

Middle East Technical University
Department of Economics
Econ 206: Statistics for Economists-II

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Recitation 5

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Question 1

Suppose we have the following data to estimate the relationship between X and Y using the linear population model:

$$y_i = \beta_0 + \beta_1 x_i + \epsilon_i \quad (1)$$

Table 1: X and Y Series

y	X
4	1
5	4
7	5
12	6

Calculate $\hat{\beta}_0$ and $\hat{\beta}_1$ using the least squares estimators.

Answer

The least squares estimators are given by

$$\hat{\beta}_1 = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sum (x_i - \bar{x})^2}, \quad \hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}.$$

We compute the sample means:

$$\bar{x} = \frac{1 + 4 + 5 + 6}{4} = 4, \quad \bar{y} = \frac{4 + 5 + 7 + 12}{4} = 7.$$

Then,

$$\sum (x_i - \bar{x})(y_i - \bar{y}) = 19, \quad \sum (x_i - \bar{x})^2 = 14.$$

Hence,

$$\hat{\beta}_1 = \frac{19}{14}, \quad \hat{\beta}_0 = 7 - \frac{19}{14} \cdot 4 = \frac{11}{7}.$$

Question 2

Suppose the population regression function is

$$y_i = \beta_0 + \beta_1(x_i - \bar{x}) + \epsilon_i.$$

Derive the least squares estimator for β_0 . Note that $\sum(x_i - \bar{x}) = 0$.

Answer

Let $z_i = x_i - \bar{x}$.

The objective function is $S(\beta_0, \beta_1) = \sum[y_i - \beta_0 - \beta_1 z_i]^2$.

Taking the partial derivative with respect to β_0 :

$$\frac{\partial S}{\partial \beta_0} = \sum 2[y_i - \beta_0 - \beta_1 z_i](-1) = 0 \implies \sum y_i - n\beta_0 - \beta_1 \sum z_i = 0 \quad (2)$$

Since $\sum z_i = \sum(x_i - \bar{x}) = 0$, we get $n\beta_0 = \sum y_i$, thus $\hat{\beta}_0 = \bar{y}$.

Question 3

Take the population model with no intercept:

$$y_i = \beta x_i + \epsilon_i.$$

- (a) Find the least squares estimator for β .
- (b) Suppose that we have the same data points for y and x as in Question 1. Calculate $\hat{\beta}$ using the data.

Answer

(a) The least squares estimator without an intercept is

$$\hat{\beta} = \frac{\sum x_i y_i}{\sum x_i^2}.$$

(b) Using the data:

$$\sum x_i y_i = 131, \quad \sum x_i^2 = 78.$$

Thus,

$$\hat{\beta} = \frac{131}{78}.$$

Question 4

Inflation forecasts for the coming year were asked to economists in the business world. The forecasts are normally distributed with standard deviation 1.8. What is the probability that the standard deviation of the forecasts of 15 selected economists is more than 2.60?

Answer

For a normal population,

$$\frac{(n-1)S^2}{\sigma^2} \sim \chi_{n-1}^2.$$

Here, $n = 15$ and $\sigma = 1.8$. We compute

$$P(S > 2.60) = P\left(\chi_{14}^2 > \frac{14 \cdot (2.60)^2}{(1.8)^2}\right) = P(\chi_{14}^2 > 29.2).$$

From chi-square tables,

$$P(\chi_{14}^2 > 29.2) \approx 0.01.$$

Question 5

Let X_1 , X_2 and X_3 be random observations from a population with mean μ and variance σ^2 . Consider the estimators:

$$\mu_1 = \frac{1}{3}(X_1 + X_2 + X_3), \quad \mu_2 = \frac{1}{2}(X_1 + X_2).$$

- **a-)** Show that each estimator is unbiased.
- **b-)** Which estimator is more efficient?
- **c-)** Find the relative efficiency of μ_1 compared to μ_2 .

Answer

(a) Since $E(X_i) = \mu$,

$$E(\mu_1) = \mu, \quad E(\mu_2) = \mu.$$

Both estimators are unbiased.

(b) Their variances are

$$\text{Var}(\mu_1) = \frac{\sigma^2}{3}, \quad \text{Var}(\mu_2) = \frac{\sigma^2}{2}.$$

Thus, μ_1 is more efficient.

(c) The relative efficiency is

$$\text{RE}(\mu_1, \mu_2) = \frac{\text{Var}(\mu_2)}{\text{Var}(\mu_1)} = \frac{3}{2}.$$