

Middle East Technical University
Department of Economics
Econ 206: Statistics for Economists-II

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Question 1

A vice president in charge of sales for a large corporation claims that salespeople are averaging no more than 15 sales contacts per week. As a check on his claim, $n = 36$ people are selected at random and the number of contacts made by each salesperson is recorded for a single randomly selected week. The mean and the variance of the 36 measurements from this random sample were 17 and 9 respectively.

1. Write down the null and alternative hypothesis. Is this a one-sided or two-sided test?
2. Construct a 95% confidence interval for the mean sales contacts per week for the whole population.
3. Does the evidence contradict the vice president's claim? Use a test with level $\alpha = 0.05$.

Solution:

1. **Hypotheses:** Since the claim is "no more than 15" ($\mu \leq 15$) and we are checking this claim using a sample mean that is higher (17), we set up the test as follows:

$$H_0 : \mu \leq 15$$

$$H_1 : \mu > 15$$

This is a **one-sided (right-tailed)** test.

2. **95% Confidence Interval (using t-distribution):** Given: $\bar{x} = 17$, $s^2 = 9 \implies s = 3$, $n = 36$. Degrees of freedom $df = n - 1 = 35$. Since the population standard deviation is unknown, we use the t-distribution. Looking up $t_{0.025}$ for $df = 35$:

$$t_{35,0.025} \approx 2.030$$

$$\bar{x} \pm t_{\alpha/2} \frac{s}{\sqrt{n}}$$

$$17 \pm 2.030 \left(\frac{3}{\sqrt{36}} \right) = 17 \pm 2.030 \left(\frac{3}{6} \right) = 17 \pm 2.030(0.5) \\ 17 \pm 1.015 \implies [15.98, 18.02]$$

3. **Test at $\alpha = 0.05$ (using t-test):**

- **Test Statistic (t):**

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{17 - 15}{3/6} = \frac{2}{0.5} = 4.0$$

- **Critical Value:** For a one-sided test at $\alpha = 0.05$ with $df = 35$, $t_{35,0.05} \approx 1.690$.
- **Conclusion:** Since $t_{stat}(4.0) > t_{critical}(1.690)$, we **reject** the null hypothesis. The evidence contradicts the vice president's claim; salespeople are making significantly more than 15 contacts on average.

Question 2

541 customers were asked about their satisfaction with a store and were asked to evaluate it on a scale from 5 (very bad) to 1 (very good). The sample mean was found to be $\bar{x} = 3.68$. From historical data, it is known that the standard deviation of the customer population is $\sigma = 1.21$. Assuming that an evaluation average of 3.75 or higher implies a store with positive customer satisfaction, test the null hypothesis that this year's population mean is 3.75 or higher against the alternative hypothesis that it is less than 3.75.

Solution:

- **Hypotheses:**

$$H_0 : \mu \geq 3.75$$

$$H_1 : \mu < 3.75$$

- **Given Data:** $\bar{x} = 3.68$, $\mu_0 = 3.75$, $n = 541$, $\sigma = 1.21$.
- **Test Statistic (Z):**

$$Z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \frac{3.68 - 3.75}{1.21/\sqrt{541}} = \frac{-0.07}{0.052} \approx -1.35$$

- **Conclusion:** The decision depends on the significance level (α). For example, the α value associated with $Z = -1.35$ is approximately 0.0885.
 - If $\alpha = 0.05$, we fail to reject H_0 (since $-1.35 > -1.645$).
 - If $\alpha = 0.10$, we reject H_0 (since $-1.35 < -1.28$).

Question 3

It is known from a fast food chain's sales that sales made in December are on average 20% higher than sales made in November. For the current year, 6 sales points were selected, and the percentage sales increases for December at these points are as follows:

$$19.2, \quad 18.4, \quad 19.8, \quad 20.2, \quad 20.4, \quad 19.0$$

Assuming the population is normally distributed, test the null hypothesis that the mean sales increase is 20% against the two-sided alternative hypothesis at a 10% significance level.

Solution:

- **Hypotheses:**

$$H_0 : \mu = 20$$

$$H_1 : \mu \neq 20$$

- **Sample Statistics:** Using the data: $\{19.2, 18.4, 19.8, 20.2, 20.4, 19.0\}$

$$\bar{x} = \frac{117}{6} = 19.5$$

$$s^2 = \frac{\sum (x_i - \bar{x})^2}{n - 1} = 0.588 \implies s = 0.767$$

- **Test Statistic (t):**

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{19.5 - 20}{0.767/\sqrt{6}} = \frac{-0.5}{0.313} \approx -1.597$$

- **Critical Value:** For a two-sided test at $\alpha = 0.10$ with $df = n - 1 = 5$:

$$t_{5,0.05} = 2.015$$

- **Conclusion:** Since $|-1.597| < 2.015$, the test statistic falls within the acceptance region. We **fail to reject** the null hypothesis. There is not enough evidence to claim the mean increase is different from 20%.

Question 4

It is required that the variance of the percentage of unwanted chemical substances in a detergent should not exceed 4%. When 20 random detergent boxes were selected, the sample variance was observed to be $s^2 = 5.62$. Test the null hypothesis that the population variance is not more than 4% at a 10% significance level.

Solution:

- **Hypotheses:**

$$H_0 : \sigma^2 \leq 4$$

$$H_1 : \sigma^2 > 4$$

- **Given Data:** $n = 20$, $s^2 = 5.62$, $\sigma_0^2 = 4$, $\alpha = 0.10$.

- **Test Statistic (χ^2):**

$$\chi^2 = \frac{(n-1)s^2}{\sigma_0^2} = \frac{(20-1) \cdot 5.62}{4} = \frac{19 \cdot 5.62}{4} \approx \mathbf{26.7}$$

- **Critical Value:** From the Chi-Square table with $df = 19$ and $\alpha = 0.10$:

$$\chi_{19,0.10}^2 = 27.2$$

- **Conclusion:** Since the test statistic (26.7) is less than the critical value (27.2), we **fail to reject** the null hypothesis. There is insufficient evidence to conclude that the variance exceeds 4%.

Question 5

In a study investigating the effects of planning on the financial performance of banks, a random sample of 6 "professional planners" showed a mean annual percentage increase in revenue of 9.972 with a standard deviation of 7.470. In another independent random sample of 9 "non-professional planners," the mean percentage increase was observed to be 2.098 with a standard deviation of 10.834. Assuming both populations are normally distributed and have equal variances, find the 90% confidence interval for the difference between the means of these populations.

Solution:

- **Given Data:**

- Sample X (Professional): $n_x = 6$, $\bar{x} = 9.972$, $s_x = 7.470$
- Sample Y (Non-professional): $n_y = 9$, $\bar{y} = 2.098$, $s_y = 10.834$
- Assumption: $\sigma_x^2 = \sigma_y^2$ (Variances are unknown but equal)

- **Calculate Pooled Variance (s_p^2):**

$$s_p^2 = \frac{(n_x - 1)s_x^2 + (n_y - 1)s_y^2}{n_x + n_y - 2} = \frac{5(7.470)^2 + 8(10.834)^2}{13} = 93.693$$

$$s_p = \sqrt{93.693} = 9.68$$

- **Critical Value:** For a 90% confidence interval ($\alpha = 0.10$) with degrees of freedom $df = n_x + n_y - 2 = 13$:

$$t_{13,0.05} = 1.771$$

- **Confidence Interval Calculation:**

$$(\bar{x} - \bar{y}) \pm t_{\alpha/2} \cdot s_p \sqrt{\frac{1}{n_x} + \frac{1}{n_y}}$$

$$(9.972 - 2.098) \pm 1.771 \cdot 9.68 \sqrt{\frac{1}{6} + \frac{1}{9}}$$

$$7.874 \pm 1.771 \cdot 9.68 \cdot \sqrt{0.277}$$

The interval is:

$$1.161 < \mu_X - \mu_Y < 16.909$$

Question 6

A market research company would like to estimate the average earnings of people with an MBA degree. The earnings are normally distributed. A random sample of 400 people with an MBA degree were selected at random and it was calculated that their mean monthly earnings are 5067 TL with a standard deviation of 985 TL.

1. Establish a 99% confidence interval for the mean earnings of all individuals with an MBA degree.
2. Suppose that Turkstat claims that the average earnings are 4961 TL in 2018 but the market research company believes that the earnings are higher. Formulate the hypothesis and test whether there is evidence supporting Turkstat's claim at a 10% significance level.

Solution:

1. **99% Confidence Interval (using t-distribution):** Given: $n = 400$, $\bar{x} = 5067$, $s = 985$. Degrees of freedom $df = n - 1 = 399$. For 99% confidence, $\alpha = 0.01$, so we look up $t_{0.005}$ for $df = 399$:

$$t_{399,0.005} \approx 2.588$$

$$\bar{x} \pm t_{0.005} \frac{s}{\sqrt{n}}$$

$$5067 \pm 2.588 \left(\frac{985}{\sqrt{400}} \right) = 5067 \pm 2.588 \left(\frac{985}{20} \right)$$

$$5067 \pm 2.588(49.25) \approx 5067 \pm 127.47$$

$$[4939.53, 5194.47]$$

2. **Hypothesis Test ($\alpha = 0.10$ using t-test):** The company believes earnings are *higher* than 4961 TL.

• **Hypotheses:**

$$H_0 : \mu = 4961$$

$$H_1 : \mu > 4961$$

• **Test Statistic (t):**

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{5067 - 4961}{49.25} = \frac{106}{49.25} \approx \mathbf{2.152}$$

- **Critical Value:** For a one-sided test at $\alpha = 0.10$ with $df = 399$, $t_{399,0.10} \approx 1.284$.

- **Conclusion:** Since $t_{stat}(2.152) > t_{critical}(1.284)$, we **reject** the null hypothesis. There is sufficient evidence at the 10% level to support the market research company's belief that earnings are higher than 4961 TL.