

Q.1  $C(q) = q^2 + 25q + 144$

a)  $P = 75$   $q^* = ?$

$$\pi(q) = 75q - q^2 - 25q - 144 = 50q - q^2 - 144$$

$$\frac{\partial \pi}{\partial q} = 0 \Rightarrow \underbrace{75}_{MR} - \underbrace{(2q+25)}_{MC} = 0 \Rightarrow q^* = 25 \quad \left( \text{note that } \frac{\partial^2 \pi(q)}{\partial q^2} < 0 \right)$$

At  $\frac{\partial \pi}{\partial q} = P - MC = 0 \Rightarrow P = MC(q) = MR$  so, we have a maximum  
 $MC = 2q + 25$   $MC(25) = 75 = P$

b)  $\pi(q) = P \cdot q - q^2 - 25q - 144 = (P - 25)q - q^2 - 144$

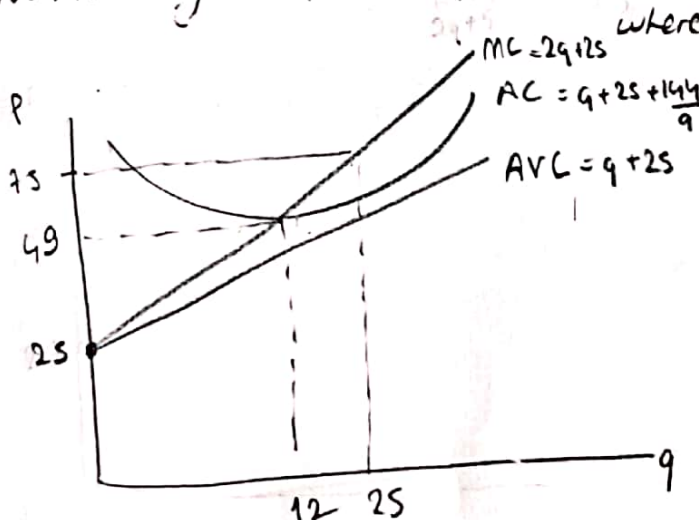
First, find the optimum quantity in terms of price.

$$\frac{\partial \pi(q)}{\partial q} = 0 \Rightarrow P - 25 - 2q = 0 \Rightarrow \boxed{q^* = \frac{P - 25}{2}}$$

$$\pi(q^*) = 0 \Rightarrow \frac{(P - 25)(P - 25)}{2} - \frac{(P - 25)^2}{2} - 144 = 0$$

$$= (P - 25)^2 = 144 \Rightarrow \boxed{P = 49}$$

c) No. As long as  $P > 25$ ,  $q^* > 0$ . Shut down price is  $P = 25$  where  $q^* = 0$ .



$$MC = 2q + 25$$

$$AC = q + 25 + \frac{144}{q}$$

$$\text{At } P = MC = AC \Rightarrow \pi^* = 0$$

But shut down price is at  $P = AVC$

$$AVC = q + 25$$

$$AVC = MC \text{ at } P = 25.$$

$$\pi^* = -144$$

As long as profits higher than fixed costs, firm continue to produce in the SR.

Q.2  $C(q) = 20 + 6q + 5q^2$

a)  $\pi(q) = pq - 20 - 6q - 5q^2 = q(p-6) - 5q^2 - 20$

$\frac{\partial \pi(q)}{\partial q} = 0 \Rightarrow p-6-10q = 0 \Rightarrow q^* = \frac{p-6}{10} \rightarrow \text{if } p > 6$

$\frac{\partial^2 \pi(q)}{\partial q^2} = -10 < 0 \Rightarrow 2^{\text{nd}} \text{ order condition is satisfied.}$

b)  $\pi(q) = (p-t-6)q - 5q^2 - 20$

$\frac{\partial \pi(q)}{\partial q} = 0 \Rightarrow q^* = \frac{p-t-6}{10}$

c)  $\frac{\partial q^*}{\partial t} = -\frac{1}{10} < 0 \Rightarrow \text{As } t \text{ increases } q \text{ decreases}$

d)  $\pi(q^*) = (p-t-6) \left( \frac{p-t-6}{10} \right) - 5 \left( \frac{p-t-6}{10} \right)^2 - 20$   
 $= \frac{(p-t-6)^2}{20} - 20$

$\frac{\partial \pi(q^*)}{\partial t} = -\frac{(p-t-6)}{10} < 0 \Rightarrow \text{Profit decreases as } t \text{ increases.}$

Q.3  $q = (L^{1/2} + K^{1/2})^2$      $\bar{q} = 121.000$      $\underbrace{P_L}_{\omega} = 10$      $\underbrace{P_K}_{r} = 1$

min  $\omega L + rK$

s.t  $\bar{q} - (L^{1/2} + K^{1/2})^2 = 0$

$\mathcal{L}(K, L, \lambda) = \omega L + rK + \lambda (\bar{q} - (L^{1/2} + K^{1/2})^2)$

FOCs

$\frac{\partial \mathcal{L}}{\partial L} = 0 \Rightarrow \omega - \lambda \cdot 2 \cdot (L^{1/2} + K^{1/2}) \cdot \frac{1}{2} \cdot L^{-1/2} = 0$

$\frac{\partial \mathcal{L}}{\partial K} = 0 \Rightarrow r - \lambda \cdot 2 \cdot (L^{1/2} + K^{1/2}) \cdot \frac{1}{2} \cdot K^{-1/2} = 0$

$\frac{\partial \mathcal{L}}{\partial \lambda} = 0 \Rightarrow \bar{q} - (L^{1/2} + K^{1/2})^2 = 0$

$\left[ \frac{r}{\omega} \cdot K^{1/2} + K^{1/2} \right]^2 = \bar{q} \Rightarrow \left[ K^{1/2} \left( 1 + \frac{r}{\omega} \right) \right]^2 = \bar{q}$   
 $\Rightarrow K = \frac{\bar{q}}{\left( 1 + \frac{r}{\omega} \right)^2}$

$r = 1, \omega = 10, \bar{q} = 121.000$

$\Rightarrow K = \frac{121.000}{\left( 1 + \frac{1}{10} \right)^2} \Rightarrow K = 100.000$   
 $L = 1000$

Q.4     $Q = 2KL + K$      $P_L = 4$      $P_K = 5$

a)  $E = 9$      $q = 45$      $L = ?$

$$45 = 2 \cdot 9 \cdot L + 9 \Rightarrow L = 2$$

b)  $TC = P_K \cdot E + P_L \cdot L$

$$= 5 \cdot 9 + 4 \cdot 2 = 53$$

If  $K$  is not fixed, cost minimization implies

$$L = P_K \cdot K + P_L \cdot L + \lambda (45 - 2KL - K)$$

FOCs

$$\frac{\partial L}{\partial K} = 0 \Rightarrow 5 - \lambda(2L + 1) = 0$$

$$\frac{\partial L}{\partial L} = 0 \Rightarrow 4 - 2\lambda K = 0$$

$$\frac{\partial L}{\partial \lambda} = 0 \Rightarrow 45 - 2KL - K = 0$$

$$\frac{5}{2} = \frac{2L+1}{2K}$$

$$\Rightarrow 15K - 4L = 2$$

$$\Rightarrow L = \frac{5K-2}{4}$$

$$45 - 2K \cdot \frac{(5K-2)}{4} - K = 0 \Rightarrow 90 - 5K^2 = 0$$

$$\Rightarrow K = 3\sqrt{2} \approx 4.24$$

$$L = 4.80$$

$$TC = 5 \cdot 4.24 + 4 \cdot 4.80 \approx 40.4$$

$$\Delta TC = 53 - 40.4 = 12.6$$

Q.5  $f(K, L) = K^{0.5} \cdot L^{0.5}$   $P_K = 10$ ,  $P_L = 40$

a)  $L(K, L, \lambda) = P_K \cdot K + P_L \cdot L + \lambda(\bar{q} - K^{0.5} \cdot L^{0.5})$

b) FOCs

$$\left. \begin{array}{l} K: P_K - \frac{1}{2} \cdot \lambda \cdot K^{-1/2} \cdot L^{1/2} = 0 \\ L: P_L - \frac{1}{2} \lambda \cdot K^{1/2} \cdot L^{-1/2} = 0 \end{array} \right\} \frac{P_K}{P_L} = \frac{L}{K} \Rightarrow \boxed{\frac{K}{L} = \frac{P_L}{P_K}}$$

$\lambda: K^{1/2} \cdot L^{1/2} = \bar{q}$

c)  $\bar{q} = 1000$   
 $\left(\frac{P_L}{P_K}\right)^{1/2} \cdot L^{1/2} = 1000 \Rightarrow L^* = 500$   
 $K^* = 2000$

d)  $\bar{q} = 2000$   $\bar{K} = 2000$

$f(\bar{K}, L) = 2000^{1/2} \cdot L^{1/2} = 2000 \Rightarrow L^* = 2000$

$TC_{SR} = 10 \cdot 2000 + 40 \cdot 2000 = \underline{100,000}$

e)  $\bar{q} = 2000$

$\left(\frac{P_L}{P_K}\right)^{1/2} \cdot L = 2000 \Rightarrow L^* = 1,000$   
 $K^* = 4,000$

$TC_{LR} = 10 \cdot 4,000 + 40 \cdot 1,000 = \underline{80,000}$

$TC_{LR}(\bar{q}) < TC_{SR}(\bar{q})$

f) In the short run, capital was fixed, she couldn't choose the optimum amount of capital and labor. But in the long run both inputs are variable and she can choose the input combination that minimizes the cost.

Q.6  $f(x,y) = x^{1/2} + y^{1/2}$

a)  $\pi = P(x^{1/2} + y^{1/2}) - P_x \cdot x - P_y \cdot y$

$$\frac{\partial \pi}{\partial x} = 0 \Rightarrow \underbrace{\frac{1}{2} \cdot P \cdot x^{-1/2}}_{P \cdot \partial f / \partial x} - P_x = 0$$

$$\frac{\partial \pi}{\partial y} = 0 \Rightarrow \frac{1}{2} \cdot P \cdot y^{-1/2} - P_y = 0$$

$$\frac{\partial \pi}{\partial x} = \overbrace{P \cdot \frac{\partial f}{\partial x}}^{MR_x} - \overbrace{P_x}^{MC_x} = 0$$

$$\frac{\partial \pi}{\partial y} = P \cdot \frac{\partial f}{\partial y} - P_y = 0$$

$$\Rightarrow x(P, P_x, P_y) = \left( \frac{P}{2P_x} \right)^2$$

$$\Rightarrow y(P, P_x, P_y) = \left( \frac{P}{2P_y} \right)^2$$

b)  $q_{LR}(P, P_x, P_y) = \frac{P}{2P_x} + \frac{P}{2P_y} = \frac{P}{2} \left( \frac{1}{P_x} + \frac{1}{P_y} \right)$

c)  $\pi_{LR}(P, P_x, P_y) = \frac{P^2}{2} \left( \frac{1}{P_x} + \frac{1}{P_y} \right) - P_x \left( \frac{P}{2P_x} \right)^2 - P_y \left( \frac{P}{2P_y} \right)^2 = \frac{P^2}{4} \left( \frac{1}{P_x} + \frac{1}{P_y} \right)$

d)  $\min P_x \cdot x + P_y \cdot y \quad \text{s.t.} \quad \bar{q} = x^{1/2} + y^{1/2}$

$$L = P_x \cdot x + P_y \cdot y + \lambda (\bar{q} - x^{1/2} + y^{1/2})$$

f.o.c.s

$$\left. \begin{array}{l} x: P_x - \frac{1}{2} \lambda \cdot x^{-1/2} = 0 \\ y: P_y - \frac{1}{2} \lambda \cdot y^{-1/2} = 0 \end{array} \right\} \frac{y}{x} = \left( \frac{P_x}{P_y} \right)^2 \Rightarrow y = \left( \frac{P_x}{P_y} \right)^2 \cdot x$$

$$\lambda: \bar{q} - x^{1/2} - y^{1/2} = 0$$

$$x^{1/2} + \frac{P_x}{P_y} \cdot x^{1/2} = \bar{q} \Rightarrow x^{1/2} \left( 1 + \frac{P_x}{P_y} \right) = \bar{q}$$

$$x^*(P_x, P_y, \bar{q}) = \frac{\bar{q}^2}{\left( 1 + \frac{P_x}{P_y} \right)^2} = \left( \frac{P_y \cdot \bar{q}}{P_x + P_y} \right)^2$$

$$y^*(P_x, P_y, \bar{q}) = \frac{\bar{q}^2}{\left( 1 + \frac{P_y}{P_x} \right)^2} = \left( \frac{P_x \cdot \bar{q}}{P_x + P_y} \right)^2$$

$$\begin{aligned}
 e) \quad C(p_x, p_y, \bar{q}) &= p_x \cdot x^*(p_x, p_y, \bar{q}) + p_y \cdot y^*(p_x, p_y, \bar{q}) \\
 &= \frac{p_x \cdot p_y^2 \bar{q}^2 + p_y \cdot p_x^2 \bar{q}^2}{(p_x + p_y)^2} = \frac{p_x \cdot p_y \cdot \bar{q}^2 (p_x + p_y)}{(p_x + p_y)^2} \\
 &= \frac{p_x \cdot p_y \cdot \bar{q}^2}{p_x + p_y}
 \end{aligned}$$

$$\bar{x} = 100 \Rightarrow f(\bar{x}, y) = 10 + y^{1/2}$$

$$f) \quad \pi = P(10 + y^{1/2}) - 100p_x - p_y \cdot y$$

$$\frac{\partial \pi}{\partial y} = 0 \Rightarrow \frac{P}{2} \cdot y^{-1/2} - p_y = 0 \Rightarrow y^*(P, p_x, p_y) = \left(\frac{P}{2p_y}\right)^2$$

$$g) \quad q_{sr}(P, p_x, p_y, \bar{x}) = 10 + (y^*(P, p_x, p_y))^{1/2} = 10 + \frac{P}{2p_y}$$

$$h) \quad \pi = P \cdot \left(10 + \frac{P}{2p_y}\right) - 100 \cdot p_x - p_y \cdot \left(\frac{P}{2p_y}\right)^2$$

$$(i) \quad \bar{q} = 10 + y^{1/2} \Rightarrow y = (\bar{q} - 10)^2 \quad \text{for } \bar{q} > 10$$

Conditional Input Demand

$$\begin{aligned}
 j) \quad C(p_x, p_y, P) &= p_x \cdot x(p_x, p_y, \bar{q}) + p_y \cdot y(p_x, p_y, \bar{q}) \\
 &= 100p_x + p_y(\bar{q} - 10)^2
 \end{aligned}$$

Q.7 a)  $f(x, y) = 2x + 3y \rightarrow$  Perfect substitutes

$$x^*(P_x, P_y, \bar{q}) = \begin{cases} 0 & \text{if } \frac{P_x}{P_y} > \frac{2}{3} \\ \frac{\bar{q}}{2} & \text{if } \frac{P_x}{P_y} < \frac{2}{3} \\ \in [0, \bar{q}] & \text{if } \frac{P_x}{P_y} = \frac{2}{3} \end{cases} \quad y^* = \begin{cases} \frac{\bar{q}}{3} & \text{if } \frac{P_x}{P_y} > \frac{2}{3} \\ 0 & \text{if } \frac{P_x}{P_y} < \frac{2}{3} \\ \in [0, \bar{q}] & \text{if } \frac{P_x}{P_y} = \frac{2}{3} \end{cases}$$

$$c(P_x, P_y, \bar{q}) = \begin{cases} P_y \cdot \frac{\bar{q}}{3} & \text{if } P_x/P_y > 2/3 \\ P_x \cdot \frac{\bar{q}}{2} & \text{if } P_x/P_y < 2/3 \\ P_x \cdot \frac{\bar{q}}{2} & \text{if } P_x/P_y = 2/3 \end{cases}$$

b)  $f(x, y, z, w) = x^\alpha y^\beta z^\gamma - w^\sigma \Rightarrow \frac{\partial f}{\partial w} < 0 \Rightarrow w = 0$  in any optimum choice

$$\min_{x, y, z} P_x \cdot x + P_y \cdot y + P_z \cdot z \quad \text{s.t.} \quad \bar{q} - x^\alpha y^\beta z^\gamma = 0$$

$$L = P_x \cdot x + P_y \cdot y + P_z \cdot z + \lambda (\bar{q} - x^\alpha y^\beta z^\gamma)$$

FOCs :

$$\begin{cases} x: & P_x - \lambda \alpha x^{\alpha-1} y^\beta z^\gamma = 0 \\ y: & P_y - \lambda \beta x^\alpha y^{\beta-1} z^\gamma = 0 \\ z: & P_z - \lambda \gamma x^\alpha y^\beta z^{\gamma-1} = 0 \\ \lambda: & \bar{q} - x^\alpha y^\beta z^\gamma = 0 \end{cases} \quad \left. \begin{aligned} \frac{P_x}{P_y} &= \frac{\alpha y}{\beta x} \Rightarrow y = x \cdot \frac{P_x}{P_y} \cdot \frac{\beta}{\alpha} \\ \frac{P_y}{P_z} &= \frac{\beta z}{\gamma y} \Rightarrow z = y \cdot \frac{P_y}{P_z} \cdot \frac{\gamma}{\beta} \\ &\Rightarrow z = x \cdot \frac{P_x}{P_z} \cdot \frac{\gamma}{\alpha} \end{aligned} \right\}$$

$$\Rightarrow \bar{q} = x^\alpha \cdot \left( x \cdot \frac{P_x}{P_y} \cdot \frac{\beta}{\alpha} \right)^\beta \cdot \left( x \cdot \frac{P_x}{P_z} \cdot \frac{\gamma}{\alpha} \right)^\gamma$$

$$\bar{q} = x^{\alpha+\beta+\gamma} \cdot \left( \frac{P_x}{P_y} \cdot \frac{\beta}{\alpha} \right)^\beta \cdot \left( \frac{P_x}{P_z} \cdot \frac{\gamma}{\alpha} \right)^\gamma \rightarrow \text{find } x(P_x, P_y, P_z, \bar{q}) \text{ and } y(\cdot), z(\cdot)$$

Then find  $c(P_x, P_y, P_z, \bar{q}) = P_x \cdot x(P_x, P_y, P_z, \bar{q}) + P_y \cdot y(\cdot) + P_z \cdot z(\cdot)$

c)  $f(x,y) = (2x+3y)^{1/2} \rightarrow$  Perfect substitutes

Same with part (a), except that  $\bar{q}$ 's will replace with  $\bar{q}^2$ .

d)  $f(x,y) = \min\{2x, 3y\} \rightarrow$  Perfect complements

at optimum  $\Rightarrow 2x = 3y = \bar{q} \rightarrow x(P_x, P_y, \bar{q}) = \frac{\bar{q}}{2}$

$y(P_x, P_y, \bar{q}) = \frac{\bar{q}}{3}$

$C(P_x, P_y, \bar{q}) = P_x \cdot \frac{\bar{q}}{2} + P_y \cdot \frac{\bar{q}}{3}$

e)  $f(x,y) = (\min\{2x, 3y\})^{1/2}$

at optimum  $\Rightarrow 2x = 3y = \bar{q}^2$

$\left. \begin{aligned} x(P_x, P_y, \bar{q}) &= \bar{q}^2/2 \\ y(P_x, P_y, \bar{q}) &= \bar{q}^2/3 \end{aligned} \right\} C(P_x, P_y, \bar{q}) = P_x \cdot \frac{\bar{q}^2}{2} + P_y \cdot \frac{\bar{q}^2}{3}$

f)  $f(a,b,c,d) = \min\{a, 2b\} + \max\{3c, 4d\}$

At optimum  $a=2b$  say  $a=2b=x$ ,  $\max\{3c, 4d\}=y$

$f(x,y) = x+y \rightarrow$  if  $P_x > P_y \rightarrow x=0, y=\bar{q}$   
if  $P_x < P_y \rightarrow x=\bar{q}, y=0$

What's  $P_x$  and  $P_y$ ?

$P_x = P_a + \frac{P_b}{2}$

$P_y = \begin{cases} P_c/3 & \text{if } P_d/P_c > 4/3 \\ P_d/4 & \text{if } P_d/P_c < 4/3 \end{cases}$

So, if  $P_a + \frac{P_b}{2} > \frac{P_c}{3}$  and  $\frac{P_d}{P_c} > \frac{4}{3}$ , then  $a = b = 0$ ,  $c = \bar{q}$ ,  $d = 0$

if  $P_a + \frac{P_b}{2} > \frac{P_d}{4}$  and  $\frac{P_d}{P_c} < \frac{4}{3}$ , then  $a = b = 0$ ,  $c = 0$ ,  $d = \bar{q}/4$   
 $a = \bar{q}$ ,  $b = \frac{\bar{q}}{2}$

if  $P_a + \frac{P_b}{2} < \frac{P_c}{3}$  and  $\frac{P_d}{P_c} > \frac{4}{3}$ , then  $a = 2b = \bar{q} \rightarrow a = \bar{q}$ ,  $b = \bar{q}/2$ ,  $c = d = 0$

if  $P_a + \frac{P_b}{2} < \frac{P_d}{4}$  and  $\frac{P_d}{P_c} < \frac{4}{3}$ , then  $a = 2b = \bar{q} \rightarrow a = \bar{q}$ ,  $b = \bar{q}/2$ ,  $c = d = 0$

Then, write:  $a = \begin{cases} \bar{q} & \text{if } \dots \\ 0 & \text{if } \dots \end{cases}$   $b = \begin{cases} \bar{q}/2 & \text{if } \dots \\ 0 & \text{if } \dots \end{cases}$

And find  $C(P_a, P_b, P_c, P_d, \bar{q}) =$

g)  $f(x, y) = \ln(xy)$

$$\mathcal{L} = P_x \cdot x + P_y \cdot y + \lambda (\bar{q} - \ln x - \ln y)$$

FOCs  $x: P_x - \frac{\lambda}{x} = 0$   $\left\{ \begin{array}{l} \frac{P_x}{P_y} = \frac{y}{x} \Rightarrow y = x \cdot \frac{P_x}{P_y} \end{array} \right.$

$y: P_y - \frac{\lambda}{y} = 0$   $\bar{q} = \ln(x \cdot x \cdot \frac{P_x}{P_y}) = \ln(x^2 \frac{P_x}{P_y})$

$\lambda = \bar{q} - \ln x - \ln y = 0$   $\Rightarrow x^2 = P_y / P_x \cdot e^{\bar{q}}$

$\Rightarrow x(P_x, P_y, \bar{q}) = \sqrt{P_y / P_x \cdot e^{\bar{q}}}$   $\left\{ \begin{array}{l} y(P_x, P_y, \bar{q}) = \sqrt{P_x / P_y \cdot e^{\bar{q}}} \end{array} \right.$   $C(P_x, P_y, \bar{q}) = P_x \cdot x(\cdot) + P_y \cdot y(\cdot) = \sqrt{2 P_x \cdot P_y e^{\bar{q}}}$

h)  $f(x, y) = \min\{x, y\} + \max\{x, y\}$

if  $x < y \Rightarrow f(x, y) = x + y$   $\left\{ \begin{array}{l} \text{if } x < y \Rightarrow f(x, y) = x + y \\ \text{if } x > y \Rightarrow f(x, y) = x + y \end{array} \right.$   $\delta(x, y) = x + y \Rightarrow x(P_x, P_y, \bar{q}) = \begin{cases} \bar{q} & \text{if } P_x < P_y \\ 0 & \text{if } P_x > P_y \\ \in [0, \bar{q}] & \text{if } P_x = P_y \end{cases}$

$y(\cdot) = \begin{cases} 0 & \text{if } P_x > P_y \\ \bar{q} & \text{if } P_x < P_y \\ \in [0, \bar{q}] & \text{if } P_x = P_y \end{cases} \Rightarrow C(\cdot) = \begin{cases} P_x \cdot \bar{q} & \text{if } P_x < P_y \\ P_y \cdot \bar{q} & \text{if } P_x > P_y \end{cases}$

Q.8  $q = 2K^{1/2} \cdot L^{1/2}$   $K=100$ ,  $v=1$ ,  $w=4$

a)  $q = 20\sqrt{L} \rightarrow L = q^2/400$

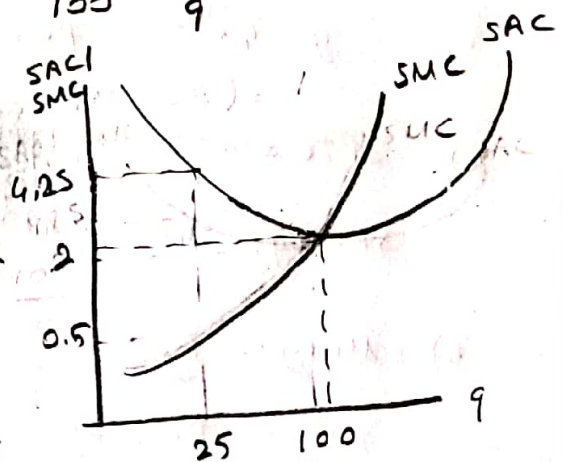
$C(v, w, q) = v \cdot 100 + w \cdot q^2/400$

$SC(q) = q^2/100 + 100 \Rightarrow SAC(q) = \frac{q}{100} + \frac{100}{q}$

1)  $SC(q) = \frac{q^2}{100} + 100 \Rightarrow SC(25) = 106,25$

$SAC(q) = \frac{q}{100} + \frac{100}{q} \Rightarrow SAC(25) = 4,25$

$SMC(q) = \frac{dSC}{dq} = \frac{q}{50} \Rightarrow SMC(25) = 0,5$



at  $SAC(q) = SMC(q)$   $\frac{q}{100} + \frac{100}{q} = \frac{q}{50} \Rightarrow q = 100$  (where SAC is min)

Q.9  $y = 8x_1^{1/2} x_2^{1/2}$   $p_1=1$ ,  $p_2=4$

min  $TC = p_1 \cdot x_1 + p_2 \cdot x_2$

s.t.  $y - 8x_1^{1/2} x_2^{1/2} = 0$

$L = x_1 + 4x_2 + \lambda (y - 8x_1^{1/2} x_2^{1/2}) = 0$

FOCs

(1)  $\frac{\partial L}{\partial x_1} = 1 - \lambda \cdot 4x_1^{-1/2} x_2^{1/2} = 0$   $\left. \begin{array}{l} \frac{x_2}{x_1} = \frac{1}{4} \Rightarrow x_1 = 4x_2 \end{array} \right\}$

(2)  $\frac{\partial L}{\partial x_2} = 4 - \lambda \cdot 4x_1^{1/2} x_2^{-1/2} = 0$   $\rightarrow$  plug this into (3)

(3)  $\frac{\partial L}{\partial \lambda} = y - 8x_1^{1/2} x_2^{1/2} = 0 \rightarrow y = 16x_2 \rightarrow x_2 = \frac{y}{16}$  and  $x_1 = \frac{y}{4}$

$$TC(y) = P_1 \cdot x_1(y) + P_2 \cdot x_2(y) = \frac{y}{4} + 4 \cdot \frac{y}{16} = \frac{y}{2}$$

$$MC(y) = \frac{dTC(y)}{dy} = \frac{1}{2} \quad \text{and} \quad AC(y) = \frac{TC(y)}{y} = \frac{1}{2}$$

Q.10  $y = 4x^{1/2}$   $P_x = 4000$   $FC = 4000$

a)  $SRTC = 4000 + 4000x$  and  $x = y^2/16$

$$SRTC(y) = 4000 + 250y^2$$

b)  $MC(y) = 6y$

$$TC(y) = FC + 3y^2$$

$$TVC(y) = 3y^2 \Rightarrow TVC(10) = 300$$

Q.11  $q = \min\{M, L^{1/2}\}$   $P_L = 1$   $P_M = 4$

$$q = M = L^{1/2} \Rightarrow M = q \text{ and } L = q^2$$

$$TC(P_L, P_M, q) = P_L \cdot q^2 + P_M \cdot q$$

$$= q^2 + 4 \cdot q$$

$$MC(q) = 2q + 4$$

$$AC(q) = q + 4$$

