

INTERMEDIATE

8TH EDITION

MICROECONOMICS

HAL R. VARIAN

20

Cost Minimization



Cost Minimization

- **A firm is a cost-minimizer if it produces any given output level $y \geq 0$ at smallest possible total cost.**
- **$c(y)$ denotes the firm's smallest possible total cost for producing y units of output.**
- **$c(y)$ is the firm's total cost function.**



Cost Minimization

- When the firm faces given input prices $w = (w_1, w_2, \dots, w_n)$ the total cost function will be written as $c(w_1, \dots, w_n, y)$.



The Cost-Minimization Problem

- Consider a firm using two inputs to make one output.
- The production function is
$$y = f(x_1, x_2).$$
- Take the output level $y \geq 0$ as given.
- Given the input prices w_1 and w_2 , the cost of an input bundle (x_1, x_2) is

$$w_1 x_1 + w_2 x_2.$$

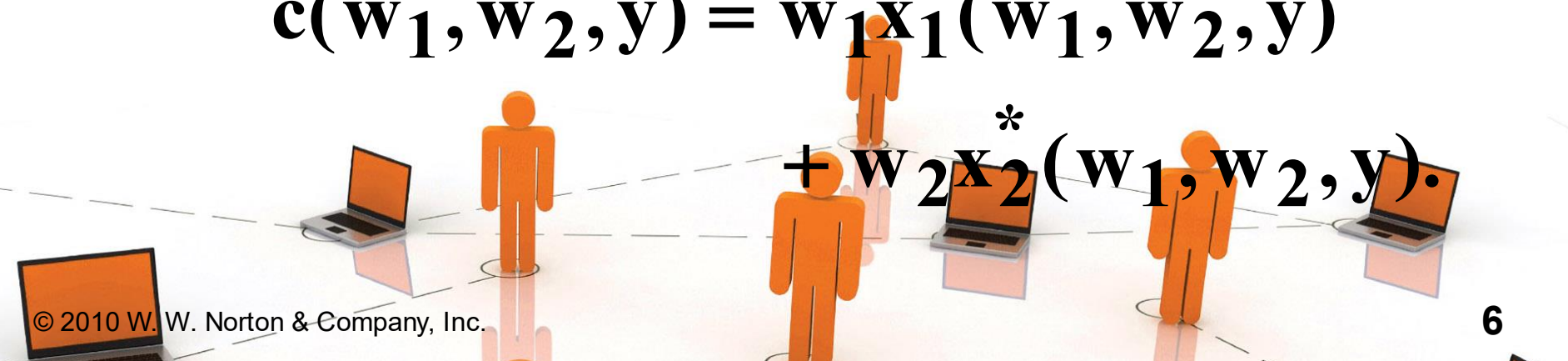
The Cost-Minimization Problem

- For given w_1 , w_2 and y , the firm's cost-minimization problem is to solve
- $$\min_{x_1, x_2 \geq 0} w_1 x_1 + w_2 x_2$$
- subject to $f(x_1, x_2) = y$.



The Cost-Minimization Problem

- The levels $x_1^*(w_1, w_2, y)$ and $x_2^*(w_1, w_2, y)$ in the least-costly input bundle are the firm's conditional demands for inputs 1 and 2.
- The (smallest possible) total cost for producing y output units is therefore

$$c(w_1, w_2, y) = w_1 x_1^*(w_1, w_2, y) + w_2 x_2^*(w_1, w_2, y).$$
An illustration at the bottom of the slide shows a network of five orange stick figures standing on a light-colored floor. Each figure is connected to a laptop by a dashed line. The laptops are positioned around the figures, and the dashed lines form a network connecting them. The equation is overlaid on this illustration.

Conditional Input Demands

- **Given w_1 , w_2 and y , how is the least costly input bundle located?**
- **And how is the total cost function computed?**



Iso-cost Lines

- A curve that contains all of the input bundles that cost the same amount is an iso-cost curve.
- E.g., given w_1 and w_2 , the \$100 iso-cost line has the equation
$$w_1x_1 + w_2x_2 = 100.$$



Iso-cost Lines

- Generally, given w_1 and w_2 , the equation of the \$c iso-cost line is
$$w_1x_1 + w_2x_2 = c$$

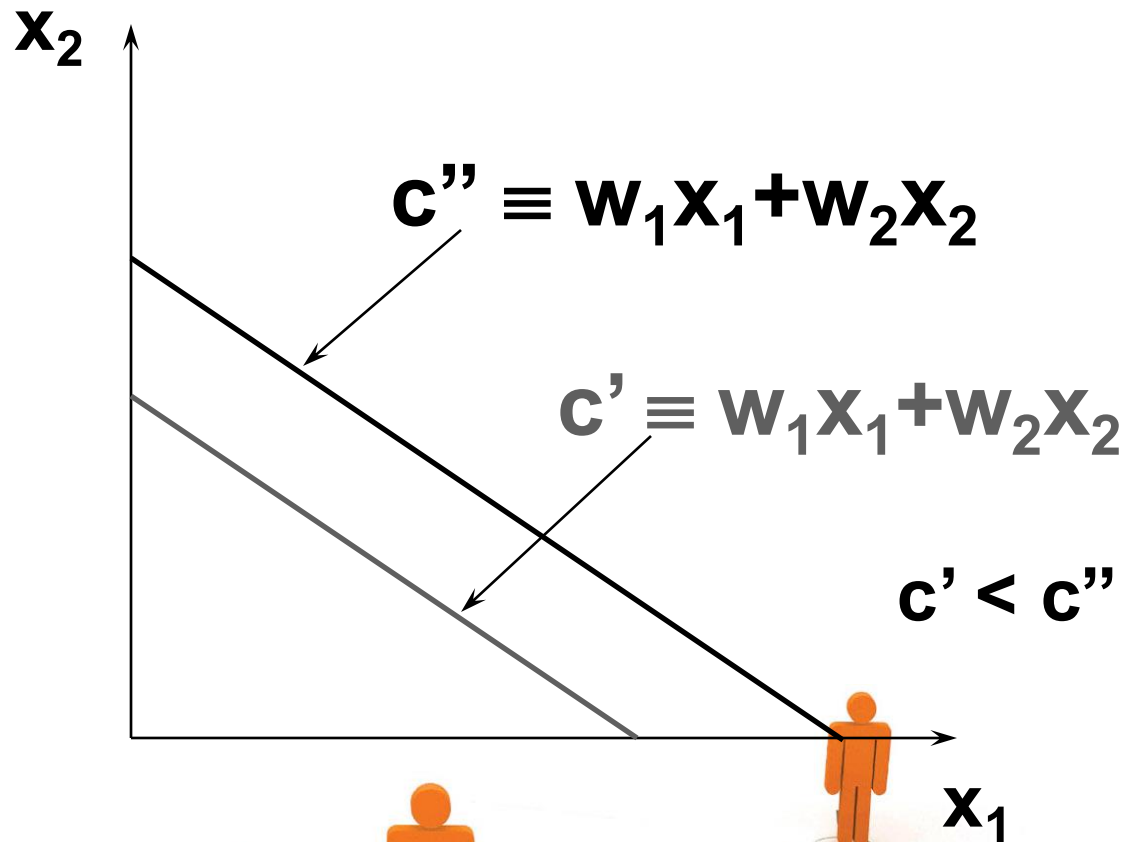
i.e.

$$x_2 = -\frac{w_1}{w_2}x_1 + \frac{c}{w_2}.$$

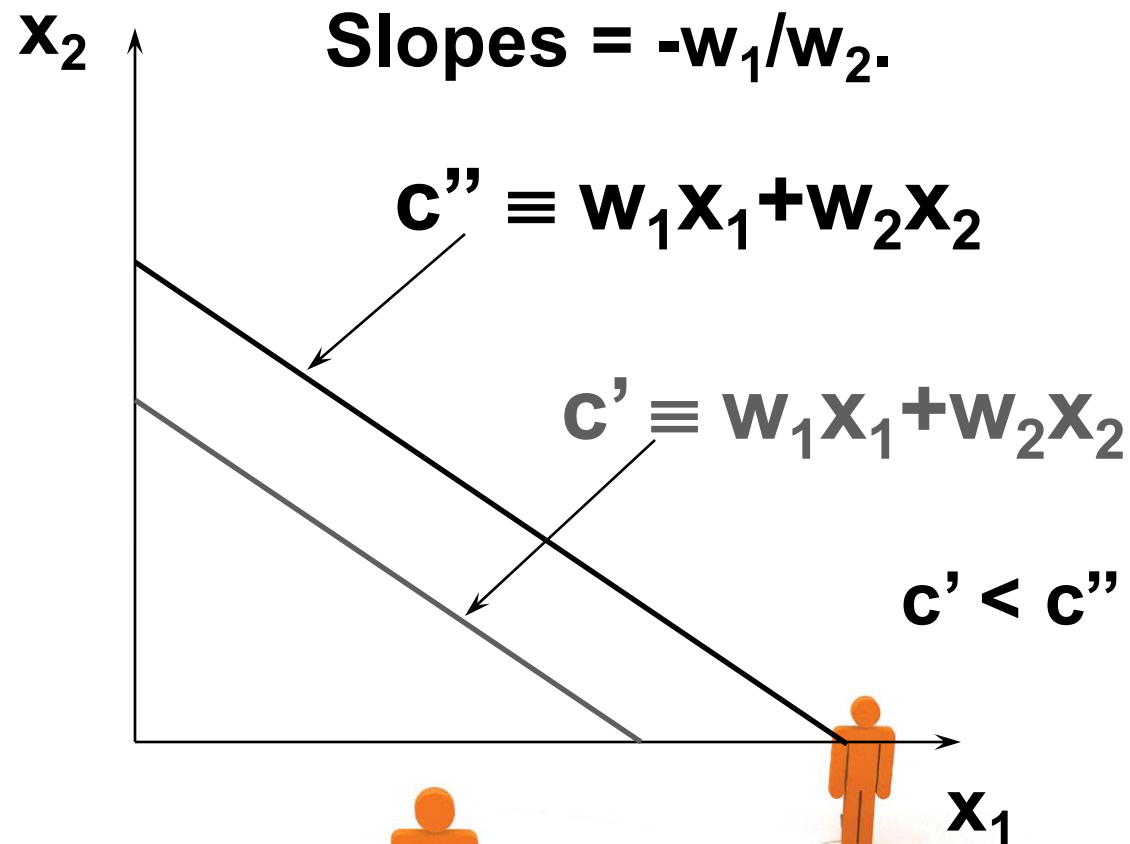
- Slope is $-w_1/w_2$.



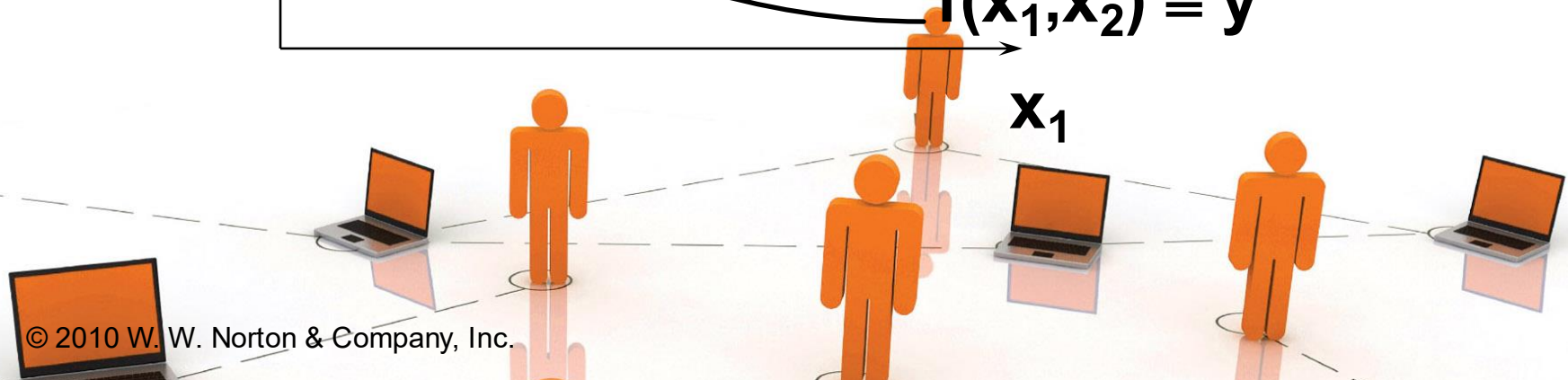
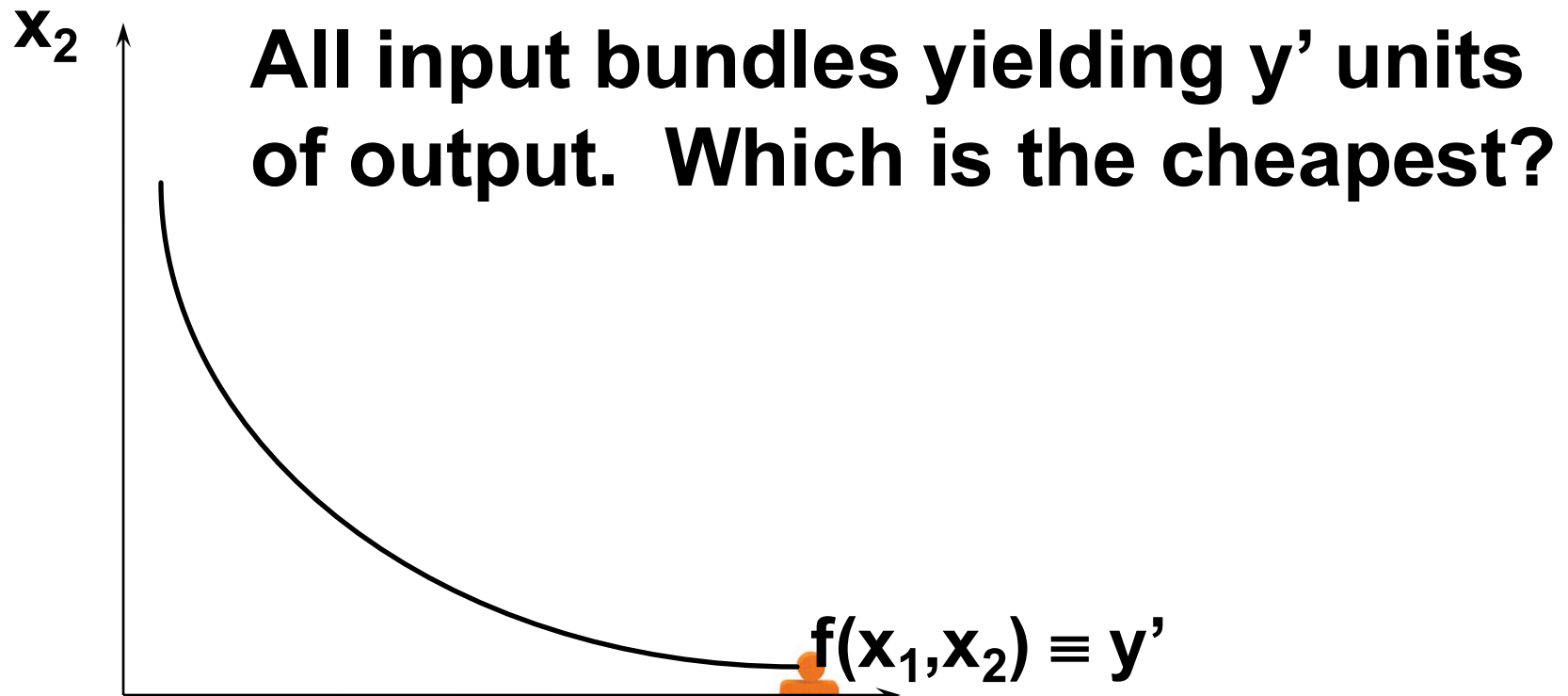
Iso-cost Lines



Iso-cost Lines

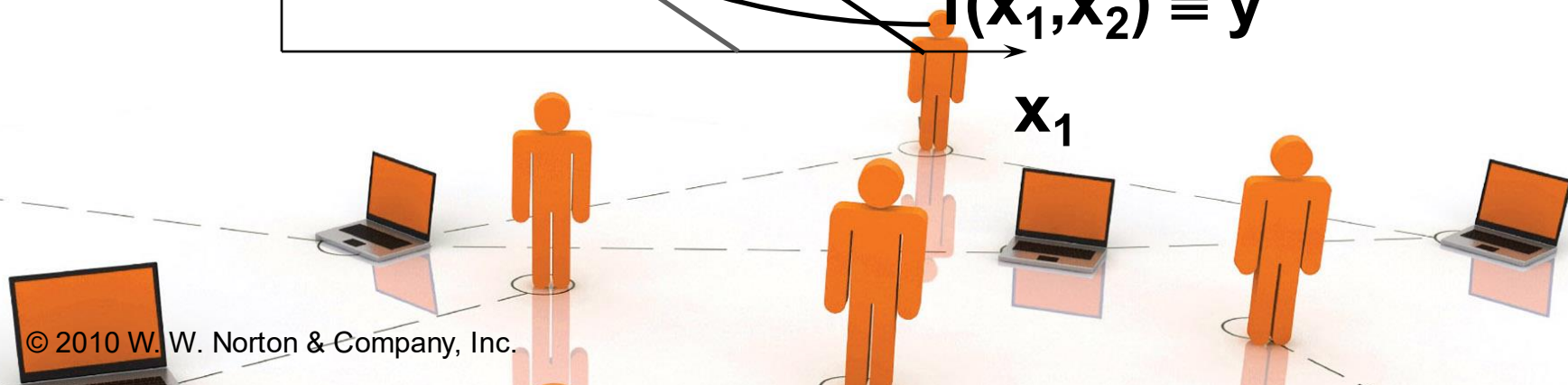
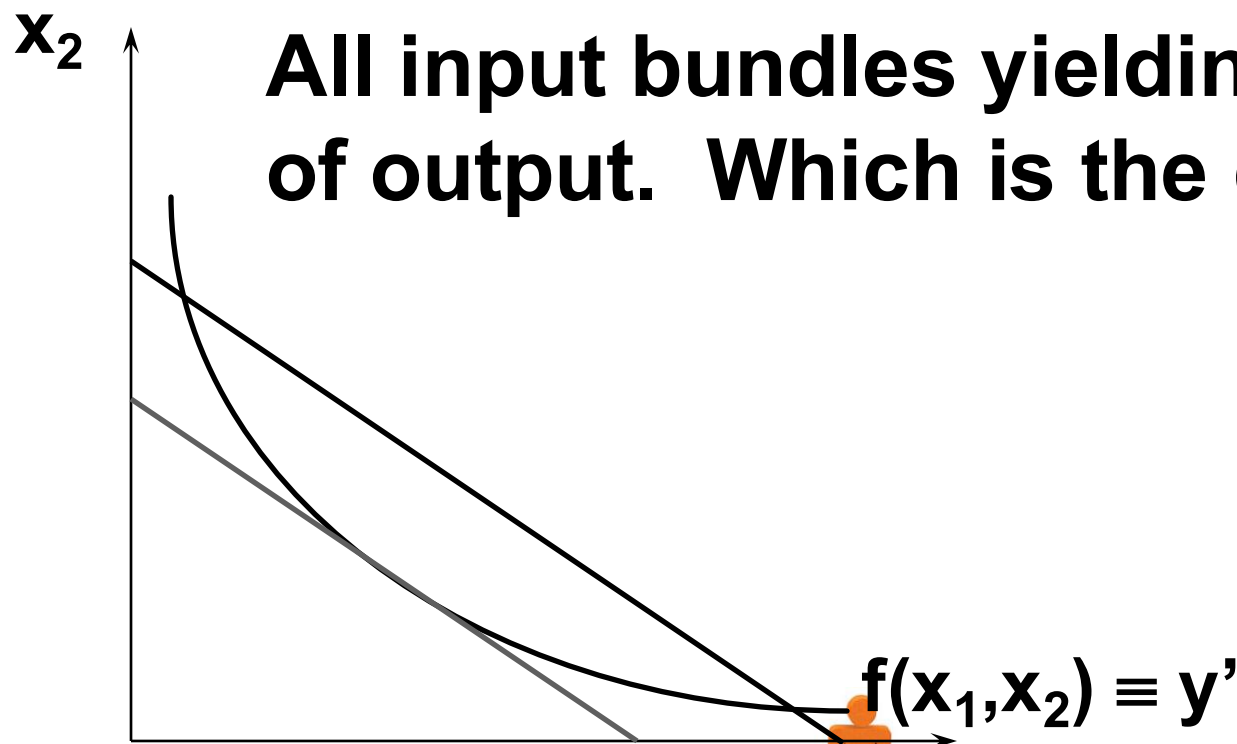


The y' -Output Unit Isoquant

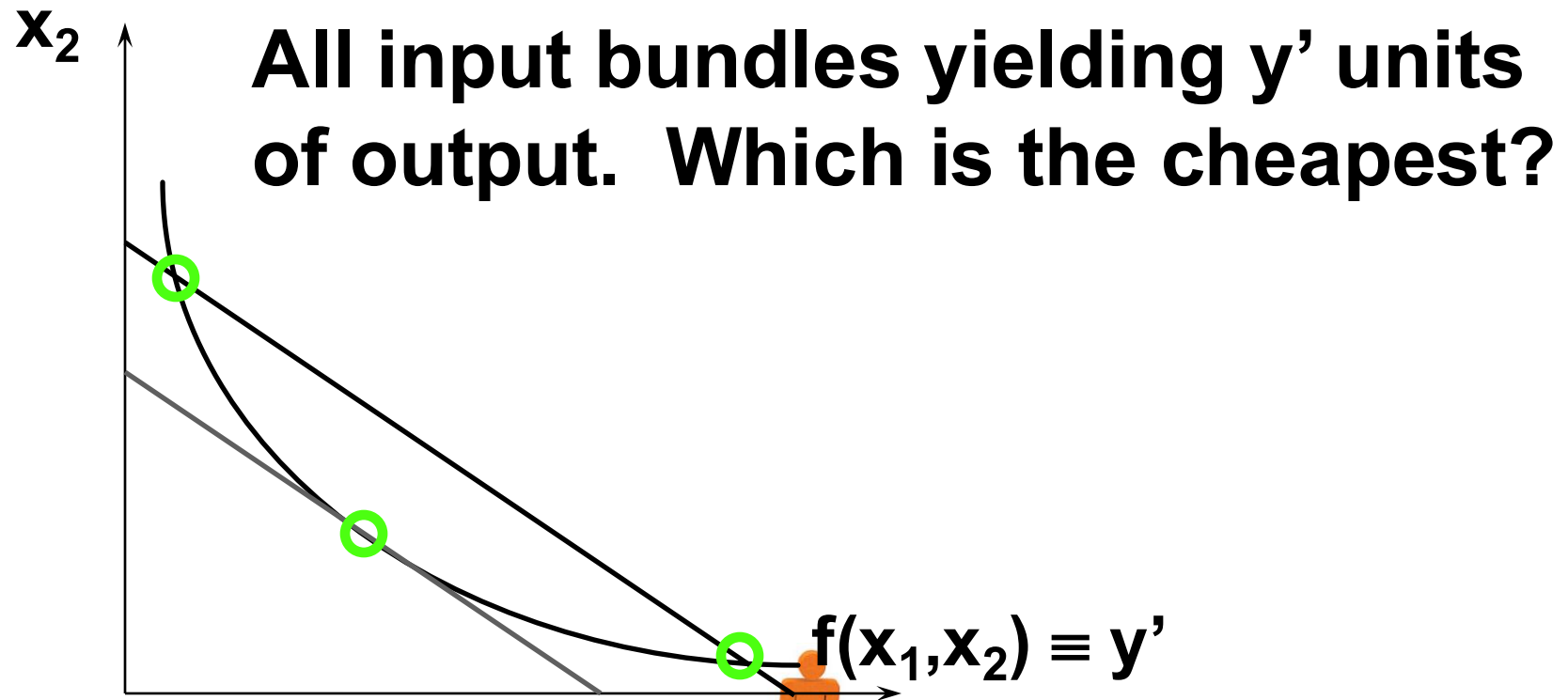


The Cost-Minimization Problem

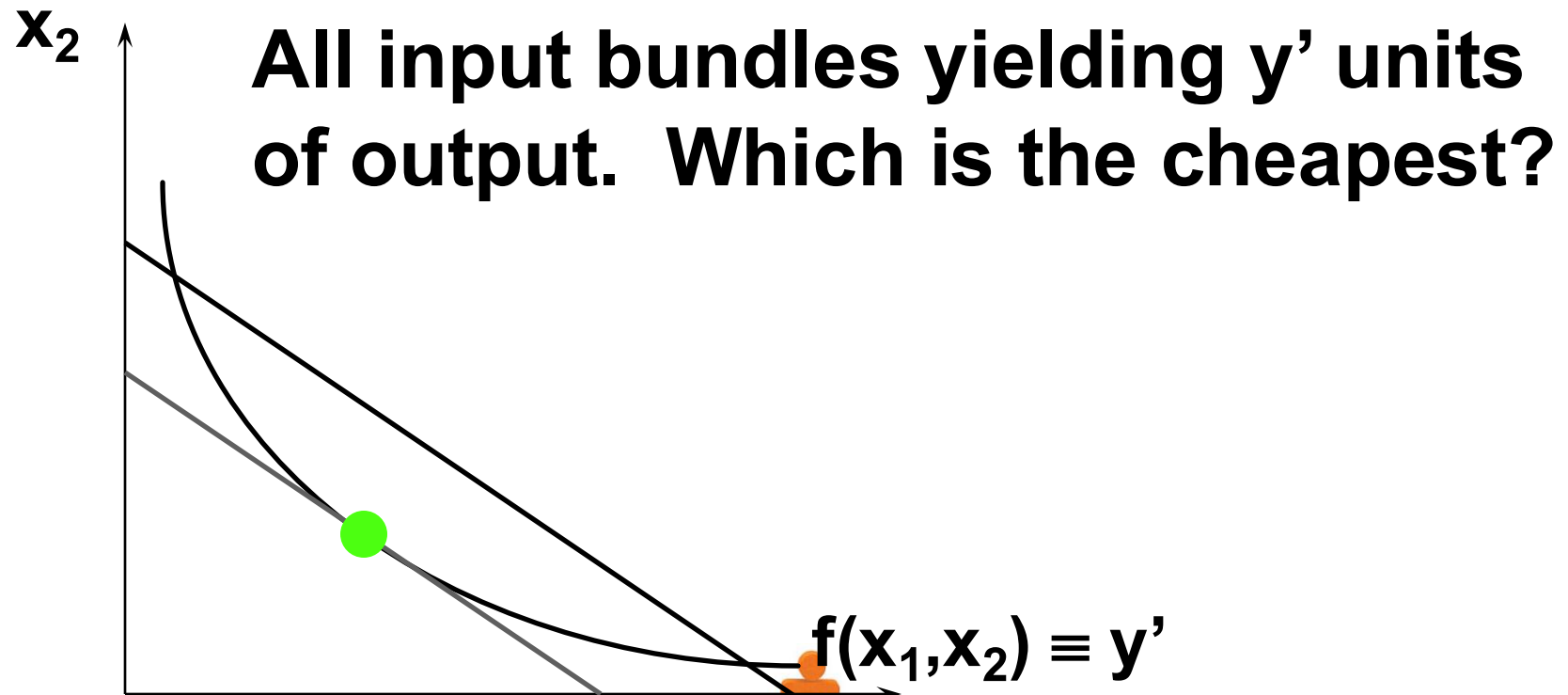
All input bundles yielding y' units of output. Which is the cheapest?



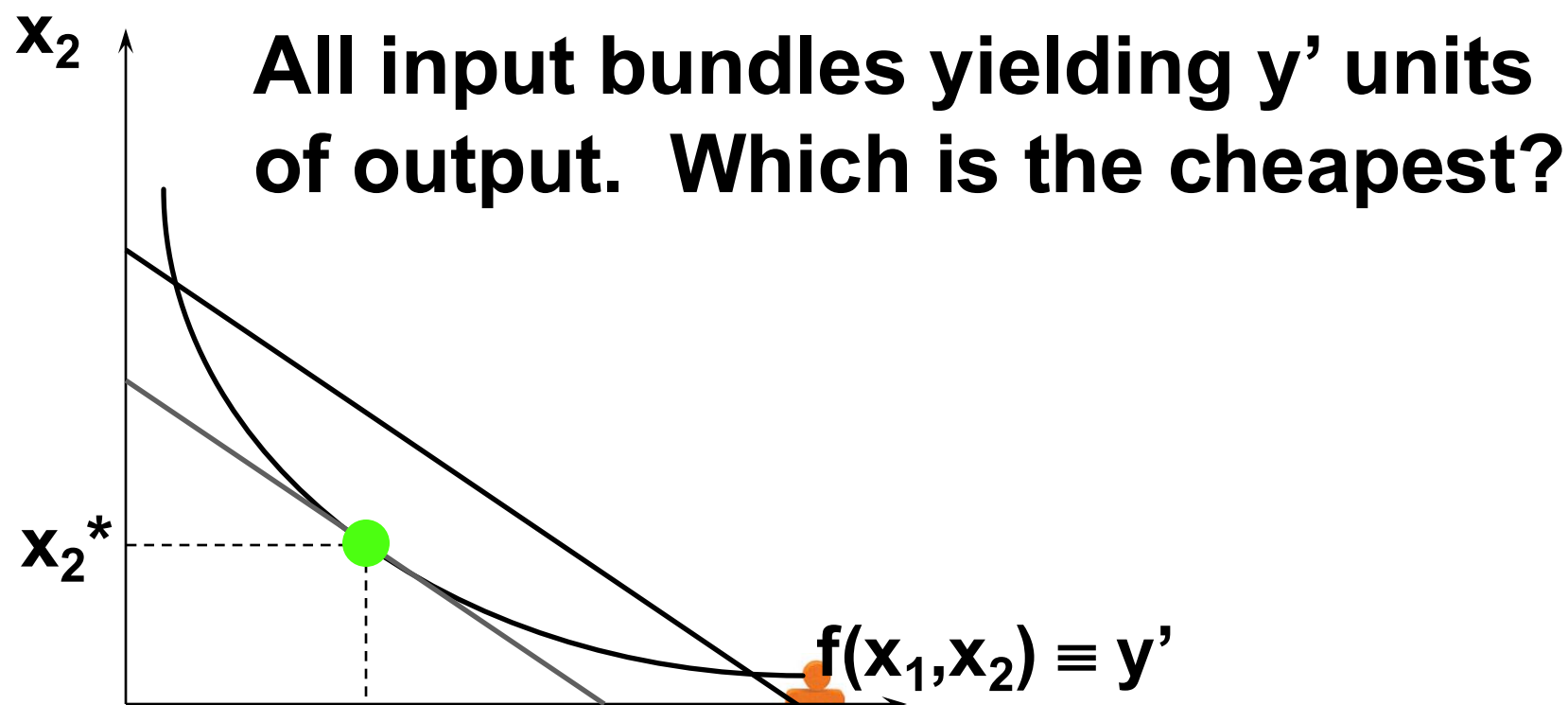
The Cost-Minimization Problem



The Cost-Minimization Problem



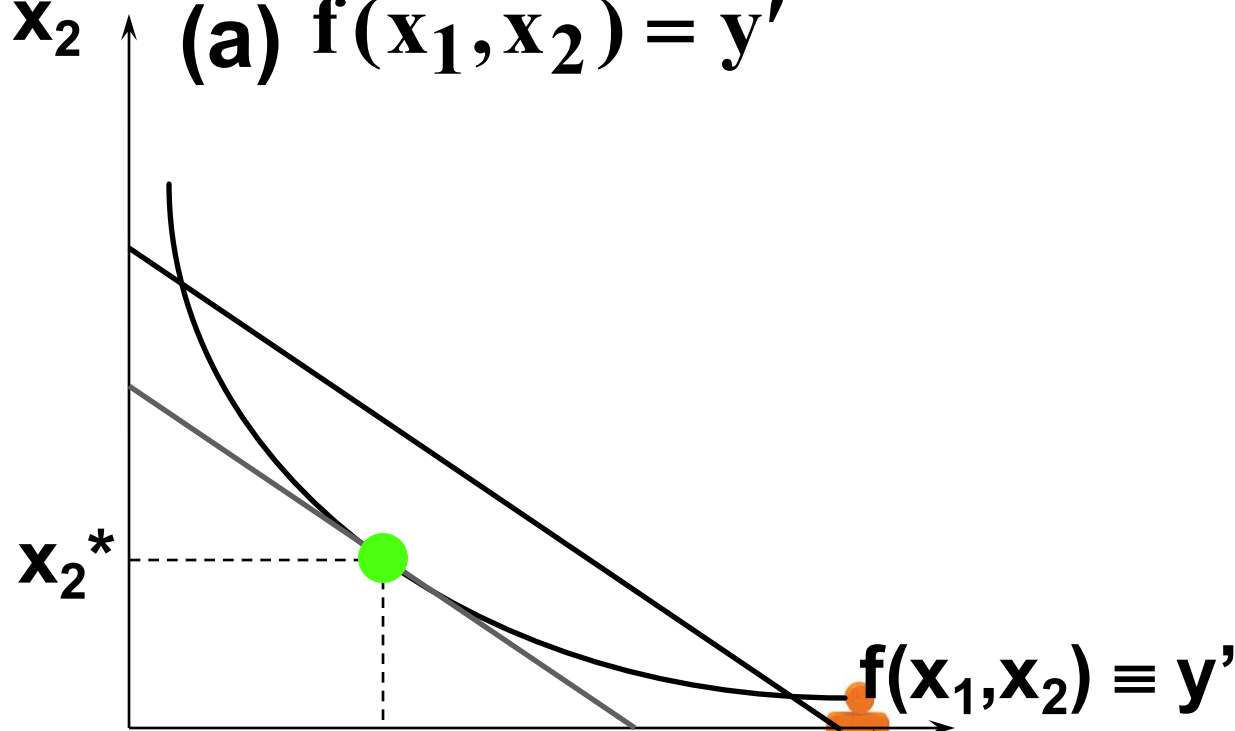
The Cost-Minimization Problem



The Cost-Minimization Problem

At an interior cost-min input bundle:

(a) $f(x_1^*, x_2^*) = y'$

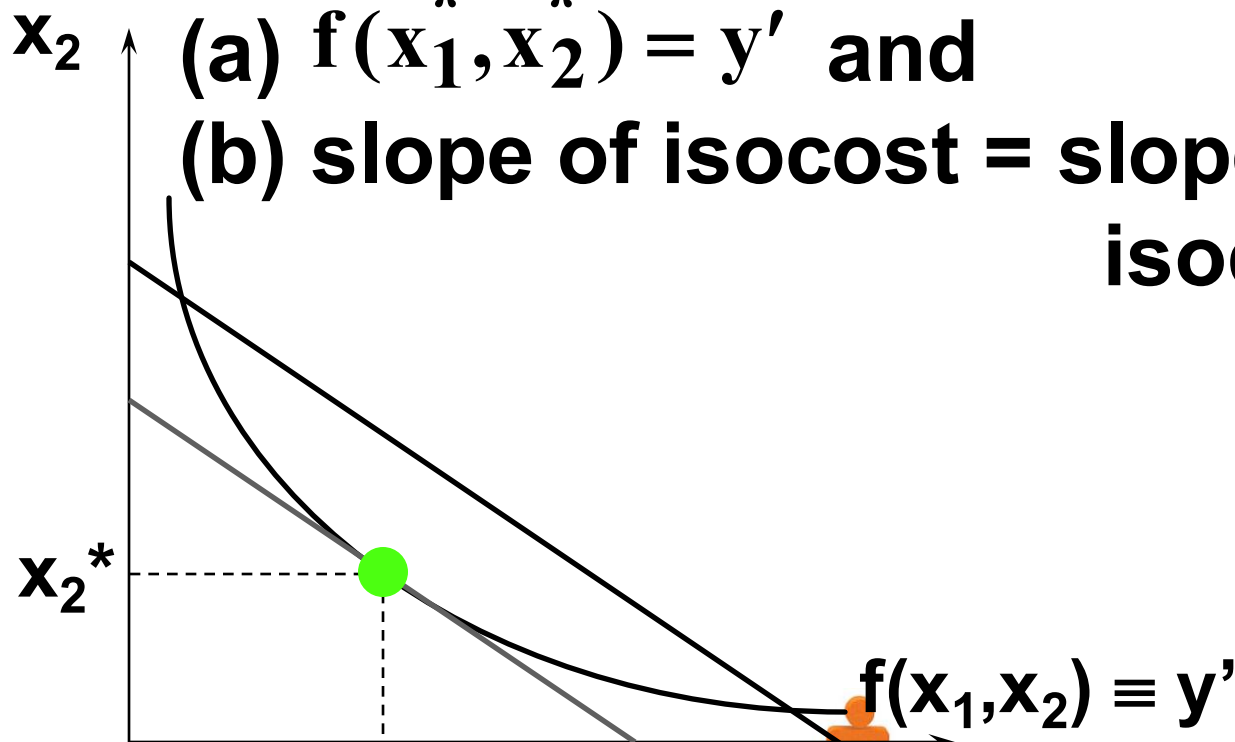


The Cost-Minimization Problem

At an interior cost-min input bundle:

(a) $f(x_1^*, x_2^*) = y'$ and

(b) slope of isocost = slope of isoquant



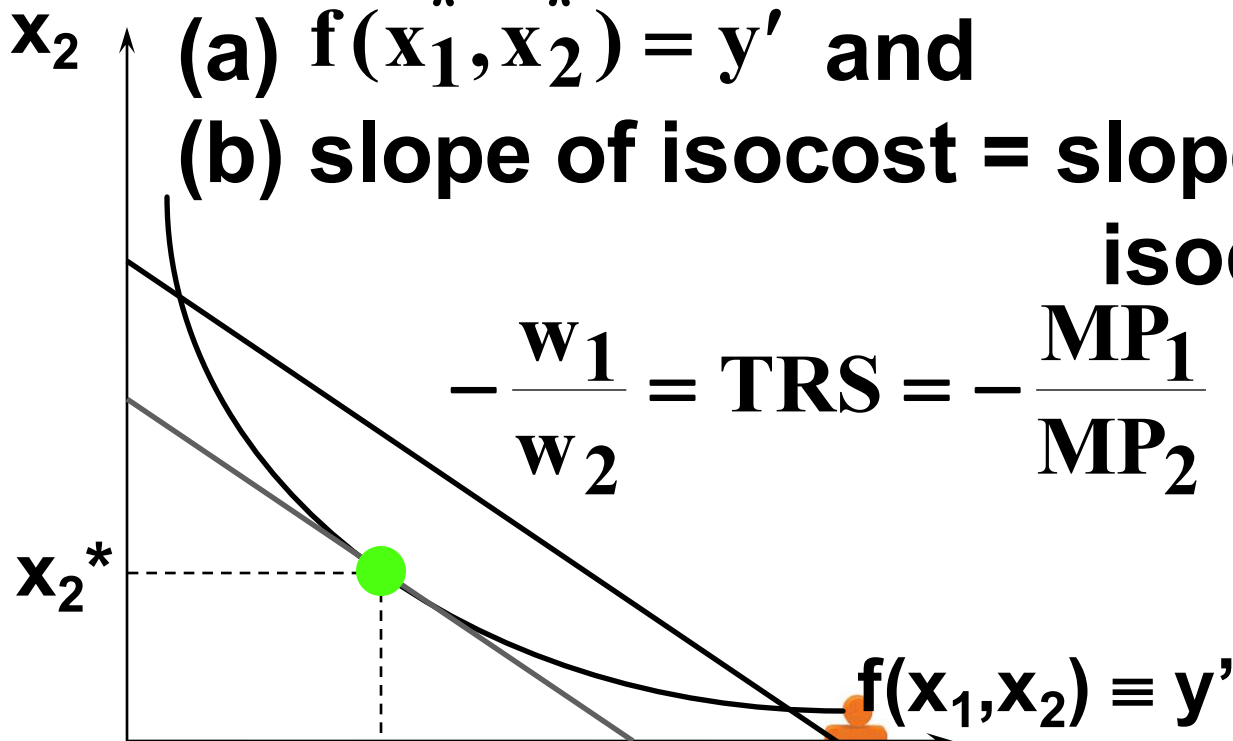
The Cost-Minimization Problem

At an interior cost-min input bundle:

(a) $f(x_1^*, x_2^*) = y'$ and

(b) slope of isocost = slope of isoquant; i.e.

$$-\frac{w_1}{w_2} = \text{TRS} = -\frac{\text{MP}_1}{\text{MP}_2} \text{ at } (x_1^*, x_2^*).$$



A Cobb-Douglas Example of Cost Minimization

- A firm's Cobb-Douglas production function is $y = f(x_1, x_2) = x_1^{1/3} x_2^{2/3}$.
- Input prices are w_1 and w_2 .
- What are the firm's conditional input demand functions?



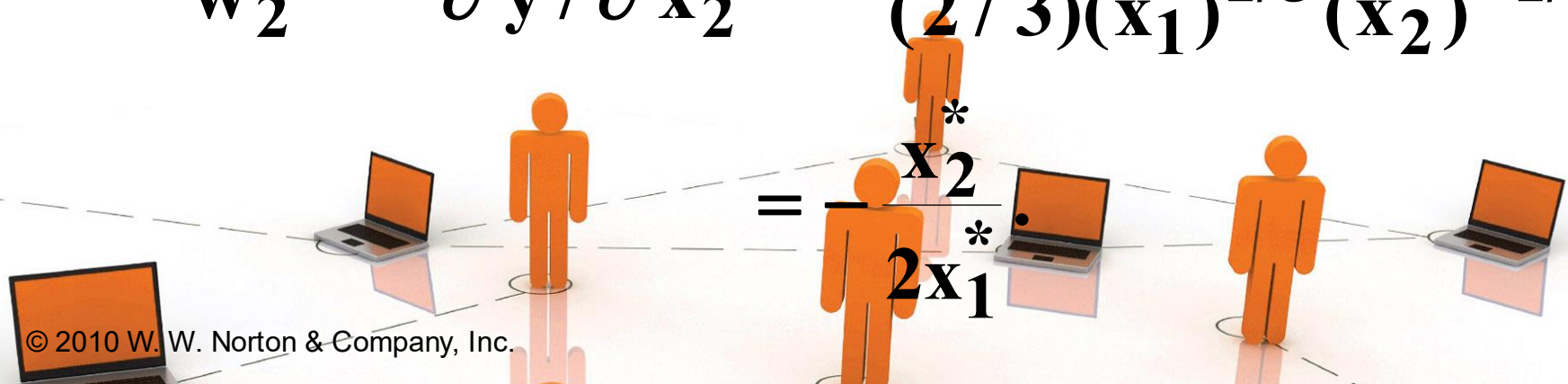
A Cobb-Douglas Example of Cost Minimization

At the input bundle (x_1^*, x_2^*) which minimizes the cost of producing y output units:

(a) $y = (x_1^*)^{1/3} (x_2^*)^{2/3}$ and

(b)
$$-\frac{w_1}{w_2} = -\frac{\partial y / \partial x_1}{\partial y / \partial x_2} = -\frac{(1/3)(x_1^*)^{-2/3} (x_2^*)^{2/3}}{(2/3)(x_1^*)^{1/3} (x_2^*)^{-1/3}}$$

$$= -\frac{x_2^*}{2x_1^*}$$



A Cobb-Douglas Example of Cost Minimization

$$(a) \ y = (x_1^*)^{1/3} (x_2^*)^{2/3} \quad (b) \ \frac{w_1}{w_2} = \frac{x_2^*}{2x_1^*}.$$



A Cobb-Douglas Example of Cost Minimization

$$(a) \ y = (x_1^*)^{1/3} (x_2^*)^{2/3} \quad (b) \ \frac{w_1}{w_2} = \frac{x_2^*}{2x_1^*}.$$

From (b), $x_2^* = \frac{2w_1}{w_2} x_1^*.$



A Cobb-Douglas Example of Cost Minimization

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From (b), $x_2^* = \frac{2w_1}{w_2} x_1^*.$

Now substitute into (a) to get

$$y = (x_1^*)^{1/3} \left(\frac{2w_1}{w_2} x_1^* \right)^{2/3}$$



A Cobb-Douglas Example of Cost Minimization

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Now substitute into (a) to get

$$y = (x_1^*)^{1/3} \left(\frac{2w_1}{w_2} x_1^* \right)^{2/3} = \left(\frac{2w_1}{w_2} \right)^{2/3} x_1^*.$$

So $x_1^* = \left(\frac{w_2}{2w_1} \right)^{2/3} y$ is the firm's conditional demand for input 1.

A Cobb-Douglas Example of Cost Minimization

Since $x_2^* = \frac{2w_1}{w_2} x_1^*$ **and** $x_1^* = \left(\frac{w_2}{2w_1} \right)^{2/3} y$

$$x_2^* = \frac{2w_1}{w_2} \left(\frac{w_2}{2w_1} \right)^{2/3} y = \left(\frac{2w_1}{w_2} \right)^{1/3} y$$

is the firm's conditional demand for input 2.



A Cobb-Douglas Example of Cost Minimization

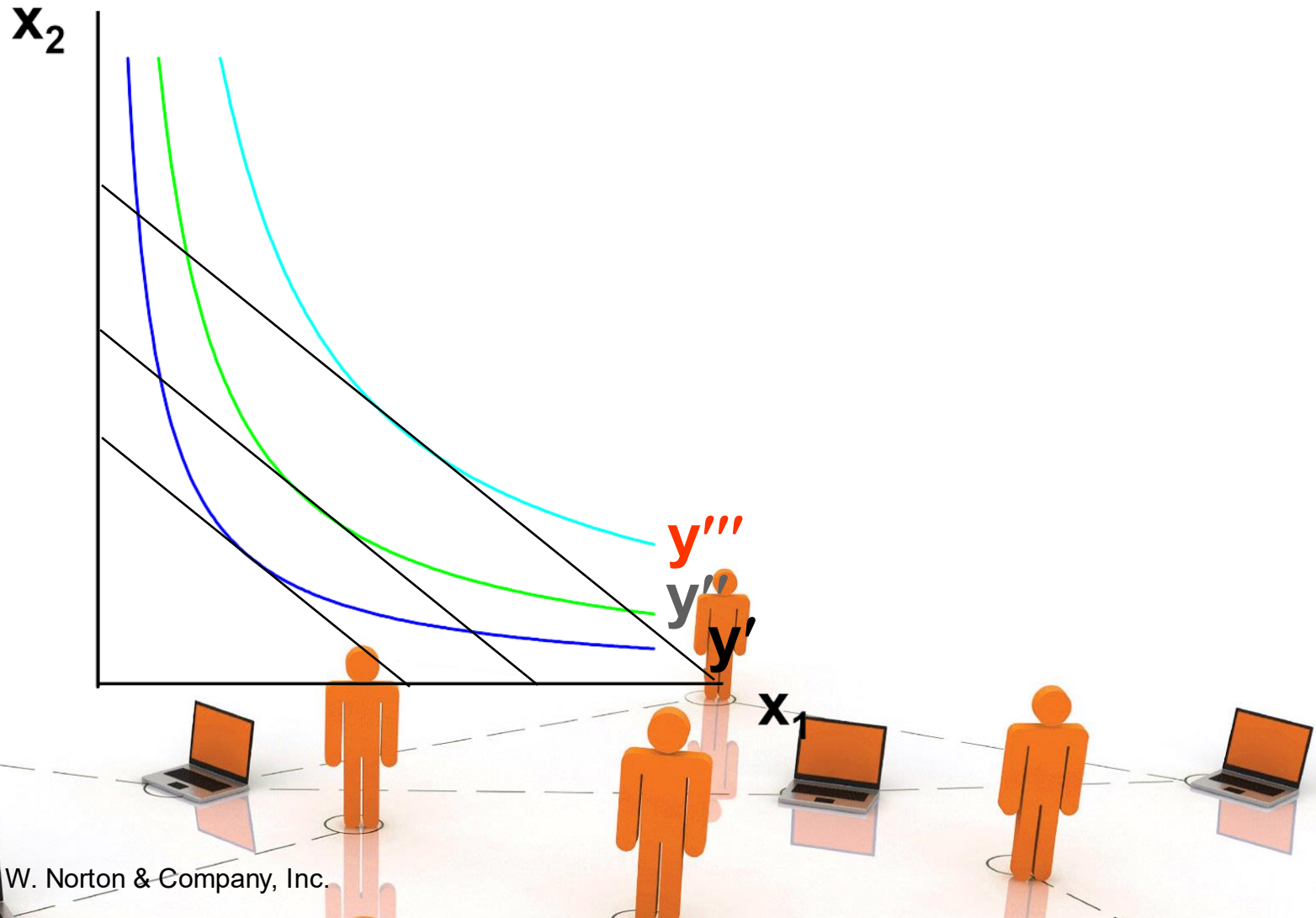
So the cheapest input bundle yielding y output units is

$$\begin{aligned} & \left(x_1^*(w_1, w_2, y), x_2^*(w_1, w_2, y) \right) \\ &= \left(\left(\frac{w_2}{2w_1} \right)^{2/3} y, \left(\frac{2w_1}{w_2} \right)^{1/3} y \right). \end{aligned}$$



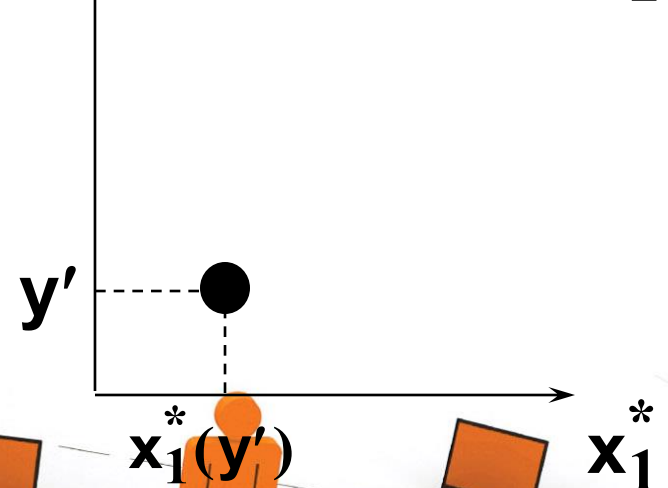
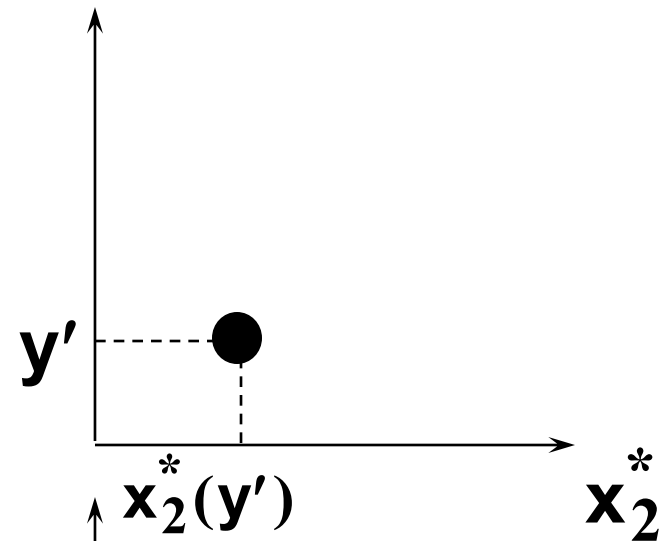
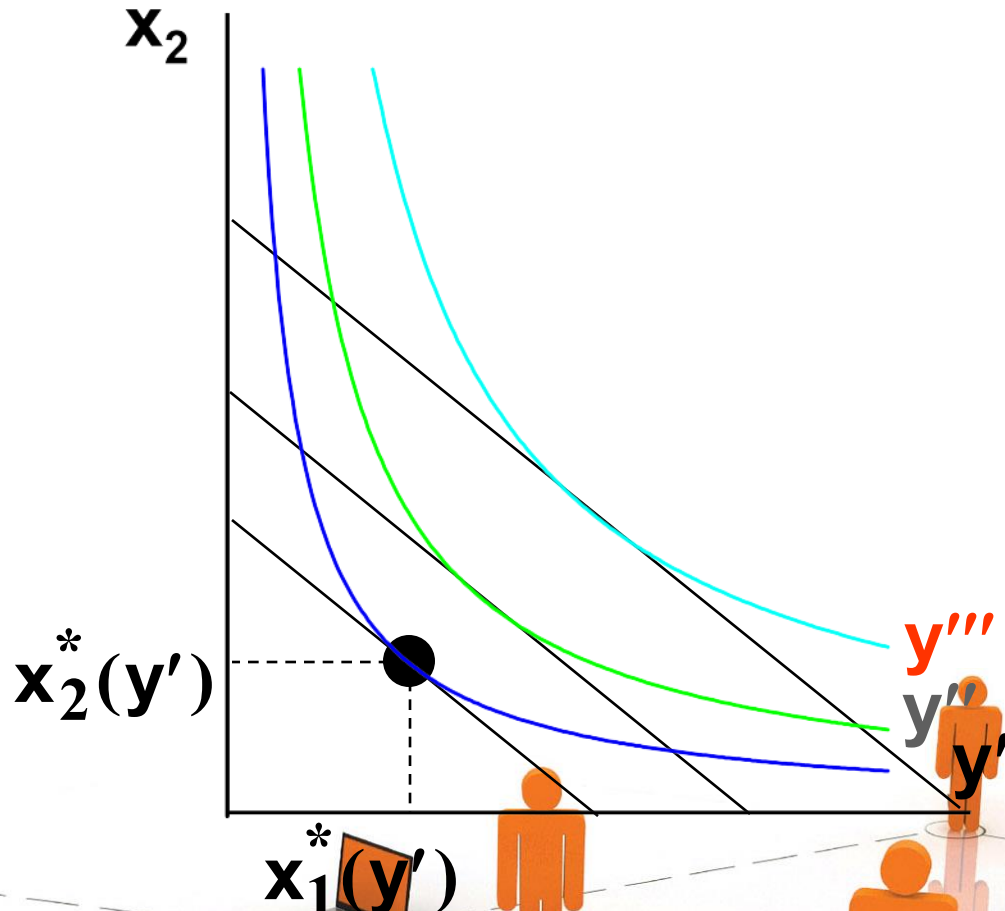
Conditional Input Demand Curves

Fixed w_1 and w_2 .



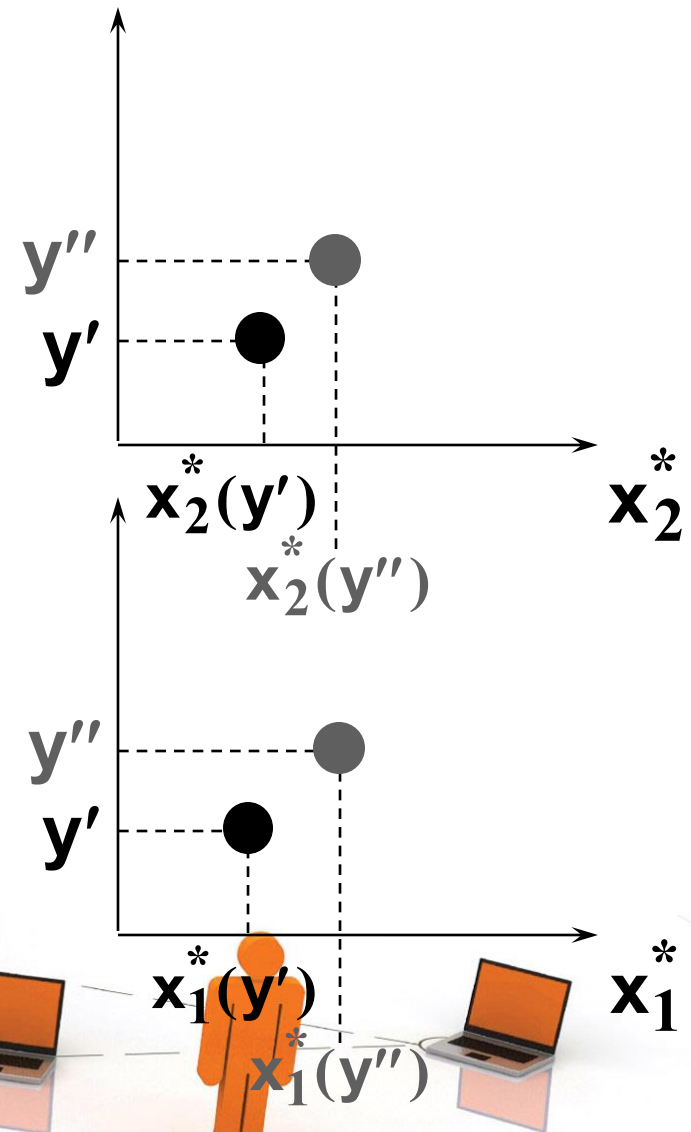
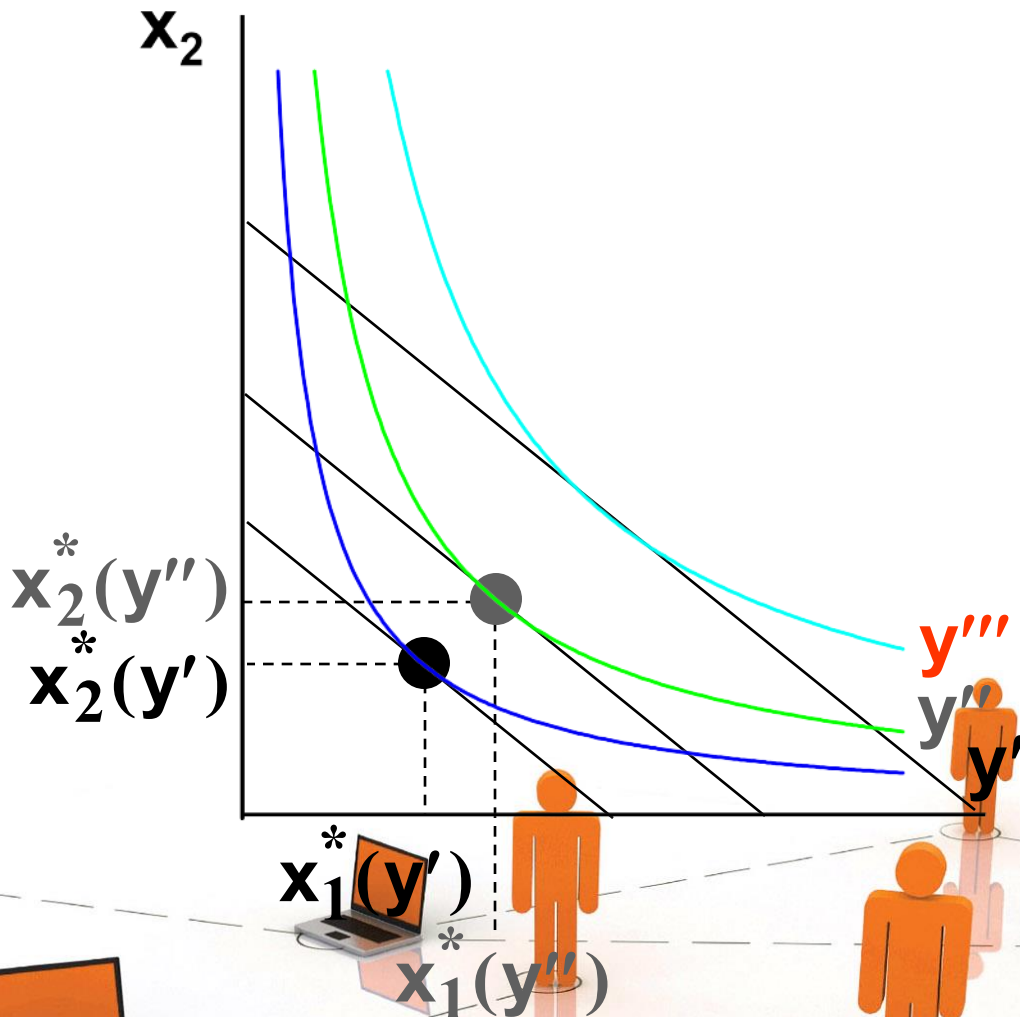
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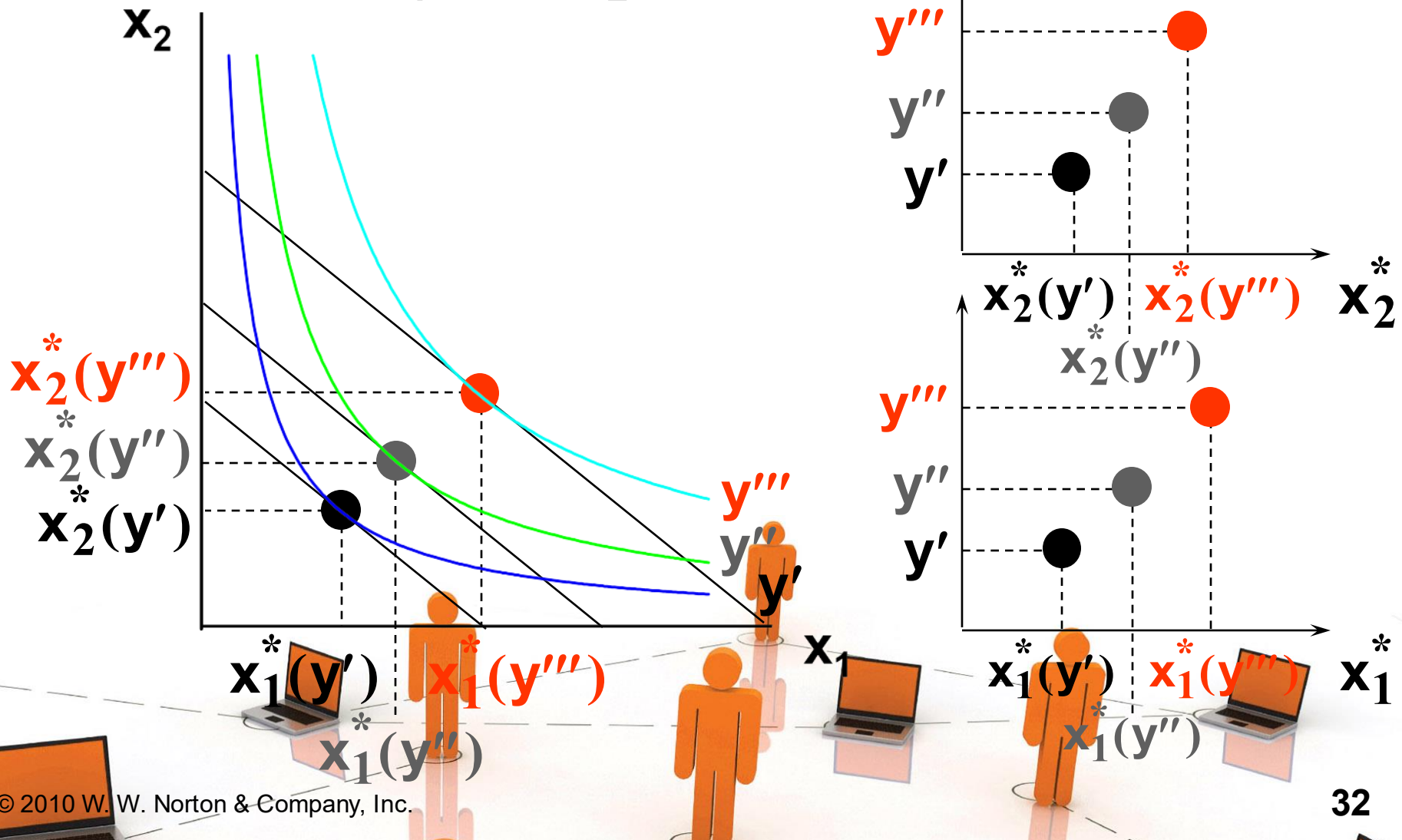
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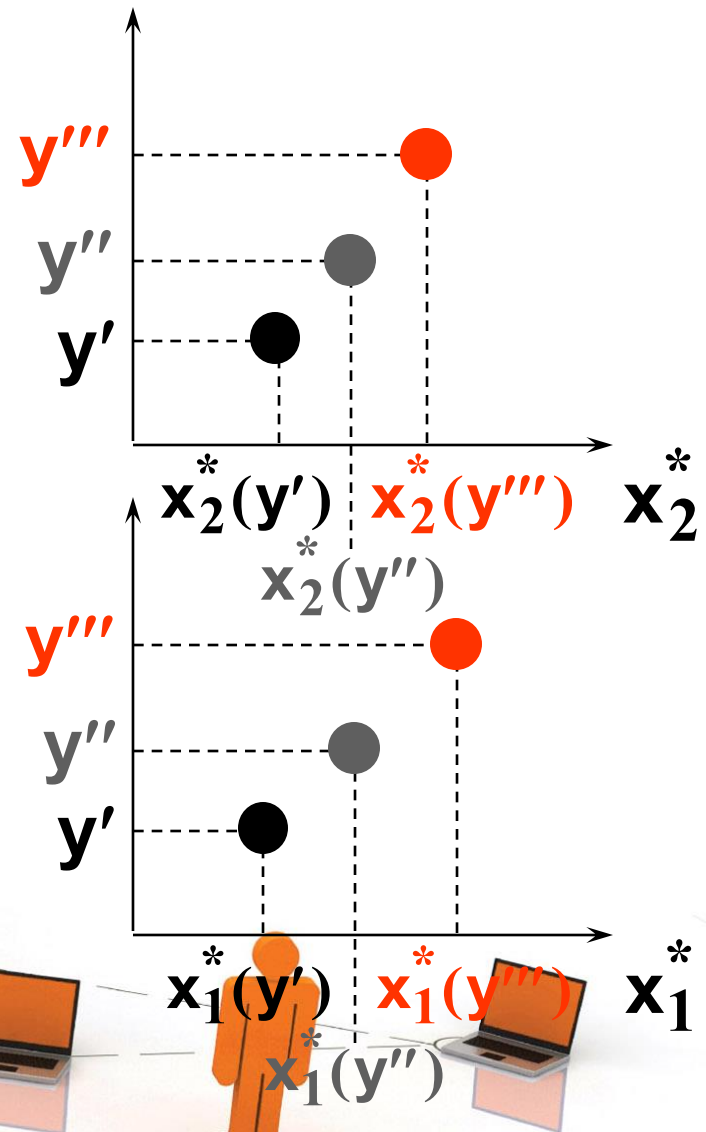
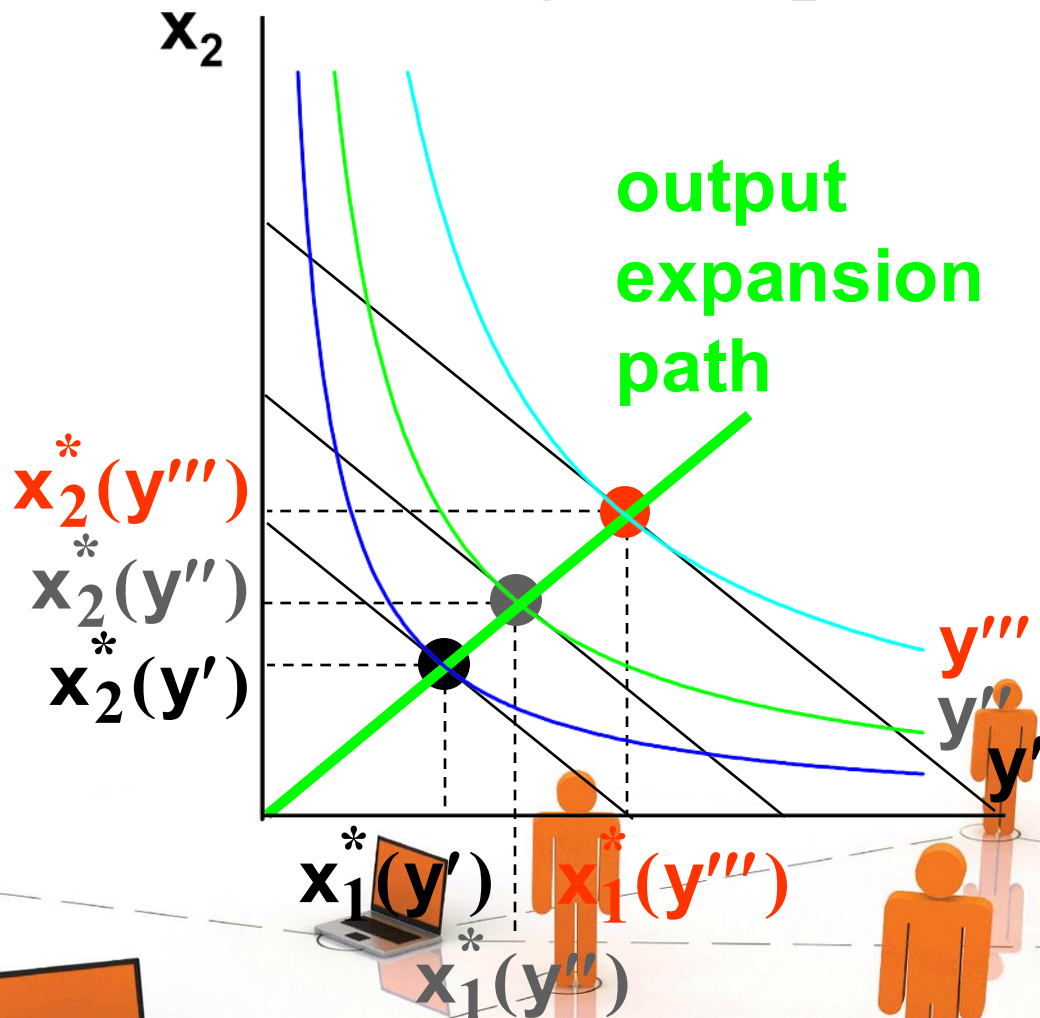
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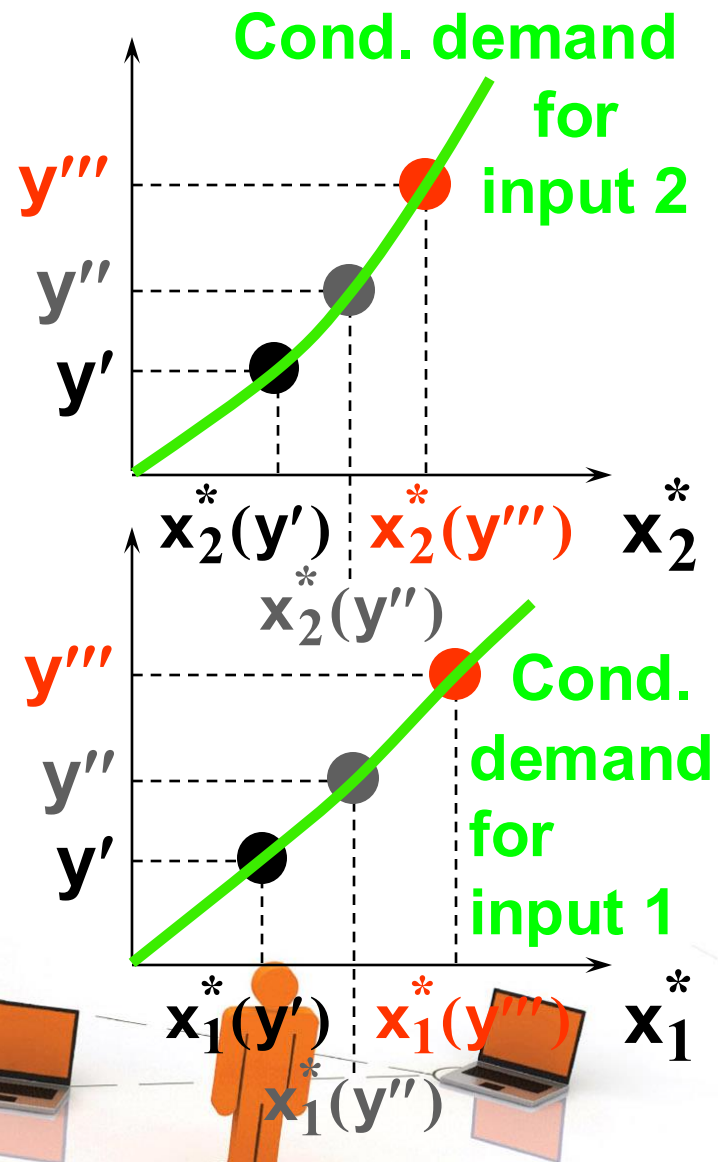
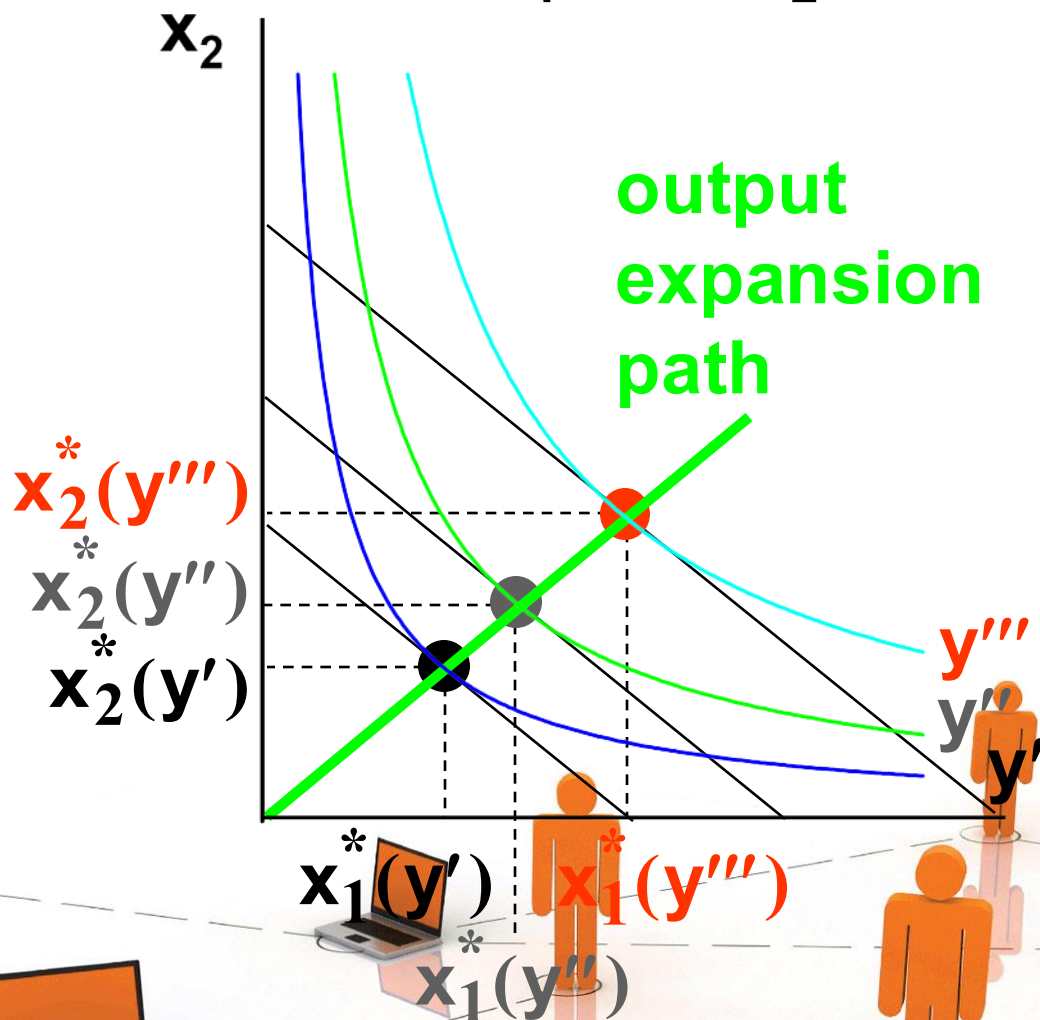
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Conditional Input Demand Curves

Fixed w_1 and w_2 .



A Cobb-Douglas Example of Cost Minimization

For the production function

$$y = f(x_1, x_2) = x_1^{1/3} x_2^{2/3}$$

the cheapest input bundle yielding y output units is

$$\left(x_1^*(w_1, w_2, y), x_2^*(w_1, w_2, y) \right)$$

$$= \left(\left(\frac{w_2}{2w_1} \right)^{2/3} y, \left(\frac{2w_1}{w_2} \right)^{1/3} y \right).$$

A Cobb-Douglas Example of Cost Minimization

So the firm's total cost function is

$$c(w_1, w_2, y) = w_1 x_1^*(w_1, w_2, y) + w_2 x_2^*(w_1, w_2, y)$$



A Cobb-Douglas Example of Cost Minimization

So the firm's total cost function is

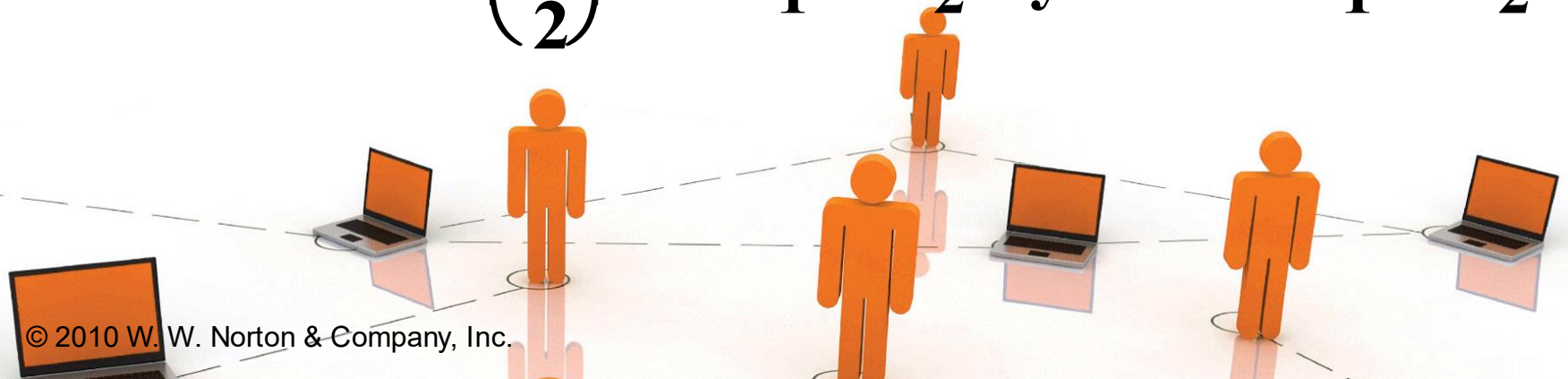
$$\begin{aligned} c(w_1, w_2, y) &= w_1 x_1^*(w_1, w_2, y) + w_2 x_2^*(w_1, w_2, y) \\ &= w_1 \left(\frac{w_2}{2w_1} \right)^{2/3} y + w_2 \left(\frac{2w_1}{w_2} \right)^{1/3} y \end{aligned}$$



A Cobb-Douglas Example of Cost Minimization

So the firm's total cost function is

$$\begin{aligned} c(w_1, w_2, y) &= w_1 x_1^*(w_1, w_2, y) + w_2 x_2^*(w_1, w_2, y) \\ &= w_1 \left(\frac{w_2}{2w_1} \right)^{2/3} y + w_2 \left(\frac{2w_1}{w_2} \right)^{1/3} y \\ &= \left(\frac{1}{2} \right)^{2/3} w_1^{1/3} w_2^{2/3} y + 2^{1/3} w_1^{1/3} w_2^{2/3} y \end{aligned}$$



A Cobb-Douglas Example of Cost Minimization

So the firm's total cost function is

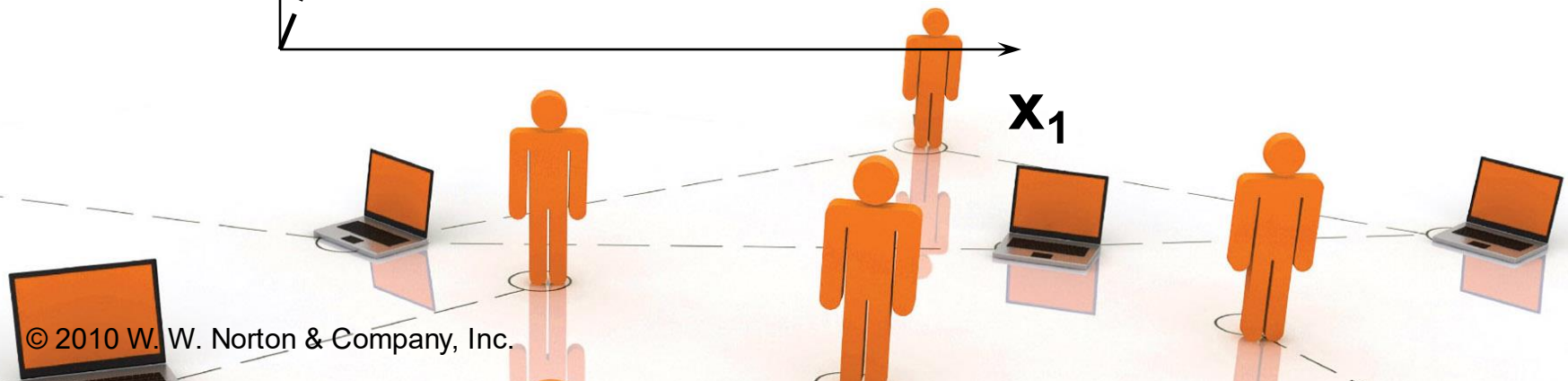
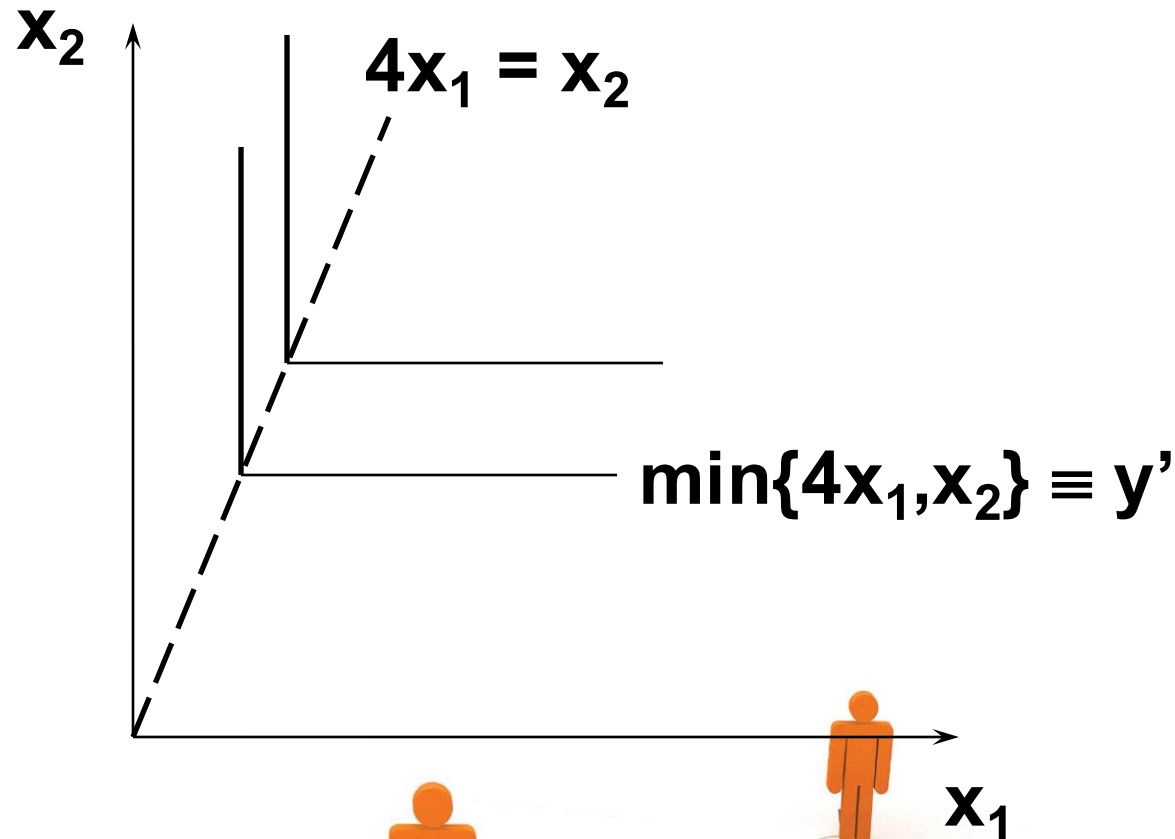
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A Perfect Complements Example of Cost Minimization

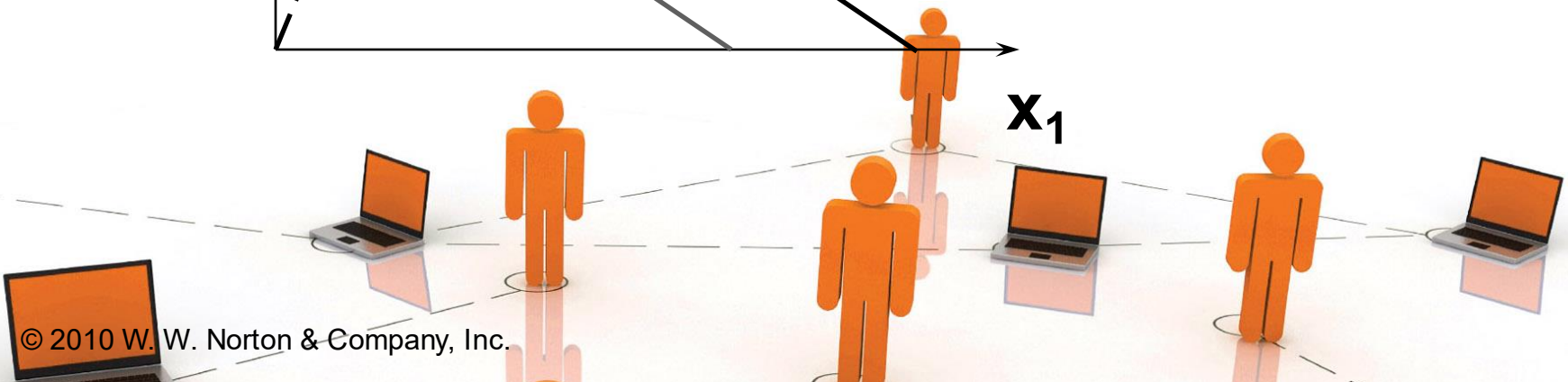
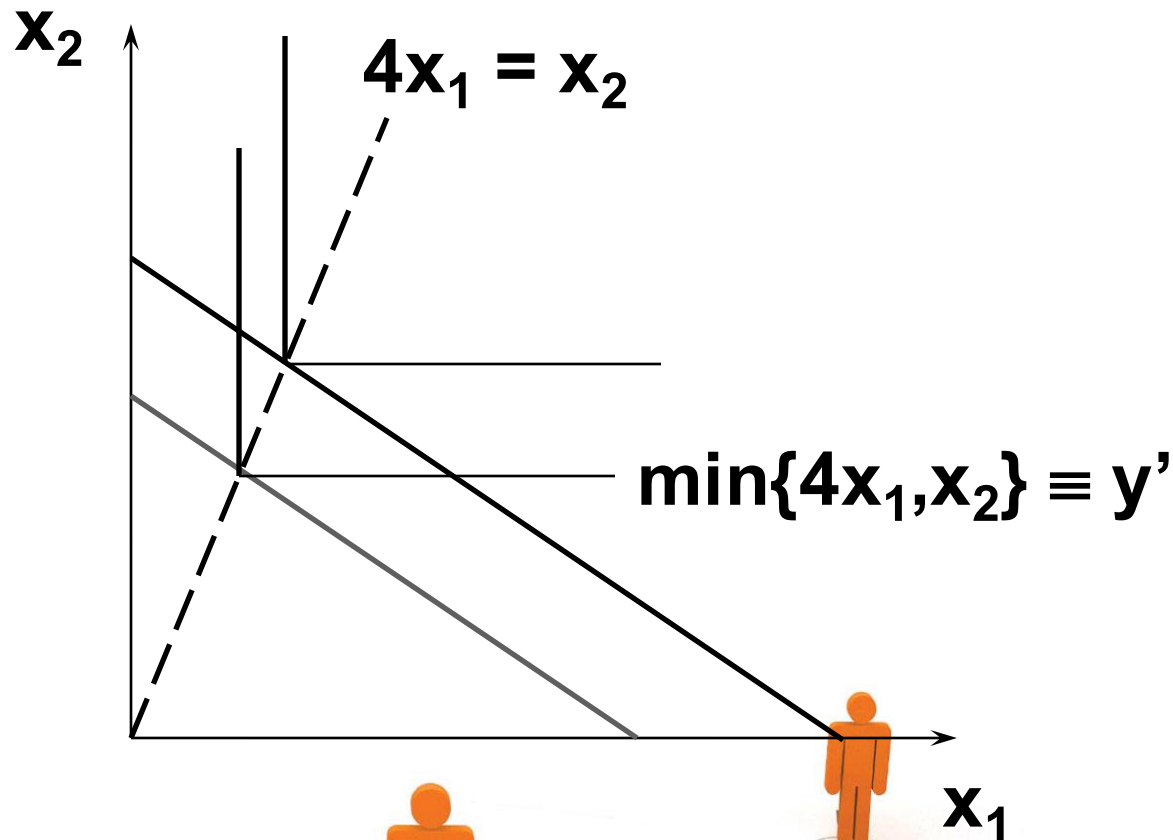
- The firm's production function is $y = \min\{4x_1, x_2\}$.
- Input prices w_1 and w_2 are given.
- What are the firm's conditional demands for inputs 1 and 2?
- What is the firm's total cost function?



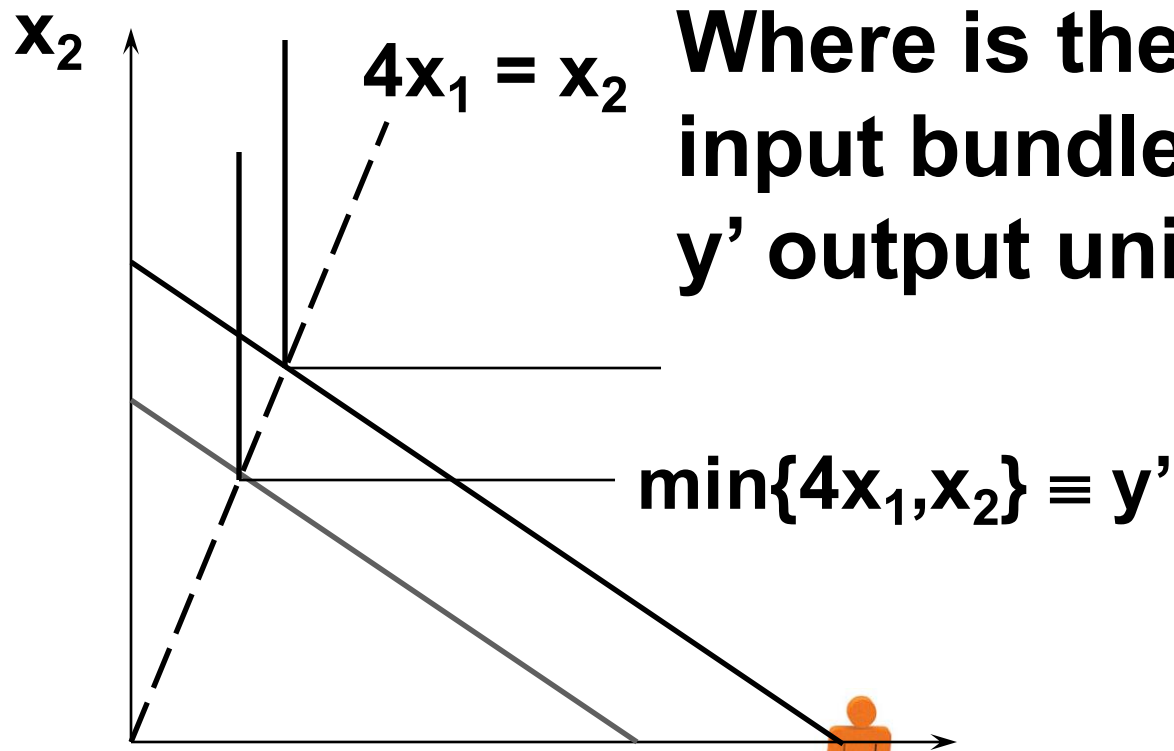
A Perfect Complements Example of Cost Minimization



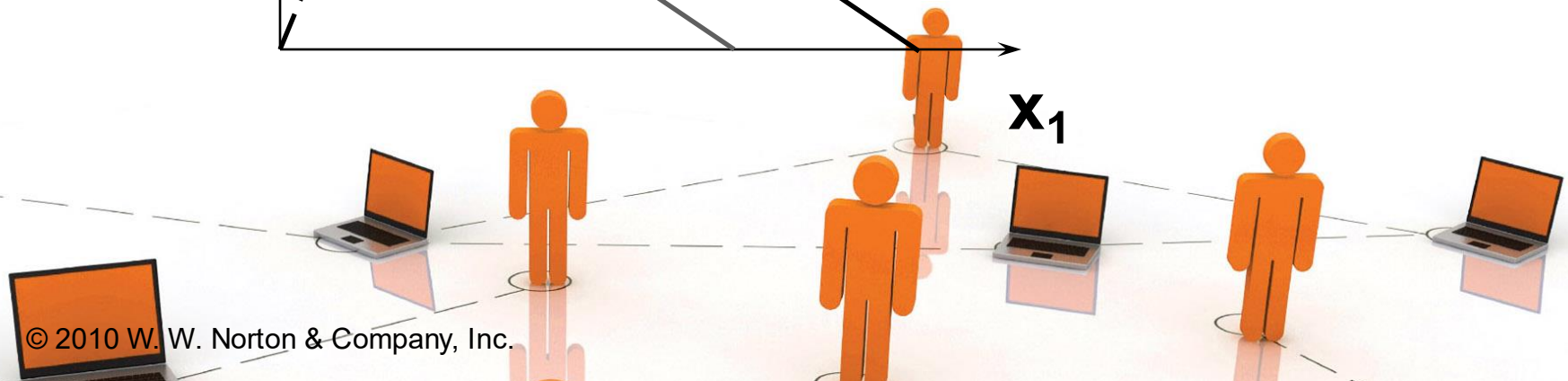
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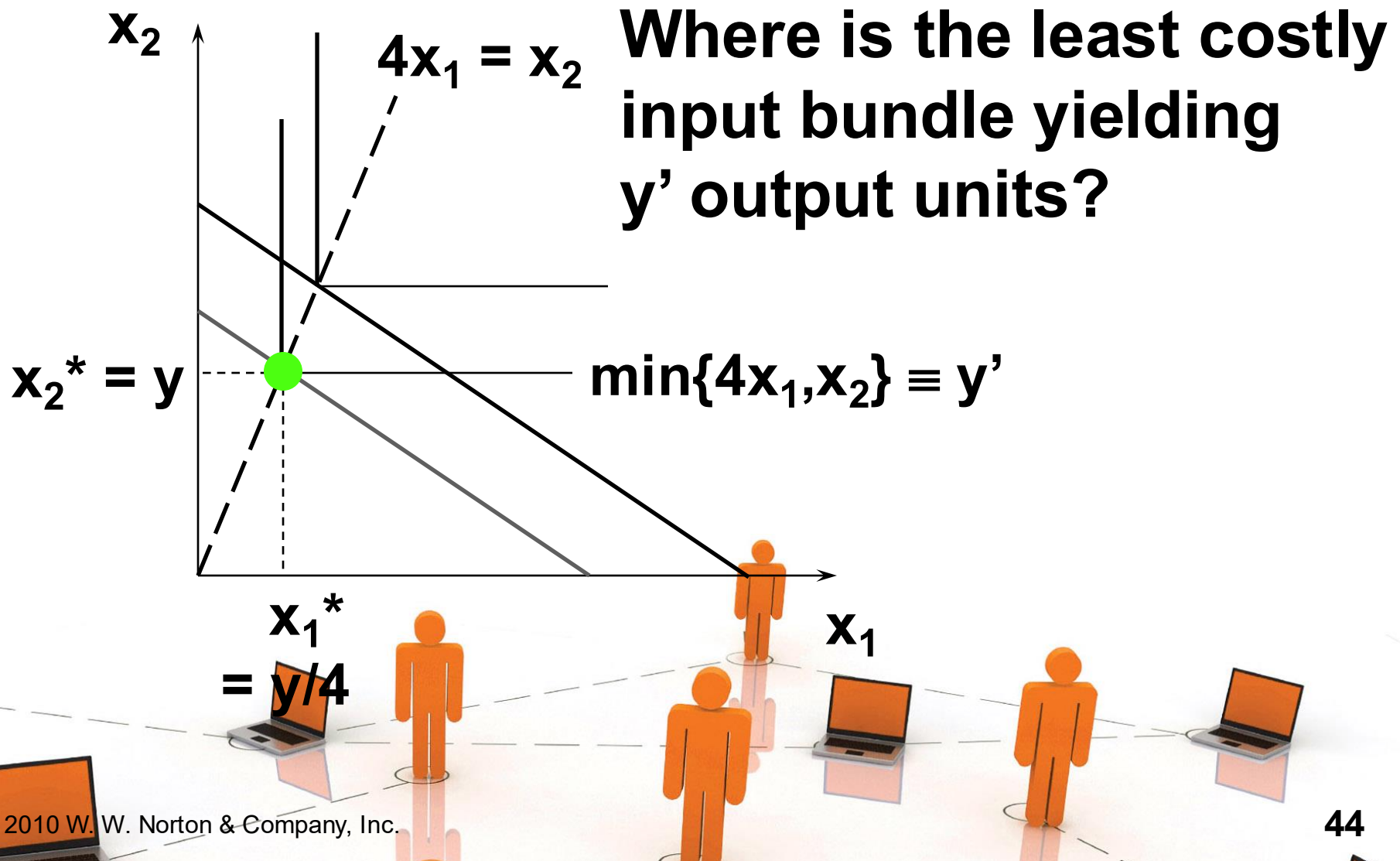
A Perfect Complements Example of Cost Minimization



**Where is the least costly
input bundle yielding
 y' output units?**



A Perfect Complements Example of Cost Minimization



A Perfect Complements Example of Cost Minimization

The firm's production function is

$$y = \min\{4x_1, x_2\}$$

and the conditional input demands are

$$x_1^*(w_1, w_2, y) = \frac{y}{4} \quad \text{and} \quad x_2^*(w_1, w_2, y) = y.$$



A Perfect Complements Example of Cost Minimization

The firm's production function is

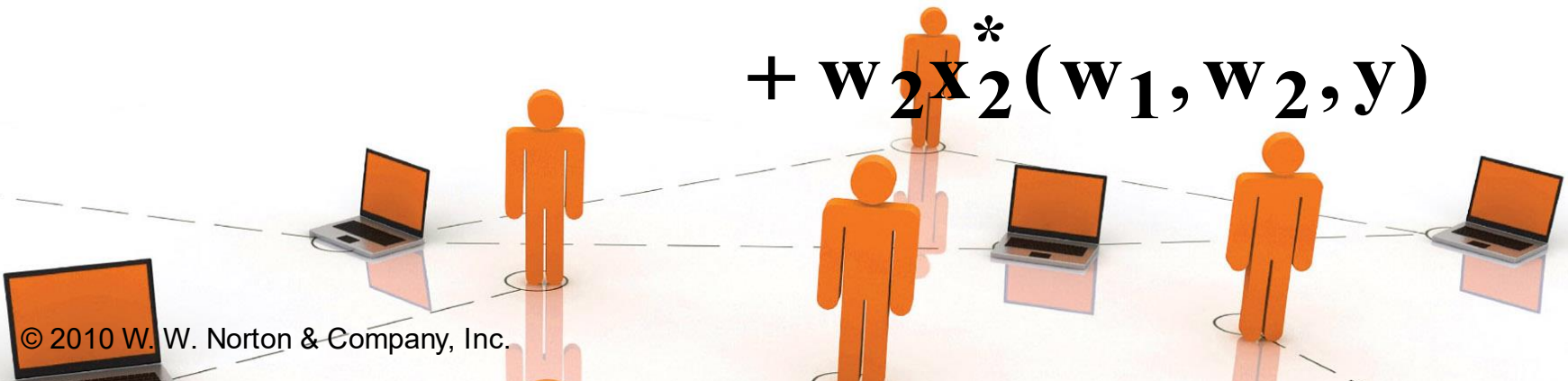
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So the firm's total cost function is

$$c(w_1, w_2, y) = w_1 x_1^*(w_1, w_2, y) + w_2 x_2^*(w_1, w_2, y)$$



A Perfect Complements Example of Cost Minimization

The firm's production function is

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So the firm's total cost function is

$$\begin{aligned} c(w_1, w_2, y) &= w_1 x_1^*(w_1, w_2, y) \\ &\quad + w_2 x_2^*(w_1, w_2, y) \\ &= w_1 \frac{y}{4} + w_2 y = \left(\frac{w_1}{4} + w_2 \right) y. \end{aligned}$$

Average Total Production Costs

- For positive output levels y , a firm's average total cost of producing y units is

$$AC(w_1, w_2, y) = \frac{c(w_1, w_2, y)}{y}.$$



Returns-to-Scale and Av. Total Costs

- **The returns-to-scale properties of a firm's technology determine how average production costs change with output level.**
- **Our firm is presently producing y ' output units.**
- **How does the firm's average production cost change if it instead produces $2y$ ' units of output?**



Constant Returns-to-Scale and Average Total Costs

- **If a firm's technology exhibits constant returns-to-scale then doubling its output level from y' to $2y'$ requires doubling all input levels.**



Constant Returns-to-Scale and Average Total Costs

- **If a firm's technology exhibits constant returns-to-scale then doubling its output level from y' to $2y'$ requires doubling all input levels.**
- **Total production cost doubles.**



Constant Returns-to-Scale and Average Total Costs

- **If a firm's technology exhibits constant returns-to-scale then doubling its output level from y' to $2y'$ requires doubling all input levels.**
- **Total production cost doubles.**
- **Average production cost does not change.**



Decreasing Returns-to-Scale and Average Total Costs

- **If a firm's technology exhibits decreasing returns-to-scale then doubling its output level from y' to $2y'$ requires more than doubling all input levels.**



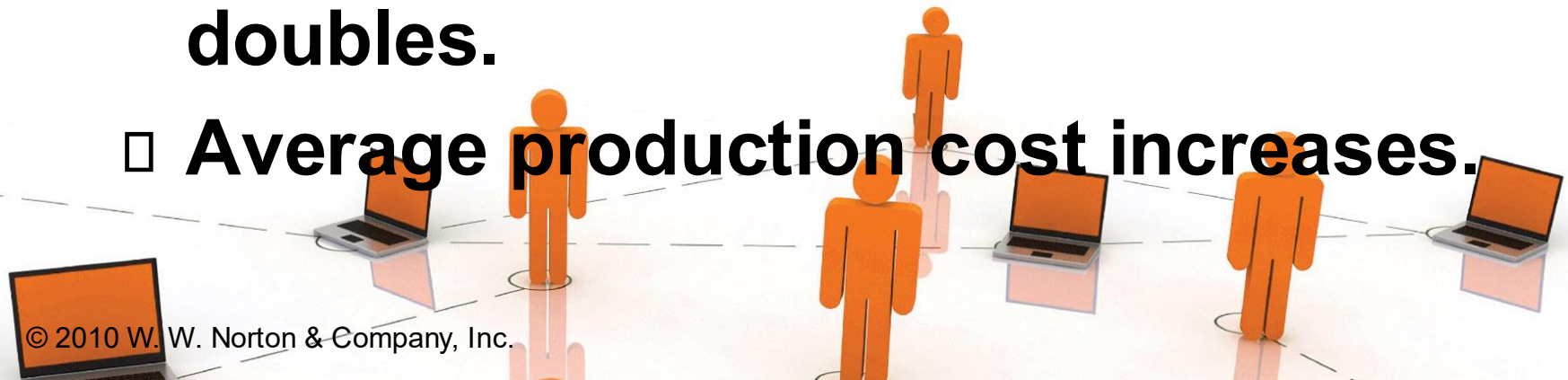
Decreasing Returns-to-Scale and Average Total Costs

- **If a firm's technology exhibits decreasing returns-to-scale then doubling its output level from y' to $2y'$ requires more than doubling all input levels.**
- **Total production cost more than doubles.**



Decreasing Returns-to-Scale and Average Total Costs

- If a firm's technology exhibits decreasing returns-to-scale then doubling its output level from y' to $2y'$ requires more than doubling all input levels.
- Total production cost more than doubles.
- Average production cost increases.



Increasing Returns-to-Scale and Average Total Costs

- **If a firm's technology exhibits increasing returns-to-scale then doubling its output level from y' to $2y'$ requires less than doubling all input levels.**



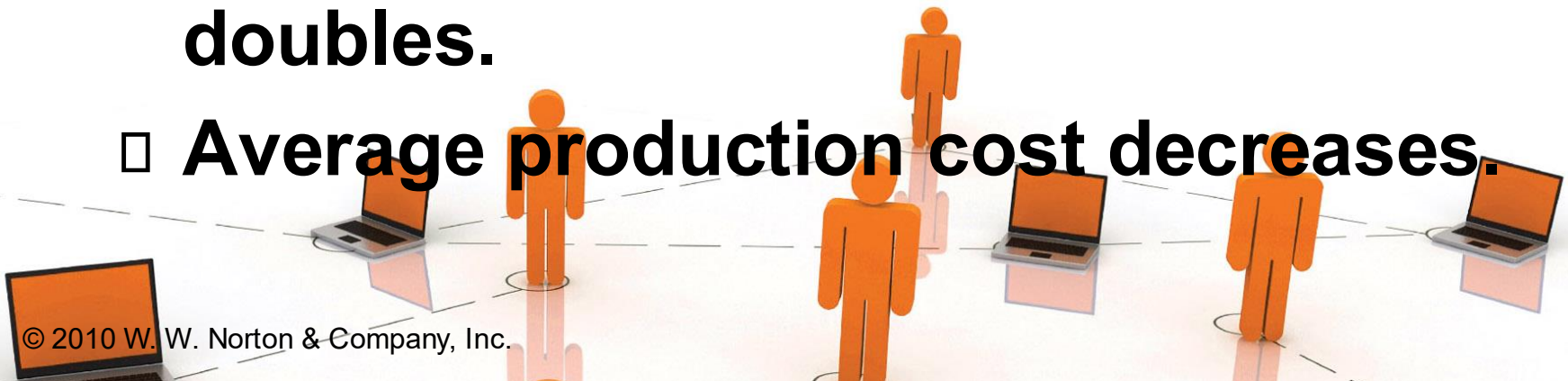
Increasing Returns-to-Scale and Average Total Costs

- If a firm's technology exhibits increasing returns-to-scale then doubling its output level from y' to $2y'$ requires less than doubling all input levels.
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Increasing Returns-to-Scale and Average Total Costs

- If a firm's technology exhibits increasing returns-to-scale then doubling its output level from y' to $2y'$ requires less than doubling all input levels.
- Total production cost less than doubles.
- Average production cost decreases.



Returns-to-Scale and Av. Total Costs

\$/output unit

AC(y)

decreasing r.t.s.

constant r.t.s.

increasing r.t.s.

y



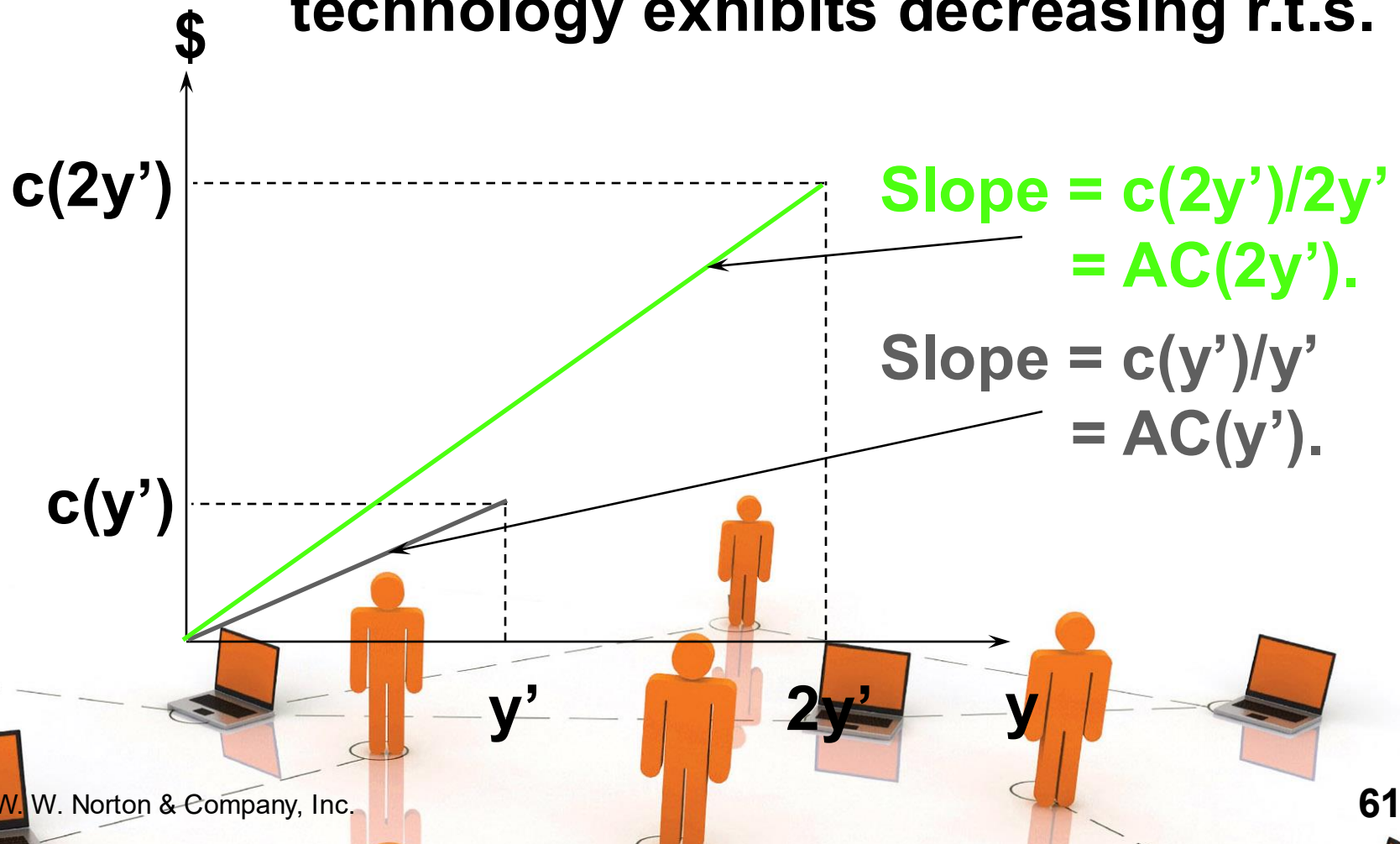
Returns-to-Scale and Total Costs

- **What does this imply for the shapes of total cost functions?**



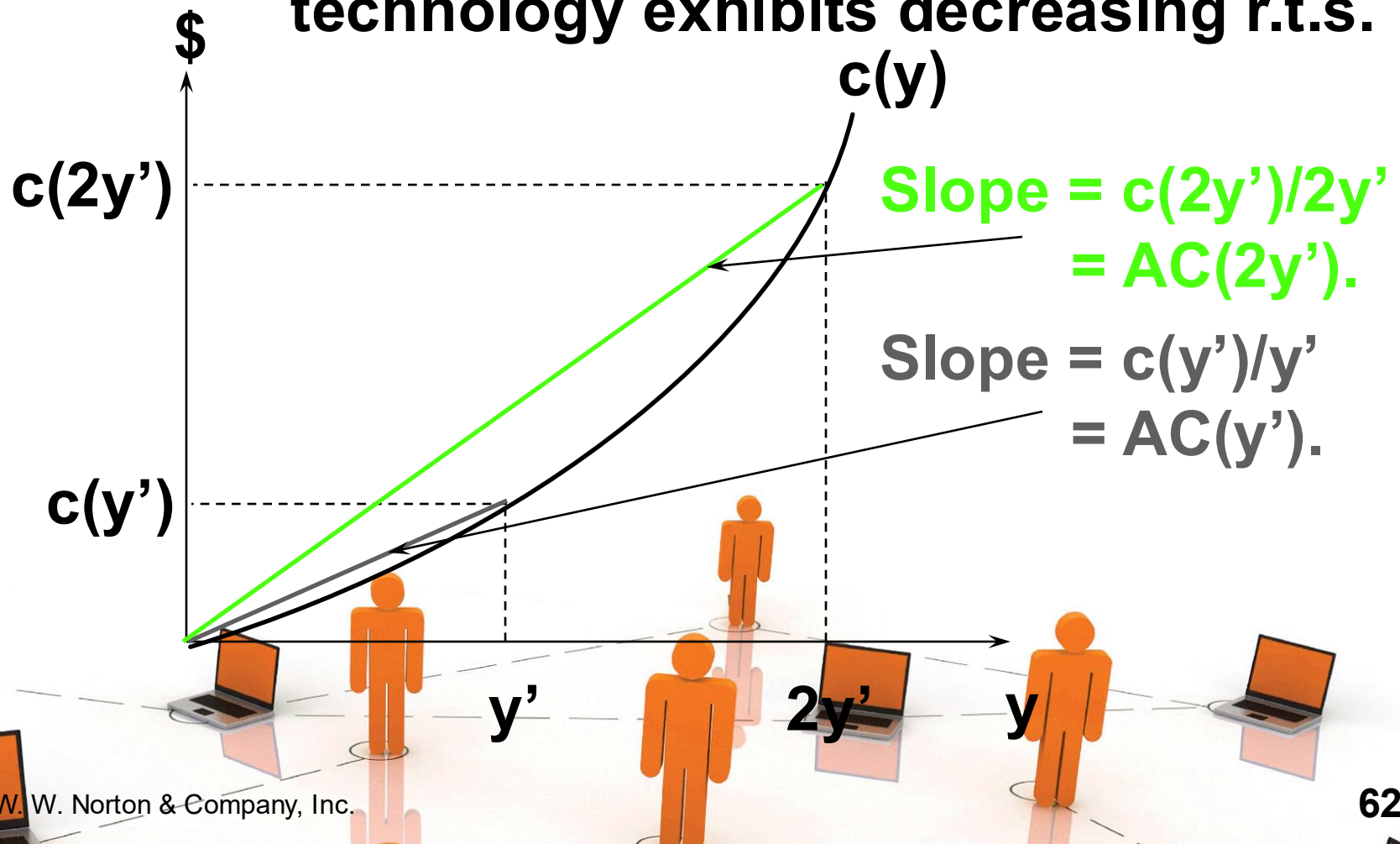
Returns-to-Scale and Total Costs

Av. cost increases with y if the firm's technology exhibits decreasing r.t.s.



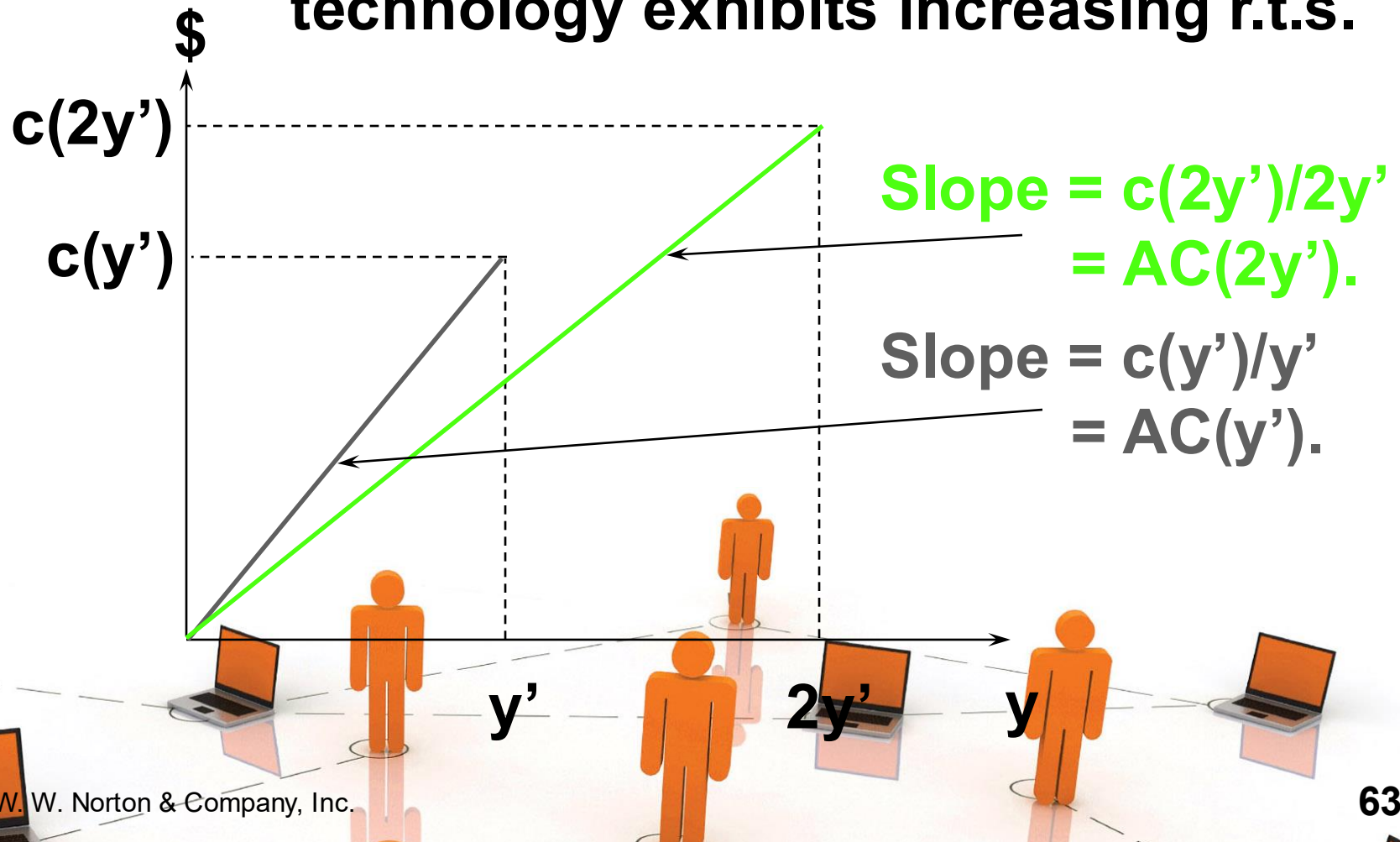
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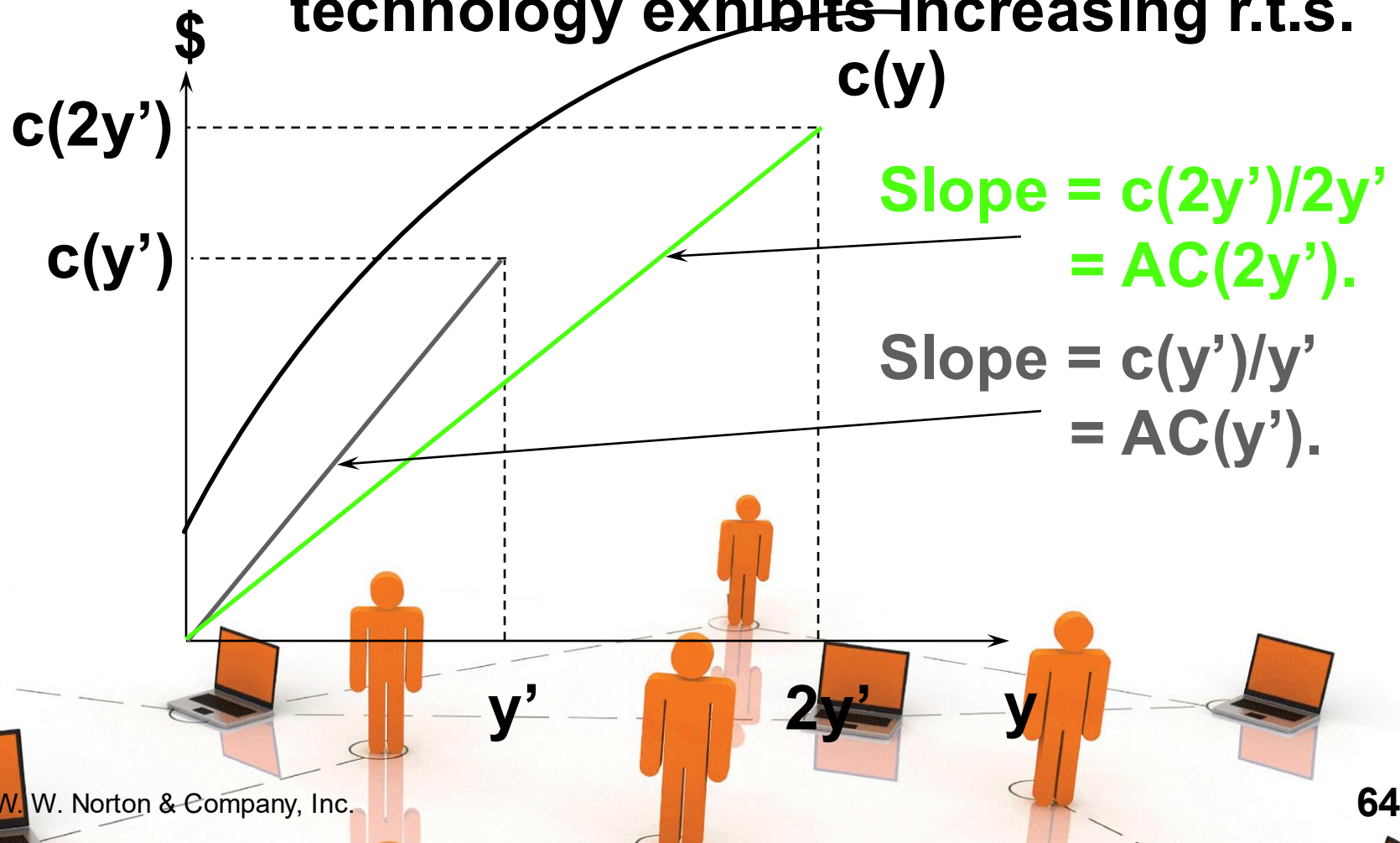
Returns-to-Scale and Total Costs

Av. cost decreases with y if the firm's technology exhibits increasing r.t.s.



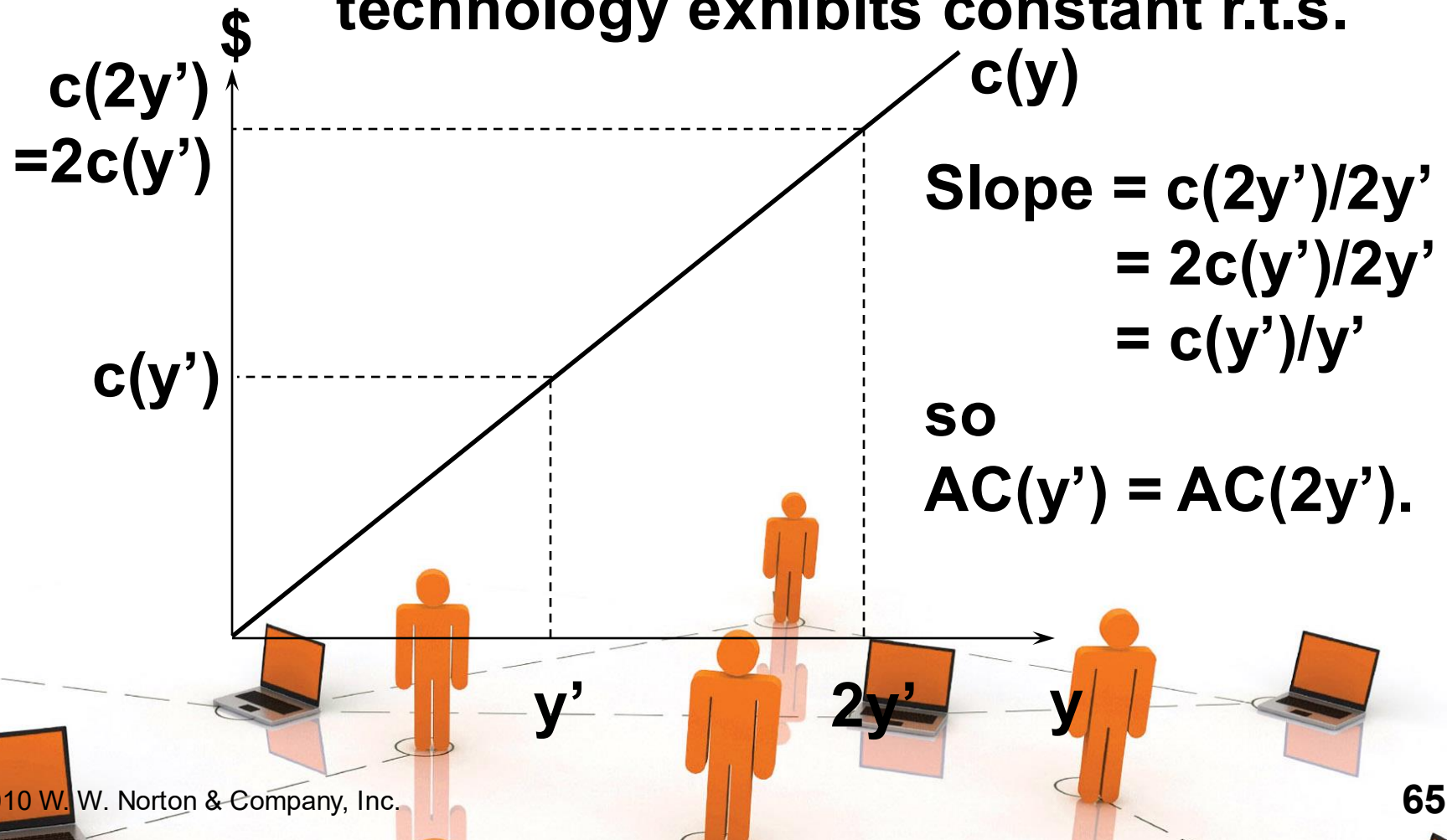
Returns-to-Scale and Total Costs

Av. cost decreases with y if the firm's technology exhibits increasing r.t.s.



Returns-to-Scale and Total Costs

Av. cost is constant when the firm's technology exhibits constant r.t.s.



Short-Run & Long-Run Total Costs

- ❑ In the long-run a firm can vary all of its input levels.
- ❑ Consider a firm that cannot change its input 2 level from x_2' units.
- ❑ How does the short-run total cost of producing y output units compare to the long-run total cost of producing y units of output?



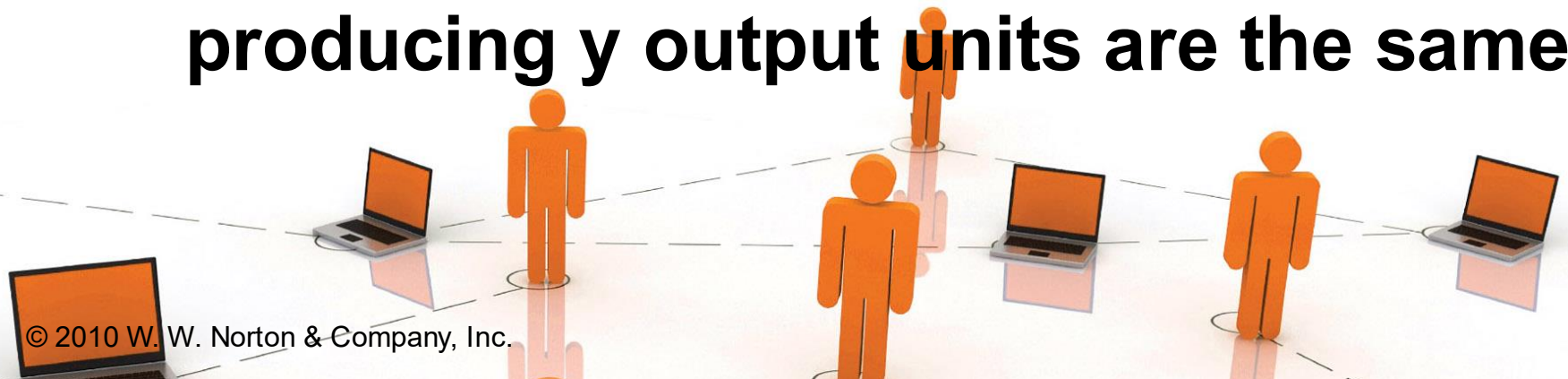
Short-Run & Long-Run Total Costs

- **The long-run cost-minimization problem is**
subject to
- **The short-run cost-minimization problem is**
subject to



Short-Run & Long-Run Total Costs

- The short-run cost-min. problem is the long-run problem subject to the extra constraint that $x_2 = x_2'$.
- If the long-run choice for x_2 was x_2' then the extra constraint $x_2 = x_2'$ is not really a constraint at all and so the long-run and short-run total costs of producing y output units are the same.

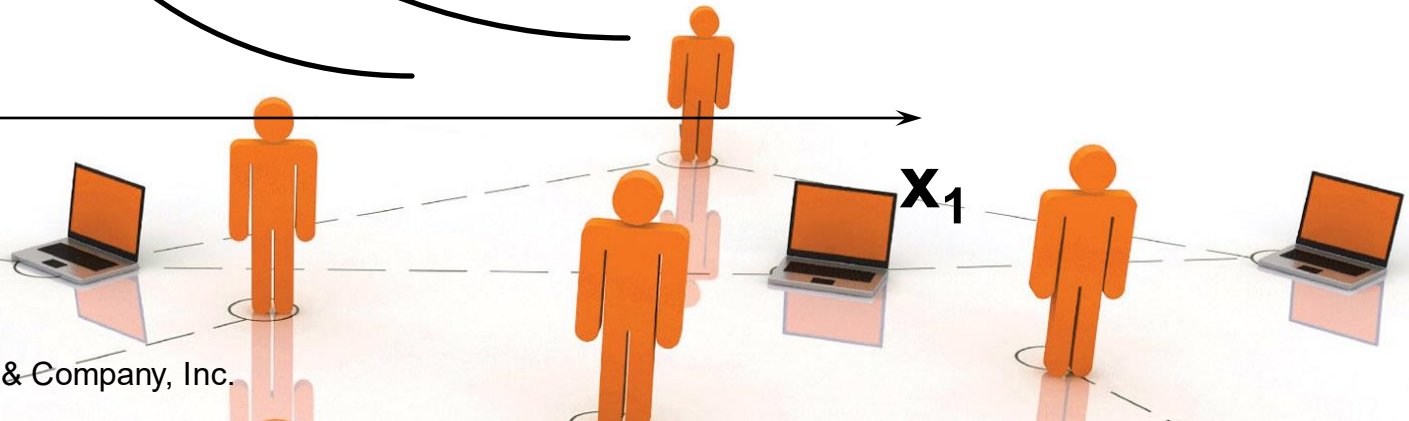
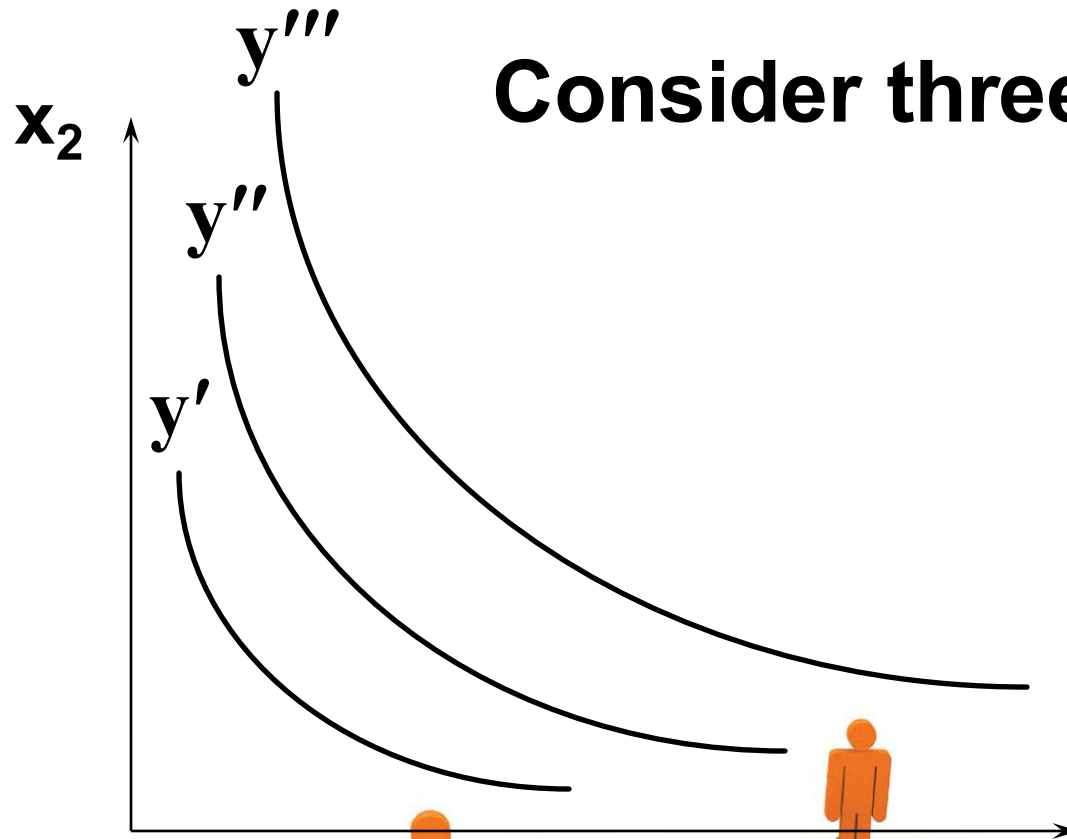


Short-Run & Long-Run Total Costs

- The short-run cost-min. problem is therefore the long-run problem subject to the extra constraint that $x_2 = x_2''$.
- But, if the long-run choice for $x_2 \neq x_2''$ then the extra constraint $x_2 = x_2''$ prevents the firm in this short-run from achieving its long-run production cost, causing the short-run total cost to exceed the long-run total cost of producing y output units.

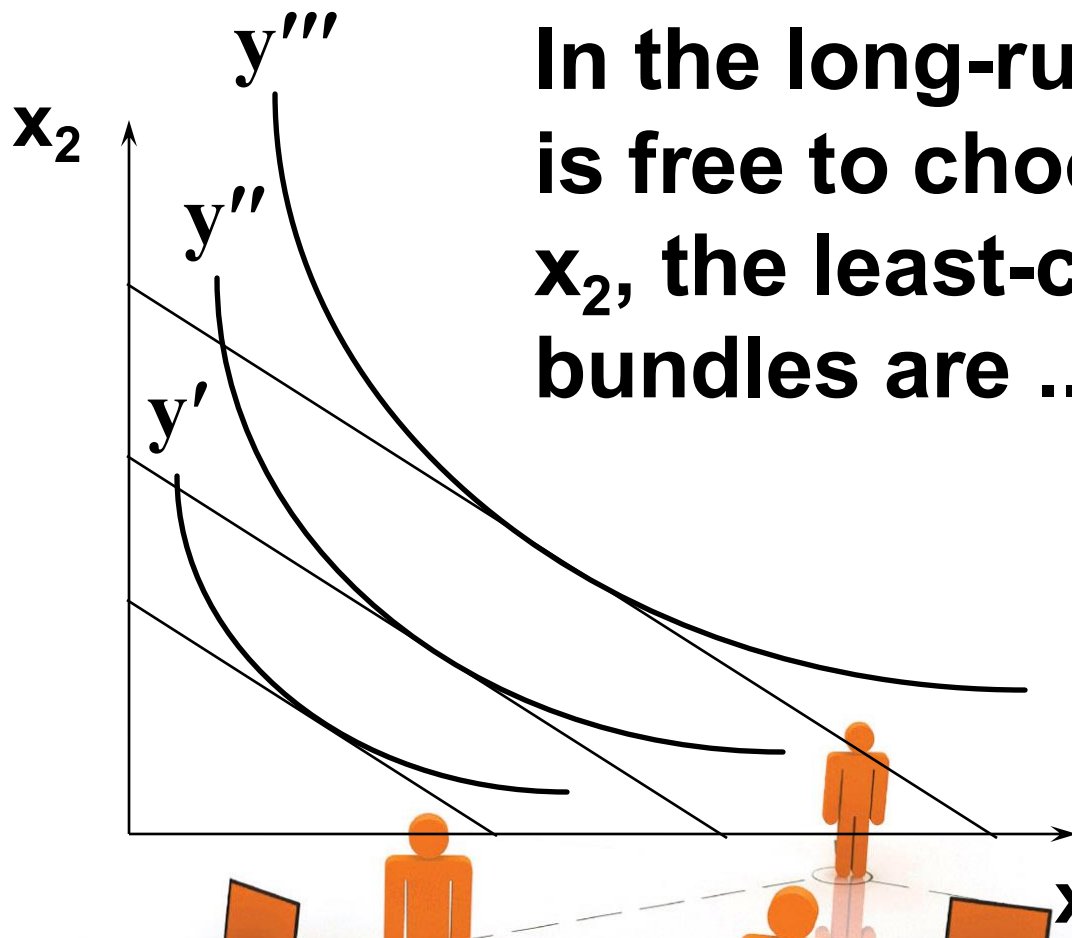
Short-Run & Long-Run Total Costs

Consider three output levels.

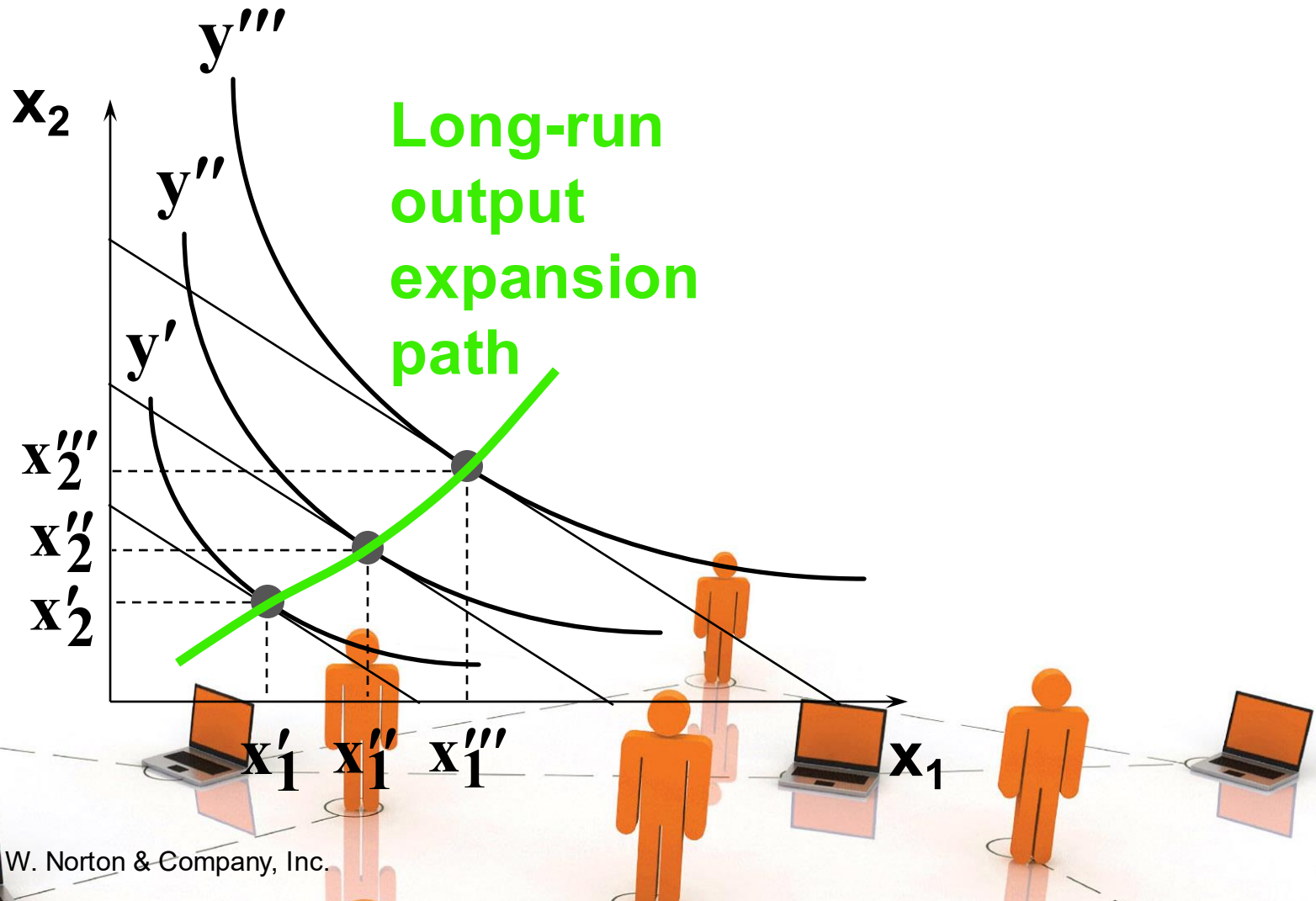


Short-Run & Long-Run Total Costs

In the long-run when the firm is free to choose both x_1 and x_2 , the least-costly input bundles are ...



Short-Run & Long-Run Total Costs



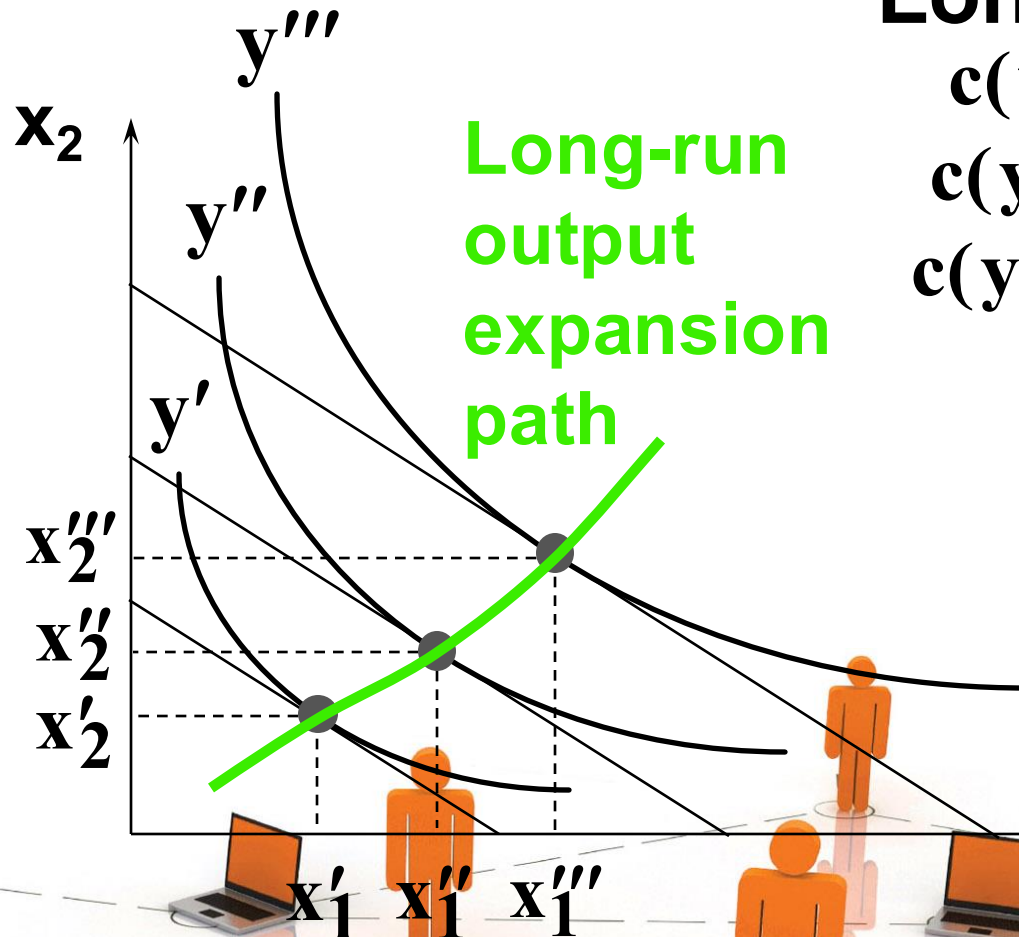
Short-Run & Long-Run Total Costs

Long-run costs are:

$$c(y') = w_1x'_1 + w_2x'_2$$

$$c(y'') = w_1x''_1 + w_2x''_2$$

$$c(y''') = w_1x'''_1 + w_2x'''_2$$



Short-Run & Long-Run Total Costs

- **Now suppose the firm becomes subject to the short-run constraint that $x_2 = x_2''$.**



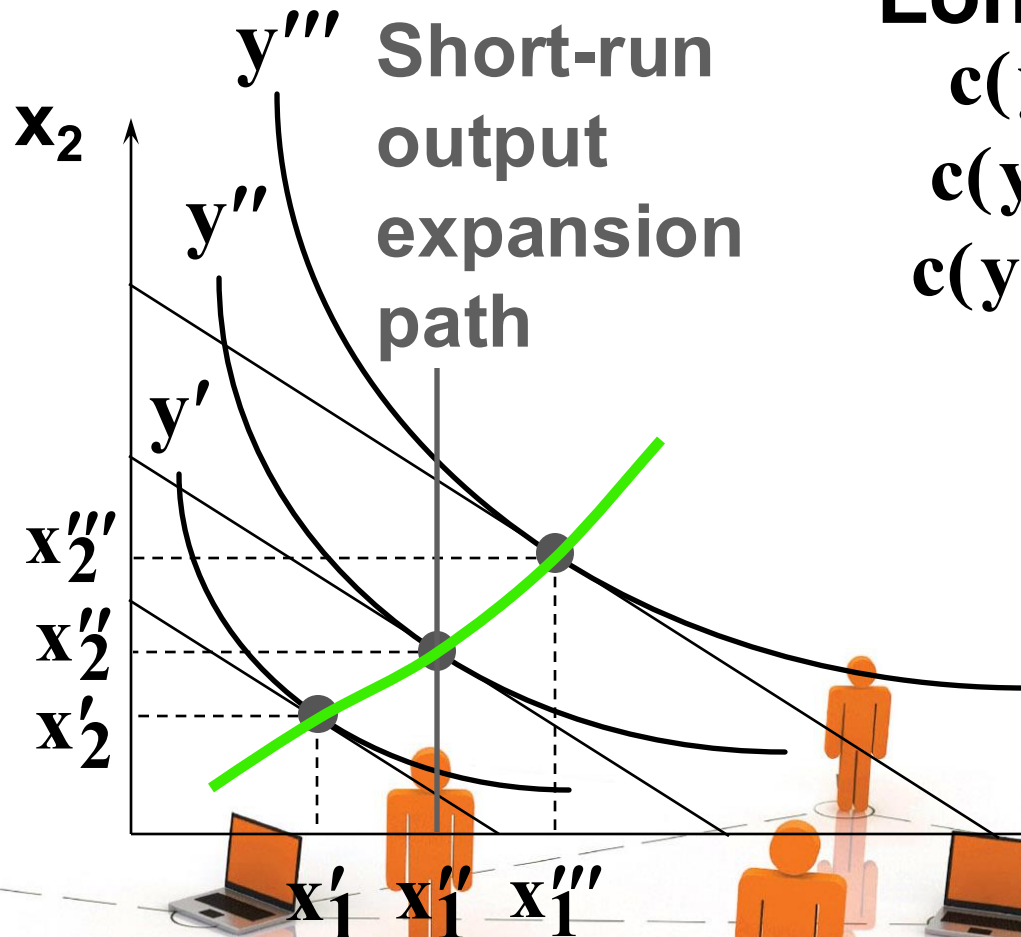
Short-Run & Long-Run Total Costs

Long-run costs are:

$$c(y') = w_1x'_1 + w_2x'_2$$

$$c(y'') = w_1x''_1 + w_2x''_2$$

$$c(y''') = w_1x'''_1 + w_2x'''_2$$



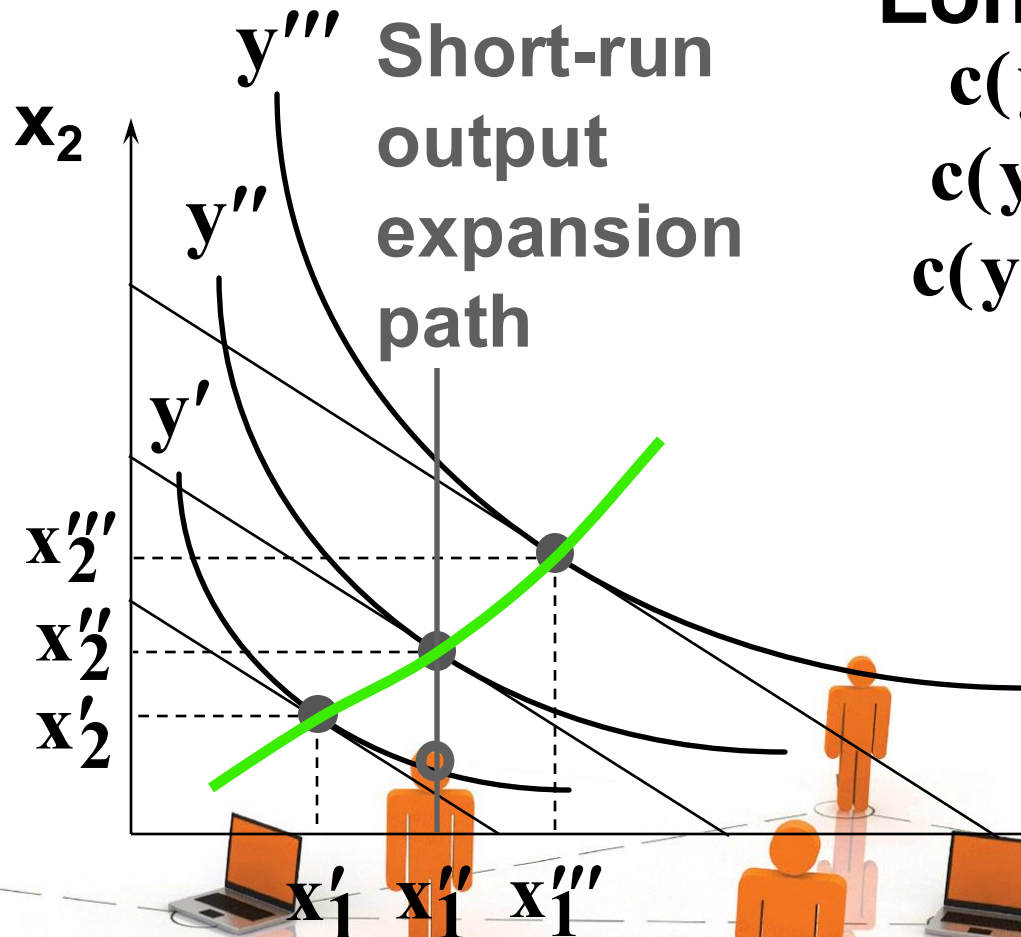
Short-Run & Long-Run Total Costs

Long-run costs are:

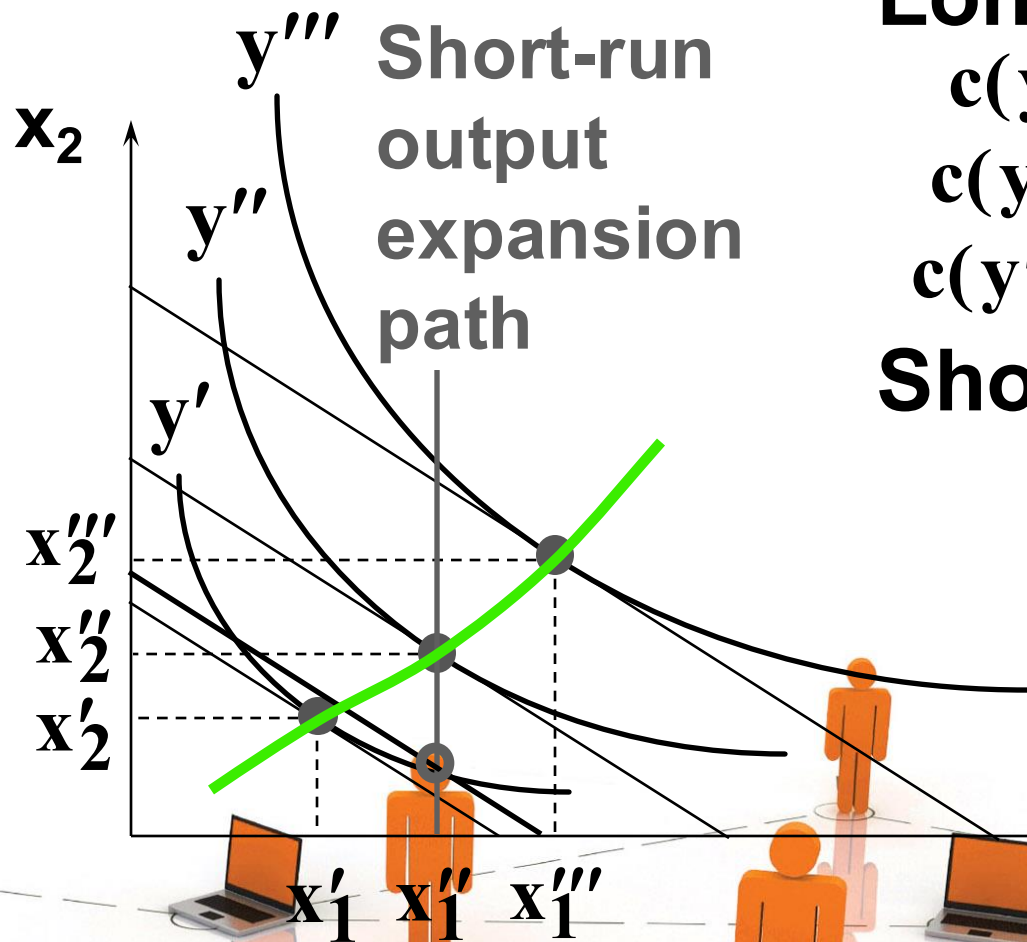
$$c(y') = w_1x'_1 + w_2x'_2$$

$$c(y'') = w_1x''_1 + w_2x''_2$$

$$c(y''') = w_1x'''_1 + w_2x'''_2$$



Short-Run & Long-Run Total Costs



Long-run costs are:

$$c(y') = w_1 x'_1 + w_2 x'_2$$

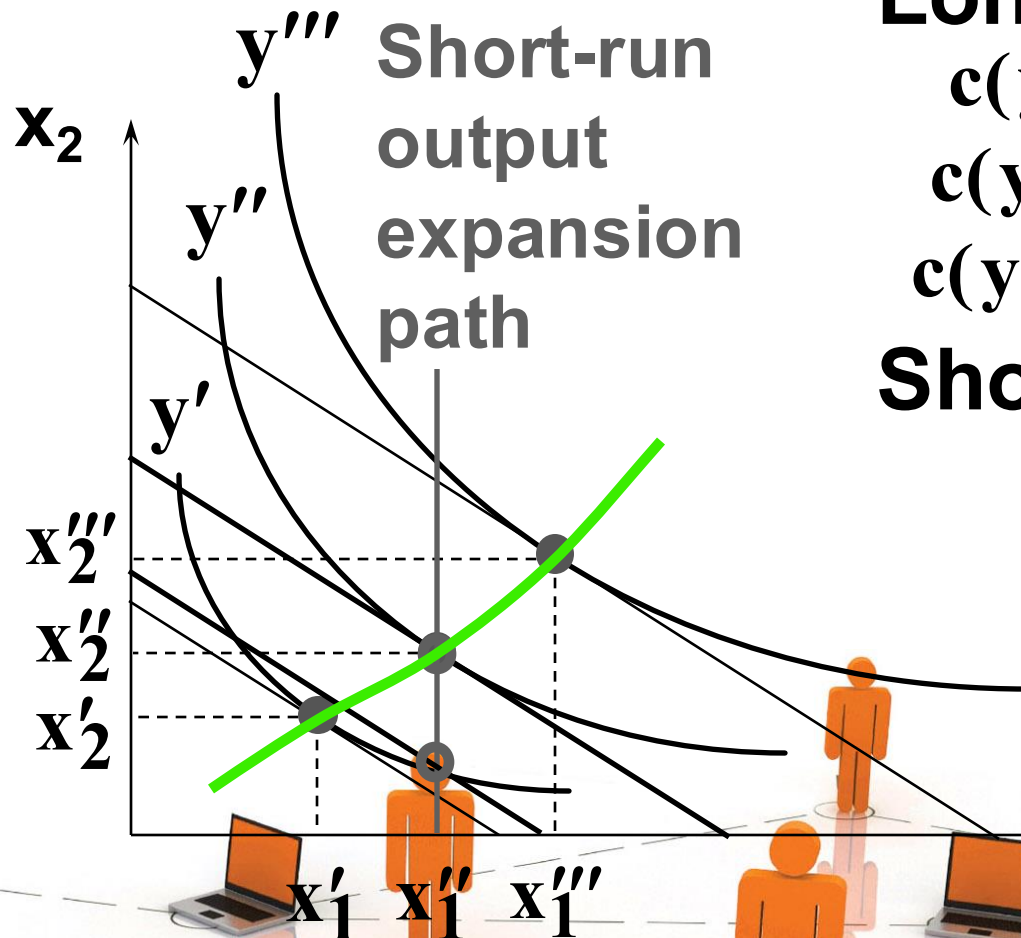
$$c(y'') = w_1 x''_1 + w_2 x''_2$$

$$c(y''') = w_1 x'''_1 + w_2 x'''_2$$

Short-run costs are:

$$c_s(y') > c(y')$$

Short-Run & Long-Run Total Costs



Long-run costs are:

$$c(y') = w_1 x_1' + w_2 x_2'$$

$$c(y'') = w_1 x_1'' + w_2 x_2''$$

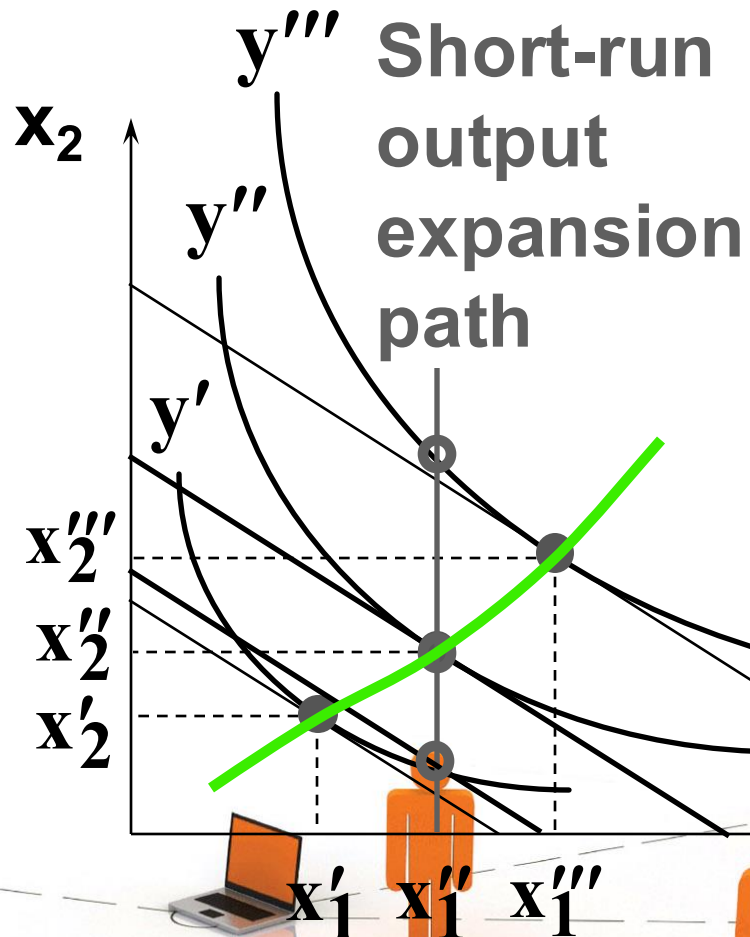
$$c(y''') = w_1 x_1''' + w_2 x_2'''$$

Short-run costs are:

$$c_s(y') > c(y')$$

$$c_s(y'') = c(y'')$$

Short-Run & Long-Run Total Costs



Long-run costs are:

$$\mathbf{c}(y') = w_1 \mathbf{x}'_1 + w_2 \mathbf{x}'_2$$

$$\mathbf{c}(y'') = \mathbf{w}_1 \mathbf{x}_1'' + \mathbf{w}_2 \mathbf{x}_2''$$

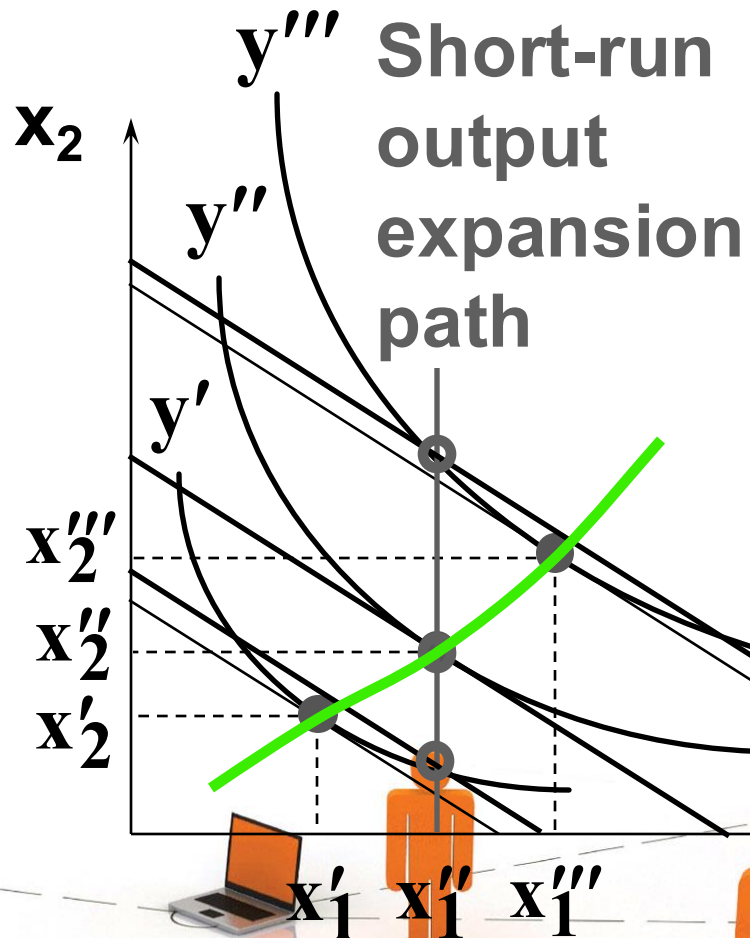
$$\mathbf{c}(y''') = \mathbf{w}_1 \mathbf{x}_1'' + \mathbf{w}_2 \mathbf{x}_2''$$

Short-run costs are:

$$\mathbf{c}_s(\mathbf{y}') > \mathbf{c}(\mathbf{y}')$$

$$\mathbf{c}_s(\mathbf{y}'') = \mathbf{c}(\mathbf{y}'')$$

Short-Run & Long-Run Total Costs



Long-run costs are:
 $c(y') = w_1 x_1' + w_2 x_2'$
 $c(y'') = w_1 x_1'' + w_2 x_2''$
~~**Short-run costs are:**~~
 $c(y''') = w_1 x_1''' + w_2 x_2'''$

$$c_s(y') > c(y')$$

$$c_s(y'') = c(y'')$$

$$c_s(y''') > c(y''')$$

Short-Run & Long-Run Total Costs

- **Short-run total cost exceeds long-run total cost except for the output level where the short-run input level restriction is the long-run input level choice.**
- **This says that the long-run total cost curve always has one point in common with any particular short-run total cost curve.**

Short-Run & Long-Run Total Costs

A short-run total cost curve always has one point in common with the long-run total cost curve, and is elsewhere higher than the long-run total cost curve.

