

Spring 2024-
2025
ECON 205

Problem Set 7

- 1) A large lot of manufactured items contains 10% with exactly one defect, 5% with more than one defect, and the remainder with no defects. Ten items are randomly selected from this lot for sale. If Y_1 denotes the number of items with one defect and Y_2 , the number with more than one defect, the repair costs are $Y_1 + 3Y_2$. Find the mean and variance of the repair costs.

2)

A learning experiment requires a rat to run a maze (a network of pathways) until it locates one of three possible exits. Exit 1 presents a reward of food, but exits 2 and 3 do not. (If the rat eventually selects exit 1 almost every time, learning may have taken place.) Let Y_i denote the number of times exit i is chosen in successive runnings. For the following, assume that the rat chooses an exit at random on each run.

- a Find the probability that $n = 6$ runs result in $Y_1 = 3$, $Y_2 = 1$, and $Y_3 = 2$.
- b For general n , find $E(Y_1)$ and $V(Y_1)$.
- c Find $\text{Cov}(Y_2, Y_3)$ for general n .
- d To check for the rat's preference between exits 2 and 3, we may look at $Y_2 - Y_3$. Find $E(Y_2 - Y_3)$ and $V(Y_2 - Y_3)$ for general n .

3)

Let the joint density of two random variables x_1 and x_2 be given by

$$f(x_1, x_2) = \begin{cases} \frac{3}{8}x_1, & 0 \leq x_2 \leq x_1 \leq 2 \\ 0, & \text{otherwise} \end{cases}$$

Find the covariance between x_1 and x_2 .

4)

Let X and Y have a joint density function

$$f(x, y) = \begin{cases} 2, & 0 < x < y < 1 \\ 0, & \text{otherwise} \end{cases}$$

- i) Find $f_X(x)$ and $f_Y(y)$.
- ii) Find $f_{Y|X}(Y|X = x)$ and $f_{X|Y}(X|Y = y)$.
- iii) Find $P(Y > 1/2|X = 1/2)$.
- iv) Find $P(X > 1/3|Y = 2/3)$.

5) The bivariate density function is defined as;

$$f(x, y) = \begin{cases} c(x^2 + xy) & \text{if } 0 \leq x \leq 1, 0 \leq y \leq 1 \\ 0, & \text{elsewhere} \end{cases}$$

- a) Find c and $P(X > Y)$.
- b) Find marginal densities of X and Y .
- c) Are X and Y independent?

6) A valid joint probability density function is given by

$$f(y_1, y_2) = \begin{cases} 4y_1y_2, & 0 \leq y_1 \leq 1, 0 \leq y_2 \leq 1 \\ 0, & \text{elsewhere} \end{cases}$$

- a) the marginal density functions for Y_1 and Y_2 .
- b) $P(Y_1 \leq 1/2 | Y_2 \geq 3/4)$.
- c) the conditional density function of Y_1 given $Y_2 = y_2$.
- d) the conditional density function of Y_2 given $Y_1 = y_1$.
- e) $P(Y_1 \leq 3/4 | Y_2 = 1/2)$.

f) Are Y_1 and Y_2 are independent?

7) For the given joint density function

$$f(y_1, y_2) = \begin{cases} 6(1 - y_2), & 0 \leq y_1 \leq y_2 \leq 1, \\ 0, & \text{elsewhere} \end{cases}$$

- a) Are Y_1 and Y_2 are independent?
- b) Find $E(Y_1)$ and $V(Y_1)$.
- c) Find $\text{Cov}(Y_1, Y_2)$.

8) For the following joint density function

$$f(y_1, y_2) = \begin{cases} e^{-y_1}, & 0 \leq y_2 \leq y_1 < \infty \\ 0, & \text{elsewhere} \end{cases}$$

- a) $P(Y_1 < 2, Y_2 > 1)$.
- b) $P(Y_1 \geq 2Y_2)$.
- c) $P(Y_1 - Y_2 \geq 1)$.
- d) Find the marginal density functions for Y_1 and Y_2 .
- e) What is the conditional density function of Y_1 given that $Y_2=y_2$? Be sure to specify the values of y_2 for which this conditional density is defined.
- f) What is the conditional density function of Y_2 given that $Y_1=y_1$? Be sure to specify the values of y_1 for which this conditional density is defined.

- 9) A health-food store stocks two different brands of a type of grain. Let X = the amount (lb) of brand A on hand and Y = the amount of brand B on hand. Suppose the joint pdf of X and Y is

$$f(x, y) = kxy \quad x \geq 0, y \geq 0, 20 \leq x + y \leq 30$$

- Draw the region of positive density and determine the value of k .
- Are X and Y independent? Answer by first deriving the marginal pdf of each variable.
- Compute $P(X + Y \leq 25)$.
- What is the expected total amount of this grain on hand?
- Compute $\text{Cov}(X, Y)$ and $\text{Corr}(X, Y)$.
- What is the variance of the total amount of grain on hand?