

ECON 201 – Problem Set 8

- 1- Lelu runs a firm that sells multipasses to intergalactic cruises. Her short-run cost function is given by $C(q) = q^2 + 25q + 144$
 - a. If the market price is \$75/pass, how many units will Lelu produce?
 - b. At what price will Lelu earn zero profits?
 - c. If the price is below the level you found in b., will Lelu shut down? If so, explain. If not, below what price will she shut down?
- 2- Consider a competitive firm with the short-run cost function $C(q) = 20 + 6q + 5q^2$. The firm faces a market price of p for its output.
 - a. Derive the firm's profit maximizing condition. Is the sufficient second order condition satisfied?
 - b. Suppose a specific tax of t ($t < p$) is levied on only this firm in the industry. What is the profit maximizing level of output as a function of p and t ? (Assume the price is high enough that the firm does not shut down)
 - c. How does the output change as the tax increases? Use calculus to determine the relevant comparative static.
 - d. How does the firm's profit change as the tax increases? Again, use calculus to determine the relevant comparative static. Show that profit decreases as t increases.
- 3- Suppose the production function of digital cameras is given by $Q = (L^{1/2} + K^{1/2})^2$. What is the cost minimizing combination of labor and capital for a manufacturer that wants to produce 121,000 digital camera when prices are 10\$ per labor and 1\$ per unit of capital?
- 4- The production function is given by $Q = 2KL + K$. The price of L is 4\$ and of K is 5\$ per unit.
 - a. In the short run, capital is fixed at $K=9$. Find the amount of labor to produce 45 units of output.
 - b. How much does the firm sacrifice by not having the ability to choose the capital level optimally?

- 5- Burcu's cupcake factory employs workers (L) and cake-machines (K) according to the following production function: $f(K, L) = K^{0.5}L^{0.5}$. The hourly cost of cake-machine is \$10 and the hourly cost of workers is \$40.
- Write out the Lagrangian for the cost-minimization problem.
 - Derive the optimal cake-machine to labor ratio.
 - Suppose Burcu wishes to produce 1000 units of cupcakes. How much labor and cake-machine should she employ? How much will it cost to produce?
 - An order arrives doubling the amount of cupcake Burcu needs to produce. Assuming she is unable to purchase more machines, how much will it cost to meet the new production level?
 - In the long-run, Burcu will be able to employ more machine as well as labor. If she continues to produce 2000 units of cupcakes, how much will it cost in the long run?
 - Explain why the cost in the long run is different than in the short run.

- 6- Suppose that a firm has the following production function:

$$f(x, y) = x^{\frac{1}{2}} + y^{\frac{1}{2}}$$

- Find the unconditional input demand of this firm for each input.
- Find the supply function of the firm.
- Find the profit function.
- Find the conditional input demand of this firm for each input.
- Find the cost function of this firm.

Suppose that in short run units of x is fixed at 100.

- Find the unconditional input demand of this firm.
- Find the supply function of the firm.
- Find the profit function.
- Find the conditional input demand of this firm for y.
- Find the cost function of this firm.

- 7- Find the conditional input demand functions and cost functions for the following production functions:

- $f(x, y) = 2x + 3y$
- $f(x, y) = x^\alpha y^\beta z^\gamma - w^\theta$
- $f(x, y) = (2x + 3y)^{\frac{1}{2}}$
- $f(x, y) = \min\{2x, 3y\}$
- $f(x, y) = (\min\{2x, 3y\})^{\frac{1}{2}}$
- $f(a, b, c, d) = \min\{a, 2b\} + \max\{3c, 4d\}$
- $f(x, y) = \ln(xy)$
- $f(x, y) = \min\{x, y\} + \max\{x, y\}$

- 8- A firm producing hockey sticks has a production function given by $q = 2K^{1/2}L^{1/2}$. In the short run, the firm's amount of capital equipment is fixed at $K=100$. Input prices are as follows: $v=1\$$, $w=4\$$.
- Calculate firm's short run total cost curve. Calculate short run average cost curve.
 - What is firm's short run marginal cost curve? What are SC, SAC, SMC for the firm if it produces 25 hockey sticks?
 - Graph the SAC and SMC curves of the firm. Indicate the point you found in part b.
- 9- The production function of a competitive firm is described by the equation $y = 8x_1^{1/2}x_2^{1/2}$. The factor prices are $p_1 = \$1$ and $p_2 = \$4$ and the firm can hire as much of either factor it wants at these prices. Find the firm's marginal cost function and average cost function.
- 10- Answer the following:
- A firm has fixed costs of \$4,000. Its short-run production function is $y = 4x^{1/2}$, where x is the amount of variable factor it uses and y is the amount of output. The price of the variable factor is \$4,000 per unit. Find the short-run total cost function in terms of output.
 - The marginal cost curve of a firm is $MC = 6y$. What is the total variable cost to produce 10 units of output?
- 11- A firm's production function is given by $q = \min\{M, L^{1/2}\}$, where M is the number of machines and L is the amount of labor that it uses. The price of labor is \$1 and the price of machines is \$4 per unit. Find the equation of the firm's long-run marginal cost and average cost curve and draw them on a graph.