaller hourly operating cost. For long time periods, however, machines of type A are more economical because they tend to be repaired less frequently.