

①

(a) Asymmetric information in this context means that the seller has more information that is relevant to the transaction than the buyer.

(b) Eqm. price must be $p = 1000x + 2000(1-x)$
 $p = 2000 - 1000x$

(c) Sellers of lemons will need \$800 at least.

Sellers of good cars " \$1600 at least.

(d) Since the buyers are willing to pay \$2000 and \$1000 for the good cars and lemons respectively, sellers can gain from trade up to \$400 and \$200.

(e) $x = 0.4$

Buyer's expected value $\Rightarrow 0.4(1000) + 0.6(2000) = \1600

— At this price, sellers of good cars are indifferent. But owners of lemons can obtain a \$800 surplus.

②

(a) $EU_{LR} = \frac{3}{4}\sqrt{16} + \frac{1}{4}\sqrt{0} = 3$

$$EU_{LR}(C1) = \frac{3}{4}\sqrt{16-7} + \frac{1}{4}\sqrt{0+8-7} = 2.5$$

$$EU_{LR}(C2) = \frac{3}{4}\sqrt{16-10} + \frac{1}{4}\sqrt{0+16-10} \approx 2.4$$

Since $EU_{LR} > EU_{LR}(C1) > EU_{LR}(C2)$, low risk customer will not participate in the insurance mkt and thus will not buy any of the offered contracts.

$$EU_{HR} = \frac{1}{2}\sqrt{16} + \frac{1}{2}\sqrt{0} = 2$$

$$EU_{HR}(C1) = \frac{1}{2}\sqrt{16-7} + \frac{1}{2}\sqrt{0+8-7} = 2$$

$$EU_{HR}(C2) = \frac{1}{2}\sqrt{16-10} + \frac{1}{2}\sqrt{0+16-10} \approx 2.4$$

$EU_{HR}(C2) > EU_{HR}(C1) = EU_{HR} \Rightarrow$ So, high risk customer will buy the second contract.

(b) Yes, the company can infer that all of the people who buy the second contract are high risk customers.

(c) $E\pi = E\pi_{HR}(C2) = 10 - \frac{1}{2}16 = 2$

③

high type: $\max_e w_h(e) - c_h(e)$

(a)

$$\max_e \alpha_1 + \alpha_2 \cdot e - e^2$$

$$\text{FOC: } \alpha_2 - 2e_h = 0 \Rightarrow e_h = \frac{\alpha_2}{2}$$

For this to be a separating eqm, low type shouldn't choose education.

$$w_h - w_l \leq c_l(e_h)$$

$$\hookrightarrow e_l = 0$$

$$\text{so, } w_l - 0 \geq w_h - c_l(e_h)$$

$$\alpha_2 \cdot e_h \leq \frac{3}{2} e_h^2$$

$$\frac{2}{3} \alpha_2 \leq e_h \Rightarrow \text{so, efficient eqm. is } e_h = \frac{2}{3} \alpha_2 \quad (\text{anything more than this would be inefficient.})$$

$$e_l = 0$$

(b) $w_h - w_l > c_h(e_h)$

$$\alpha_2 e_h > e_h^2$$

$$\alpha_2 > e_h = \frac{2}{3} \alpha_2 \leadsto \text{high type will not deviate.}$$

(c) Yes, in perfect information case, the high type would obtain the lower education level. This level maximizes utility of high type but it would be preferred by low types as well, if they could get higher wage.

④ (a) Set $e_1=0$. The incentive constraints for agent 1 are:

$$u_1(a_1, 0) = 1 \geq 2 - e_2 = u_1(a_2, e_2)$$

$$u_1(a_1, 0) = 1 \geq 3 - e_3 = u_1(a_3, e_3)$$

for agent 2 we have:

$$u_2(a_2, e_2) = 2 - \frac{e_2}{4} \geq 1 = u_2(a_1, e_1)$$

$$u_2(a_2, e_2) = 2 - \frac{e_2}{4} \geq 3 - \frac{e_3}{4} = u_2(a_3, e_3)$$

and for agent 3:

$$u_3(a_3, e_3) = 3 - \frac{e_3}{9} \geq 1 = u_3(a_1, e_1)$$

$$u_3(a_3, e_3) = 3 - \frac{e_3}{9} \geq 2 - \frac{e_2}{9} = u_3(a_2, e_2)$$

It is not difficult to check that $e_1=0$; $e_2=1$ and $e_3=5$ satisfy all those inequalities. (and $e_2=1$ is the min. value satisfying it)

The beliefs of the company are

$$\mu(a_3 | e \geq 5) = 1 ; \mu(a_3 | e < 5) = 0$$

$$\mu(a_2 | e \geq 5) = 0 ; \mu(a_2 | e < 1) = 0 ; \mu(a_2 | 1 \leq e < 5) = 1$$

$$\mu(a_1 | e > 1) = 0 ; \mu(a_1 | e < 1) = 1$$

(b) In this case agents 1 and 2 are pooled together and the firm offers two wages $w_1 = w_2 = 1.5$ (the average of a_1 and a_2) and $w_3 = 3$. To have an eqm. of this type it must be true that

$$u_1(1.5, 0) = 1.5 \geq 3 - e_3 = u_1(a_3, e_3)$$

$$u_2(1.5, 0) = 1.5 \geq 3 - \frac{e_3}{4} = u_2(a_3, e_3)$$

$$u_3(a_3, 3) = 3 - \frac{e_3}{9} \geq 1.5 = u_3(1.5, 0)$$

It is easy to see that the minimum e_3 satisfying those conditions is $e_3=6$. The beliefs that support this eqm. are:

$$\mu(a_3 | e \geq 6) = 1 ; \mu(a_3 | e < 6) = 0$$

$$\mu(a_1 | e < 6) = \frac{1}{2} ; \mu(a_2 | e < 6) = \frac{1}{2}$$

In this case, type 1 agent is better off than in the case above. However, type 2 and type 3 are both worse off.

⑤ $N = \{1, 2\} \rightarrow \text{Players}$

$T_1 = \{c\}, T_2 = \{c_a, c_b\} \rightarrow \text{types: cost fncs}$

Utility fncs: $\pi_1(q_1, q_2, c) = (a - q_1 - q_2)q_1 - c \cdot q_1$

$\pi_2(q_1, q_a, c_a) = (a - q_1 - q_a)q_a - c q_a$

$\pi_2(q_1, q_b, c_b) = (a - q_1 - q_b)q_b - c q_b$

For firm 2:

- If cost fnc is $c_a \Rightarrow \max_{q_a} (a - q_1 - q_a)q_a - c q_a$

$$\text{FOC: } a - q_1 - 2q_a - c = 0 \Rightarrow q_a = \frac{a - q_1 - c_a}{2}$$

- If cost fnc is $c_b \Rightarrow q_b = \frac{a - q_1 - c_b}{2}$

For firm 1: $\max_{q_1} \underbrace{\theta(a - q_1 - q_2(c_a) - c)q_1}_{\text{EP if firm 2's cost is } c_a} + (1 - \theta) \underbrace{(a - q_1 - q_2(c_b) - c)q_1}_{\text{EP if firm 2's cost is } c_b}$

$$\text{FOC: } \theta(a - 2q_1 - q_2(c_a) - c) + (1 - \theta)(a - 2q_1 - q_2(c_b) - c) = 0$$

$$2q_1 = q_1 = \frac{\theta(a - q_2(c_a) - c) + (1 - \theta)(a - q_2(c_b) - c)}{2}$$

Solving for $q_2(c_a)$ and $q_2(c_b)$; we obtain

$$2q_1 = \theta \left(a - \frac{a - q_1 - c_a}{2} - c \right) + (1 - \theta) \left(a - \frac{a - q_1 - c_b}{2} - c \right)$$

$$q_1^* = \frac{a - 2c + \theta c_a + (1 - \theta)c_b}{3}$$

$$q_2^*(c_a) = \frac{a - 2c_a + c}{3} + \frac{(1 - \theta)}{6} (c_a - c_b)$$

$$q_2^*(c_b) = \frac{a - 2c_b + c}{3} + \frac{\theta}{6} (c_a - c_b)$$