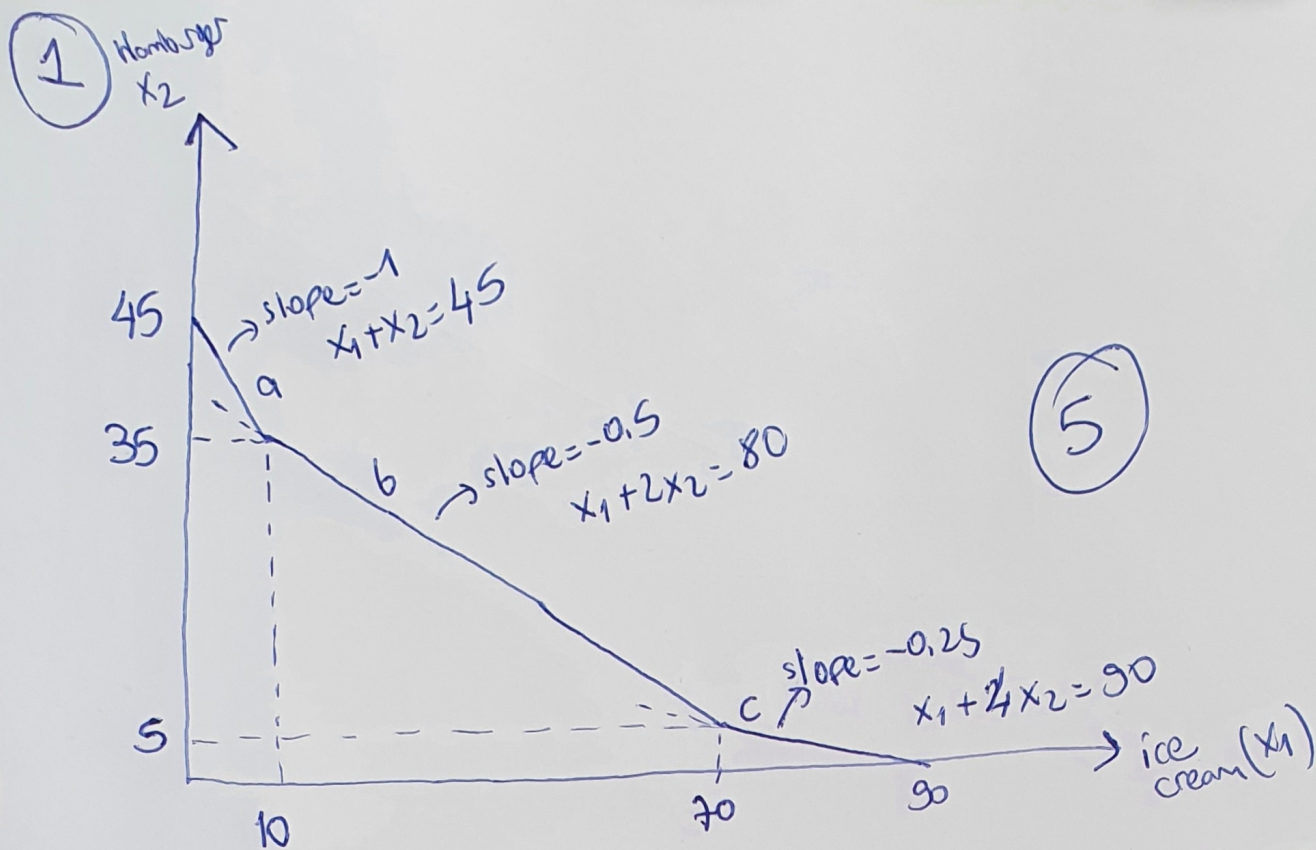




⑤

$$MRS^a = \frac{x_2}{x_1} = \frac{P_1}{P_2} = 1 \text{ s.t. } x_1 + x_2 = 45 \Rightarrow x_1^* = x_2^* = 22.5 \quad \times$$

①⑤

$$MRS^b = \frac{x_2}{x_1} = \frac{1}{2} \text{ s.t. } x_1 + 2x_2 = 80 \Rightarrow \begin{matrix} x_2^* = 20 \\ x_1^* = 40 \end{matrix} \quad \checkmark$$

$$MRS^c = \frac{x_2}{x_1} = \frac{1}{4} \text{ s.t. } x_1 + 4x_2 = 90 \Rightarrow \begin{matrix} x_2^* = \frac{90}{8} \\ x_1^* = 45 \end{matrix} \quad \times$$

$$2) \quad u(x) = 2 \underbrace{\min\{x_1, x_2\}}_{x_5, P_5 = P_1 + P_2} + 3 \underbrace{\max\{x_3, x_4\}}_{x_6, P_6 = \min\{P_3, P_4\}}$$

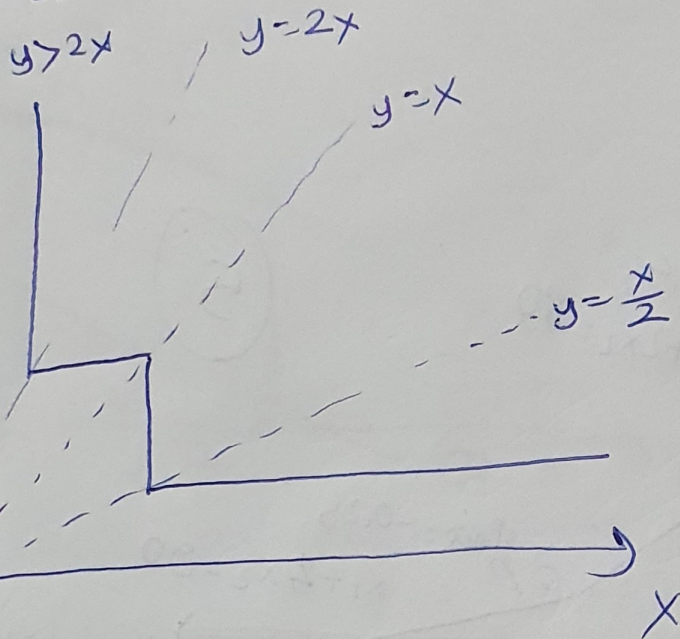
$$\frac{2}{3} > \frac{P_1 + P_2}{\min\{P_3, P_4\}} \Rightarrow x_6 = 0 = x_3 = x_4 \quad \wedge \quad x_1 = x_2 = \frac{I}{P_1 + P_2}$$

$$\frac{2}{3} < \frac{P_1 + P_2}{\min\{P_3, P_4\}} \Rightarrow x_1 = x_2 = 0 \quad \wedge \quad \begin{matrix} P_3 > P_4 \Rightarrow x_3 = 0, x_4 = \frac{I}{P_4} \\ P_4 > P_3 \Rightarrow x_3 = \frac{I}{P_3}, x_4 = 0 \\ P_3 = P_4 \Rightarrow \left(\frac{I}{P_3}, 0\right) \text{ or } \left(0, \frac{I}{P_4}\right) \end{matrix}$$

$$\frac{2}{3} = \frac{P_1 + P_2}{\min\{P_3, P_4\}} \Rightarrow x_1 = x_2 = \frac{I}{P_1 + P_2} \quad \wedge \quad \begin{matrix} P_3 > P_4 \Rightarrow x_3 = 0, x_4 = \frac{I - (P_1 + P_2) \cdot \frac{I}{P_1 + P_2}}{P_4} \\ P_4 > P_3 \Rightarrow x_3 = \frac{I - (P_1 + P_2) \cdot \frac{I}{P_1 + P_2}}{P_3}, x_4 = 0 \\ P_4 = P_3 \Rightarrow \text{both} \end{matrix}$$

3) $U(x,y) = \max \left\{ \min \left\{ x, \frac{y}{2} \right\}, \min \left\{ \frac{x}{2}, y \right\} \right\}$

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$$y > 2x \Rightarrow \max \left\{ x, \frac{x}{2} \right\} = x$$

$$2x > y > x \Rightarrow \max \left\{ \frac{y}{2}, \frac{x}{2} \right\} = \frac{y}{2}$$

$$x > y > \frac{x}{2} \Rightarrow \max \left\{ \frac{y}{2}, \frac{x}{2} \right\} = \frac{x}{2}$$

$$\frac{x}{2} > y \Rightarrow \max \left\{ \frac{y}{2}, y \right\} = y$$

4) $\max (E+1)^2(J+2) + \lambda(1760 - 240E - 80J)$

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$$\frac{\partial L}{\partial E} = 2(E+1)(J+2) - 240\lambda = 0$$

$$\frac{\partial L}{\partial J} = (E+1)^2 - 80\lambda = 0$$

$$\left. \begin{array}{l} \frac{\partial L}{\partial E} = 2(E+1)(J+2) - 240\lambda = 0 \\ \frac{\partial L}{\partial J} = (E+1)^2 - 80\lambda = 0 \end{array} \right\} \frac{2(J+2)}{E+1} = 3$$

$$\Downarrow$$

$$J = \frac{3E-1}{2}$$

$$1760 = 240E + 80\left(\frac{3E-1}{2}\right) = 360E - 40$$

$$\Rightarrow E^* = 5$$

5) a) utility function

MRS

$$MRS^V = \frac{2MU_x}{2MU_y} = MRS^U$$

False

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b) $\varepsilon = \frac{\% \Delta Q}{\% \Delta P}$

for giften goods $\varepsilon > 0$

False

5

$$6) \max \sqrt{x_1} + x_2 \quad \text{s.t.} \quad x_1 + 2x_2 \leq I \quad x_1 \geq 0 \\ x_2 \geq 0$$

a) (10) $L = \sqrt{x_1} + x_2 + \lambda(I - x_1 - 2x_2)$

$$L_{x_1} = \frac{1}{2\sqrt{x_1}} - \lambda \leq 0 \quad x_1 \geq 0 \quad x_1 \left(\frac{1}{2\sqrt{x_1}} - \lambda \right) = 0$$

$$L_{x_2} = 1 - 2\lambda \leq 0 \quad x_2 \geq 0 \quad x_2(1 - 2\lambda) = 0$$

$$L_{\lambda} = I - x_1 - 2x_2 \geq 0 \quad \lambda \geq 0 \quad \lambda(I - x_1 - 2x_2) = 0$$

b) (10) 2(iii) $x_2(1 - 2\lambda) = 0 \Rightarrow x_2 = 0$ or $\lambda = \frac{1}{2}$

Suppose $\lambda = \frac{1}{2}$. 1(ii) $\Rightarrow \frac{1}{2\sqrt{x_1}} \leq \frac{1}{2} \Rightarrow 1 \leq \sqrt{x_1} \Rightarrow$

$x_1 > 0$. Then, 1(iii) $\Rightarrow x_1 = 1$. 3(iii) $\Rightarrow$

$x_2 = \frac{I - 1}{2}$ and $I \geq 1$. Note that

2(iii) $\Rightarrow x_2 > 0$.

Suppose $x_2 = 0$. 2(i) $\Rightarrow \lambda > \frac{1}{2}$. Then, 3(iii) $\Rightarrow$

$x_1 = I$. 1(iii) $\Rightarrow \frac{1}{2\sqrt{I}} = \lambda > \frac{1}{2} \Rightarrow I < 1$

and $x_1 < 1$.

$\Rightarrow x_1^* = \min\{I, 1\}$