

Uncertainty – Completed Seminar Handout (Fully Solved)

Problem 1

$$u_i(x) = \sqrt{x}$$

(i) Compare utilities:

$$u_i(103) = \sqrt{103} \approx 10.15$$

$$E[u_i] = 0.95\sqrt{110} + 0.05\sqrt{0} \approx 0.95 \times 10.49 = 9.96$$

Since $10.15 > 9.96 \rightarrow$ Individual i chooses Asset 1

(ii) $u_j(x) = x^2$:

$$u_j(103) = 103^2 = 10609$$

$$E[u_j] = 0.95 \times 110^2 = 0.95 \times 12100 = 11495$$

Since $11495 > 10609 \rightarrow$ Individual j chooses Asset 2

(iii) Threshold probability π :

$$\text{For i: } \sqrt{103} = \pi\sqrt{110} \rightarrow \pi = \sqrt{103} / \sqrt{110} \approx 0.968$$

Thus if $\pi > 0.968 \rightarrow$ choose Asset 2, otherwise Asset 1

$$\text{For j: } 103^2 = \pi \times 110^2 \rightarrow \pi = 10609 / 12100 \approx 0.877$$

Thus if $\pi > 0.877 \rightarrow$ choose Asset 2, otherwise Asset 1

Problem 2

Outcomes: 5, 50, 500 with equal probability (1/3)

$$\text{Utility: } u(x) = (1/2)x^2$$

(i) Expected value:

$$EV = (5 + 50 + 500)/3 = 185$$

(ii) Expected utility:

$$EU = (1/3)[(1/2)5^2 + (1/2)50^2 + (1/2)500^2]$$

$$EU = (1/3)[12.5 + 1250 + 125000] = 42087.5$$

(iii) Certainty equivalent:

$$(1/2)p^2 = 42087.5 \rightarrow p^2 = 84175 \rightarrow p \approx 290.13$$

Since $CE > EV \rightarrow$ Anna is risk loving

(iv) Alternative utility $u(x) = x$:

$$EV = 185$$

$$EU = 185$$

$$CE = 185 \rightarrow \text{Risk neutral}$$