

①

	Excludable	Non-Excludable
Rival	Private Goods (Ex: Ice cream, cheese, houses, cars)	Common-Resources (Fresh water, fish, timber, pasture)
Non-Rival	Club Goods (Cable TV, cinemas, wifi, tollroads)	Public Goods (fresh air, knowledge, national defense)

Private Goods: - sold to consumers at a certain price before they can be used.
 - anyone who cannot afford to buy them is excluded from their consumption
 - likewise, the consumption of private goods by an individual prevents other individuals from consuming the same goods.

Public Goods: No one can be prevented from consuming them, and consumers can use them w/o reducing their availability to other consumers

Common Resources: - Virtually anyone can use them
 - If one individual consumes them, their availability to other consumers is reduced.

Club Goods: - Individuals can be prevented from consuming them (i.e., access can be restricted), but their consumption does not reduce their availability to other individuals (at least not until a point of overuse or congestion is reached).

ECON 203 — PS #3 Suggested Solutions

② $u_1(x_1, y) = x_1 + 6y$; $u_2(x_2, y_2) = x_2 y$; $C(y) = y^2$; $\omega_1 = (10, 0)$ & $\omega_2 = (20, 0)$

(a) MRS_1 & $MRS_2 = MC$ where $MRS_1 = 6$ and $MRS_2 = x_2/y$ & $MC = 2y$

$$\Rightarrow 6 + \frac{x_2}{y} = 2y \quad y = 4 \Rightarrow (\text{given in the question})$$

$$6 + \frac{x_2}{4} = 2 \cdot 4 \Rightarrow x_2 = 8 \longrightarrow x_2 + y_2 = 20 \text{ and } y_1 + y_2 = 16$$

Thus, $y_2 = 12$, $y_1 = 4$ & $x_1 = 6$

Note: $MRS_{y,x}$ — # of private good the consumer is willing to give up for 1 unit of the public good.

At the consumer side: $MRS_{y,x} = P_y/P_x$

At the ~~firm~~ (supply) side: $MRT_{y,x} = MC_y/MC_x = P_y/P_x$

Let's $P_x = 1 \Rightarrow MRS = MRT = P_y = MC_y$

(b) let y_1 and y_2 be the contribution to public good provision.

① max $x_1 + 6y$ s.t. $y_1 + y_2 = y^2$ and $x_1 + y_1 = 10$
 $\{x_1, y_1\}$

$$\max (10 - y_1) + 6\sqrt{y_1 + y_2}$$

$$\partial u / \partial y_1 = -1 + \frac{6}{2\sqrt{y_1 + y_2}} = 0 \Rightarrow y_1 + y_2 = 9$$

② max $(20 - y_2)\sqrt{y_1 + y_2} \Rightarrow \partial u / \partial y_2 = -\sqrt{y_1 + y_2} + \frac{(20 - y_2)}{2\sqrt{y_1 + y_2}} = 0$
 $\{x_2, y_2\}$

$$20 - y_2 = 2(\underbrace{y_1 + y_2}_{=9})$$

$$y_2 = 2 \text{ and } y_1 = 7$$

Thus, $(x_1^*, x_2^*, y^*) = (3, 18, 3)$

③ $\sum_i mrs_i = mc$ where $mrs_i = \frac{\partial u_i / \partial y}{\partial u_i / \partial x_i}$

$u_1(x_1, y) = x_1 + \frac{y}{2}$, $u_2(x_2, y) = x_2 + \sqrt{y}$, $u_3(x_3, y) = x_3 y$, $u_4(x_4, y) = x_4^2 y$

Then, $mrs_1 = \frac{1}{2}$

$mrs_2 = \frac{1}{2\sqrt{y}}$

$mrs_3 = \frac{x_3}{y}$

$mrs_4 = \frac{x_4}{2y}$

and $mc = 1$

$\frac{1}{2} + \frac{1}{2\sqrt{y}} + \frac{x_3}{y} + \frac{x_4}{2y} = 1 \Rightarrow y + \sqrt{y} + 2x_3 + x_4 = 2y$
 $2x_3 + x_4 = y - \sqrt{y}$ — set of Pareto optimal allocations

④ $u_1(x_1, y) = \min \{x_1, y\}$; $w_1 = (6, 0)$

$u_2(x_2, y) = \min \{x_2, y\}$; $w_2 = (4, 0)$

(a) Let y_1 and y_2 be the contribution to public good provision for each agent 1 and 2 respectively. Note that $y = y_1 + y_2$; $x_1 + y_1 = 6$ and $x_2 + y_2 = 4$

Agent 1: $u_1(x_1, y) = \min \{6 - y_1, y_1 + y_2\}$

utility is maximized when; $6 - y_1 = y_1 + y_2$

$y_1^* = \frac{6 - y_2}{2} \rightsquigarrow$ Best-response function for agent 1 (BR_1)

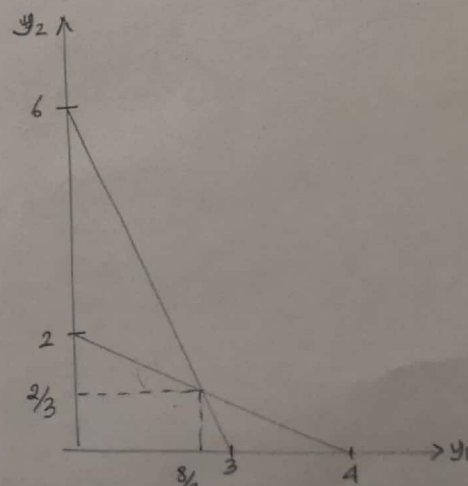
Agent 2: $u_2(x_2, y) = \min \{4 - y_2, y_1 + y_2\}$

utility is maximized when $4 - y_2 = y_1 + y_2$

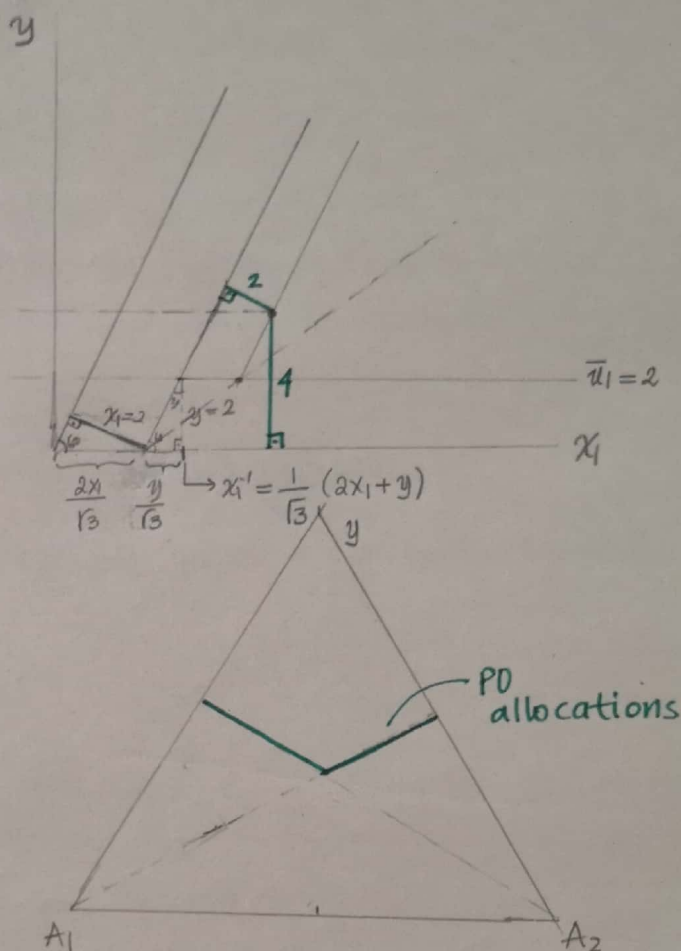
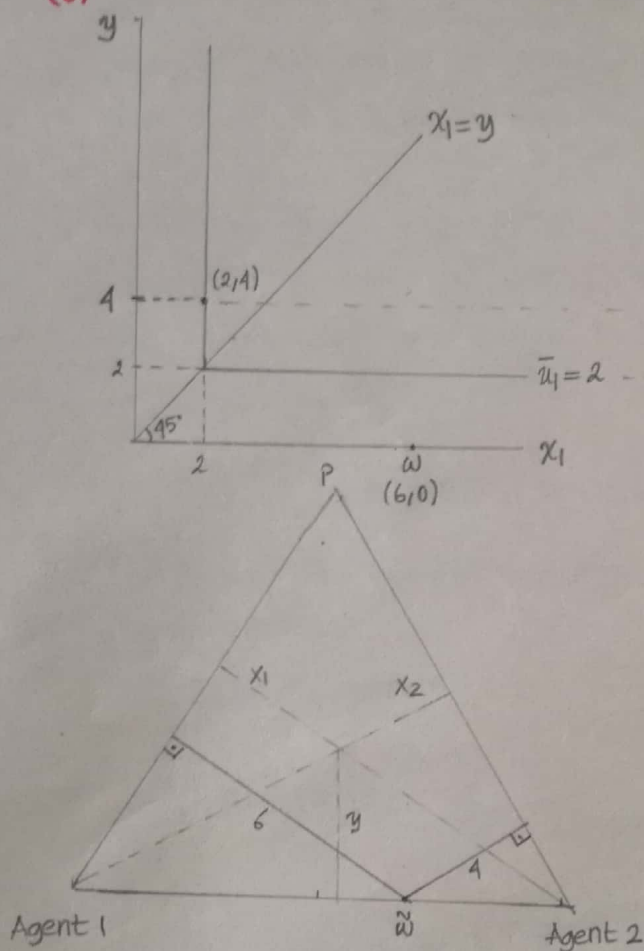
$y_2^* = \frac{4 - y_1}{2} \rightsquigarrow$ Best-response function for agent 2 (BR_2)

Intersection of BR_1 and BR_2 yields:

$y_1^* = \frac{8}{3}$, $y_2^* = \frac{2}{3}$, $x_1^* = \frac{10}{3}$, $x_2^* = \frac{10}{3}$



(b)



⑤

(a) Each resident $i \in \{\text{Ann, Bob}\}$ maximizes utility subject to her/his budget constraint, taking as given the amount of firemen provided by the other resident (denoted by j).

$$\max \{2 \ln(x_i) + \ln(M_i + M_j)\} \text{ s.t. } \{x_i + M_i \leq 200\}$$

The residents want to spend all of their money (it has no other use) so the budget constraint can be solved for $M_i = 200 - x_i$, leaving us with a maximization objective with just one variable:

$$2 \ln(x_i) + \ln(200 - x_i + M_j)$$

$$\text{The FOC is } \Rightarrow \frac{2}{x_i} - \frac{1}{200 - x_i + M_j} = 0$$

$$\text{solving for } x_i \text{ yields } x_i = \frac{400}{3} + \frac{2}{3} M_j.$$

$$\text{Plugging this into the budget constraint gives: } M_i = \frac{200}{3} - \frac{2}{3} M_j.$$

Given the symmetry of the problem, we know that $M_i = M_j$ in equilibrium, so $M_i = 40$, and the total number of firemen provided is $M^* = (M_i + M_j) = 80$. Residents use the rest of their budget to consume $X_i = X_j = 160$.

(b) To find the socially optimal number of firemen, we maximize the entire surplus of ~~Economy~~, which is simply the sum of the two residents' utilities subject to the budget constraint based on the income of both residents. Given the symmetry of the problem we can set $X_i = X_j = X$:

$$\max \{4 \ln(X) + 2 \ln(M)\} \text{ subject to } \{2X + M \leq 400\}$$

substituting $M = 400 - 2X$ from the budget constraint leaves a maximization objective with one variable:

$$2 \ln(X) + \ln(400 - 2X)$$

$$\text{The FOC is: } \frac{2}{X} - \frac{2}{400 - 2X} = 0$$

$$\text{solving for } X \text{ yields } X = \frac{400}{3} = 133 \frac{1}{3}$$

Plugging this back into the budget constraint gives:

$$M^{**} = 400 - 2 \frac{400}{3} = 133 \frac{1}{3}$$

Now, the free riding problem is solved, as the sum of the marginal rates of substitution (as opposed to the individual's marginal rate of substitution) is equated with the marginal rate of transformation. That is, the positive externality that each consumer has on the other is internalized.

(c) The tax reduces the income of each resident by 10. Residents must also take into account that the govt is now making sure that at least 20 firemen are provided. The individual maximization problem is now

$$\max \{2 \ln(X_i) + \ln(M_i + M_j + 20)\} \text{ s.t. } \{X_i + M_i \leq 190\}$$

substituting in M_i from the budget constraint, this becomes

$$2 \ln(X_i) + \ln(210 - X_i - M_j)$$

The FOC is: $\frac{2}{X_i} - \frac{1}{210 - X_i + M_j} = 0$

Solving for X yields: $X = \frac{420}{3} + \frac{2}{3} M_j$

Plugging this into the budget constraint gives $M_i = 50 - \frac{2}{3} M_j$.

Given the symmetry of the problem, we know that $M_i = M_j$ in eqn. So, $M_i = 30$, and the total number of firemen provided is $M^* = (30 + 30 + 20) = 80$.

This is the same as the answer in part (a), which is below the social optimum, so intervention has not done anything to solve the free rider problem. The tax reduces the residents' income, but does not lead them to internalize externality.

(d) Notice that the different tax for Ann and Bob means that the problem is no longer symmetric for the two residents.

Taking into account that the govt provides 75 firemen with the tax income, the maximization problem of Bob is:

$$\max \{2 \ln(X_B) + \ln(M_A + M_B + 75)\} \quad \text{s.t.} \quad \{X_B + M_B \leq 175\}$$

substituting in M_B from the budget constraint, this becomes

$$2 \ln(X_B) + \ln(250 - X_B + M_A)$$

The FOC is $\frac{2}{X_B} - \frac{1}{250 - X_B + M_A} = 0$

Solving for X yields $X_B = \frac{500}{3} + \frac{2}{3} M_A$.

Plugging this into the budget constraint gives $M_B = \frac{25}{3} - \frac{2}{3} M_A$

The maximization problem of Ann is:

$$\max \{2 \ln(X_A) + \ln(M_A + M_B + 75)\} \quad \text{subject to} \quad \{X_A + M_A \leq 150\}$$

Substituting in M_A from the budget constraint, this becomes

$$2 \ln(X_A) + \ln(225 - X_A + M_B)$$

The FOC is: $\frac{2}{X_A} - \frac{1}{225 - X_A + M_B} = 0$

Solving for X yields: $X_A = 150 + \frac{2}{3} M_B$.

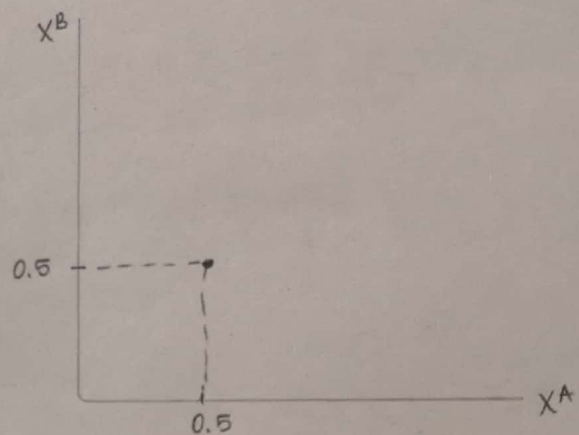
Plugging this into the budget constraint gives $M_A = -\frac{2}{3} M_B$. But it is impossible to have a negative contribution, so $M_A = 0$, resulting in $M_B = \frac{25}{3} = 8\frac{1}{3}$ and total $M = 8\frac{1}{3}$.

The problem would have been mathematically the same if Bob were richer than Ann by 25 to begin with, and the govt was taxing both equally. Then, with the govt. providing 75 firemen only Bob would find it worthwhile to contribute voluntarily for additional firemen.

(5) (a) Firm A maximizes profit function (suppose $x^B = 0.5$)

$$\pi^A = \ln(x^A + 0.5) - x^A$$

$$\frac{\partial \pi^A}{\partial x^A} = \frac{1}{x^A + 0.5} - 1 = 0 \Rightarrow x^A = 0.5$$



(b) Best-response function: $BR^A(x^B)$

$$\pi^A = \ln(x^A + x^B) - x^A$$

$$\frac{\partial \pi^A}{\partial x^A} = \frac{1}{x^A + x^B} - 1 = 0$$

$$x^A = BR^A(x^B) = 1 - x^B \text{ for } x^B \leq 1$$

$$BR^A(x^B) = 0 \text{ for } x^B > 1$$

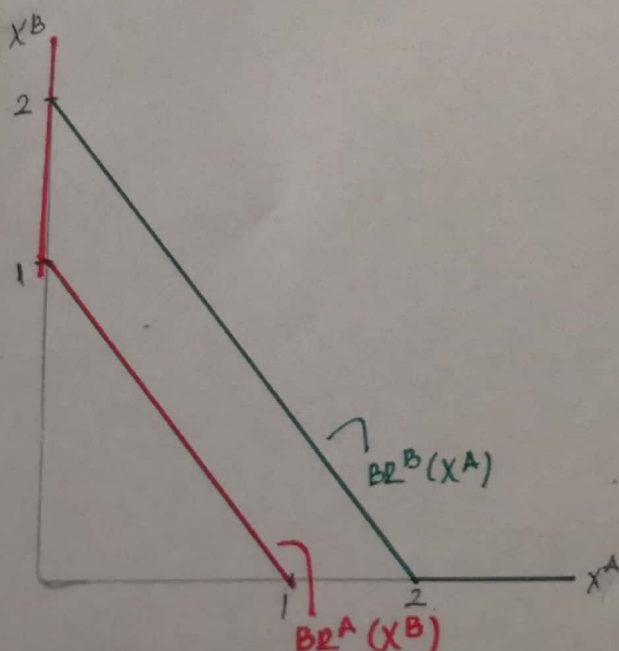
(c) $BR^B(x^A)$:

$$\pi^B = 2 \ln(x^A + x^B) - x^B$$

$$\frac{\partial \pi^B}{\partial x^B} = \frac{2}{x^A + x^B} - 1 = 0$$

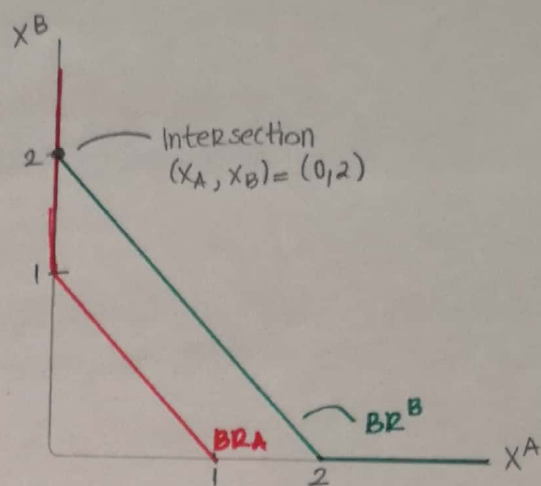
$$\Rightarrow x^B = BR^B(x^A) = 2 - x^A \text{ for } x^A \leq 2$$

$$BR^B(x^A) = 0 \text{ for } x^A > 2$$



(d) Intersection of best response functions represents the NE and there is only one such point: $(0, 2)$.

Total amount spent: $0 + 2 = 2$



(e) Yes, firm A is free riding on firm B's investment in the area - its customers enjoy the space and firm A's property worths more but it does not contribute to it.

(f) Pareto efficient investment can be determined by maximizing the joint / total profit of both firms:

$$\pi^{A+B} = \pi^A + \pi^B = [\ln(X^A + X^B) - X^A + 2 \ln(X^A + X^B) - X^B]$$

$$\text{FOC: } \frac{\partial \pi^{A+B}}{\partial X^A} = \frac{1}{X^A + X^B} - 1 - \frac{2}{X^A + X^B} = 0$$

$$\frac{3}{X^A + X^B} = 1 \Rightarrow X^A + X^B = 3$$

Optimal level of investment is $X=3$ which is greater than the one we found in part (d) ($X=2$), where agents underinvest.

⑧ (a) Net valuations For each person :

$$\left. \begin{array}{l} \text{Buket : } 60 - 70 = -10 \\ \text{Ceren : } 120 - 70 = 50 \\ \text{Demet : } 50 - 70 = -20 \end{array} \right\} \begin{array}{l} \text{If sum of net valuations} > 0, \text{ then it is efficient.} \\ \text{since } -10 + 50 - 20 > 0 \end{array}$$

(b) If we exclude Buket :

$$\text{Net value of Ceren} + \text{Net value of Demet} = 50 - 20 = 30 > 0$$

- Then, cleaning is still efficient. Outcome does not change. So, Buket is not pivotal.

• If we exclude Ceren:

$$NV_{\text{Buket}} + NV_{\text{Demet}} = -10 - 20 = -30 < 0, \text{ the outcome changes so, Ceren is pivotal}$$

• If we exclude Demet:

$$NV_{\text{Buket}} + NV_{\text{Ceren}} = -10 + 50 = 40 > 0, \text{ the outcome does not change. Demet is not pivotal.}$$

$$\text{The amount of tax: } \underbrace{60 - 70}_{\text{Externality for Buket}} + \underbrace{50 - 70}_{\text{Ext. for Demet}} = -30 \rightarrow \text{Ceren pays 30 as tax.}$$

(c) We need to check if this mechanism is truthful. Could any of these roommates increase payoff by lying?

Given Ceren & Demet tell the truth, what is Buket's incentive?

• Status-quo : If Buket is truthful $\Rightarrow$ they have the cleaning done, since Buket is (valuation = 60) not pivotal, she does not pay tax. The net value = -10

• If Buket states $V_B > 60 \Rightarrow$ they still get the cleaning done, Buket is not pivotal.

• " $60 > V_B > 40 \Rightarrow$

• If Buket states $V_B < 40 \Rightarrow$ they will not have the cleaning done, Buket is pivotal & $(60 - (2 \cdot 30 - 2 \cdot 10))$ imposes externality on others. The outcome has changed.

Buket gets 0 net value b/c the house is not cleaned. She imposes externality on others. She helps Demet by 20 ($= 50 - 70$) but deprives Ceren by 50 ($= 120 - 70$).

So, she pays a tax of 30.

$0 - 30 < -10 \rightarrow$ she won't lie!

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(a) Net valuations of each person:

$$\left. \begin{array}{l} \text{Zengin: } 110 - 90 = 20 \\ \text{Burcu: } 60 - 90 = -30 \\ \text{Duygu: } 125 - 90 = 35 \\ \text{Zeki: } 70 - 90 = -20 \\ \text{Ayşenur: } 100 - 90 = 10 \end{array} \right\}$$

If sum of net valuations > 0 , then it is efficient to buy.

Then, since

$$20 - 30 + 35 - 20 + 10 = 15 > 0$$

it is efficient to buy the machine.

(b) If we exclude:

- Zengin $\Rightarrow$ The total net value for the rest of T.A.'s is;

$$-30 + 35 - 20 + 10 = -5 < 0 \Rightarrow \text{Buying the machine is inefficient. Since the outcome changes, Zengin is pivotal.}$$

- Burcu $\Rightarrow nv_Z + nv_D + nv_Z + nv_A = 20 + 35 - 20 + 10 = 45 > 0$ the outcome does not change
Burcu is not pivotal.

- Duygu $\Rightarrow nv_Z + nv_B + nv_Z + nv_A = 20 - 30 - 20 + 10 = -20 < 0$ outcome changes
Duygu is pivotal.

- Zeki $\Rightarrow nv_Z + nv_B + nv_D + nv_A = 20 - 30 + 35 + 10 = 35 > 0$ outcome does not change
Zeki is not pivotal.

- Ayşenur $\Rightarrow nv_Z + nv_B + nv_D + nv_Z = 20 - 30 + 35 - 20 = 5 > 0$ outcome does not change
Ayşenur is not pivotal.

T.A.	Valuation	Pivotal or not	Tax
Zengin	110	pivotal	tax
Burcu	60	not pivotal	no tax
Duygu	125	pivotal	tax
Zeki	70	not pivotal	no tax
Ayşenur	100	not pivotal	no tax