

Q.1 $n=100$

(Chp 12)

$$C(q) = \frac{1}{300} q^3 + 0.2q^2 + 4q + 10$$

a) In the short run $P(q) = MC(q) \rightarrow$ Group from
 $\pi = TR - TC$
 $\frac{\partial \pi}{\partial q} = MR(q) - MC(q) = 0$

$$\frac{\partial C(q)}{\partial q} = MC(q) = 0.01q^2 + 0.4q + 4$$

$\frac{\partial}{\partial q}$

$$P(q) = 0.01q^2 + 0.4q + 4$$

firms in every market
chooses q^* where $MC(q) = MR(q)$

We need to find q in terms of P to find the supply curve of a firm in the market.

$$100P = q^2 + 40q + 400$$

$$100P - (q+20)^2 \Rightarrow \boxed{q = 10\sqrt{P} - 20}$$

b) To find industry's supply curve, we need to sum all these identical firms' supply function

$$Q_s = q \cdot n = 100(10\sqrt{P} - 20)$$

$$\boxed{Q_s = 1000\sqrt{P} - 2000}$$

c) We have found market supply. Now we're given market demand. We can find eqm price and quantity.

$$Q_d = Q_s$$

$$-200P + 8000 = 1000\sqrt{P} - 2000$$

$$200P + 1000\sqrt{P} = 10000$$

$$P + 5\sqrt{P} - 50 = 0$$

$$\begin{array}{cc} \sqrt{P} & 10 \\ \sqrt{P} & -5 \end{array}$$

$$\Rightarrow (\sqrt{P} + 10)(\sqrt{P} - 5) = 0$$

$$\Rightarrow \sqrt{P} = 5$$

$$P^* = 25$$

$$Q^* = 3000$$

$$q_i^* = \frac{3000}{100} = 30$$

$$C_i = 420$$

$$AC = 13,3$$

$$\pi = TR - TC$$

$$= P \cdot q - AC \cdot q$$

$$= q(P - AC) = 30(25 - 13,3) \Rightarrow \boxed{\pi = 351}$$

$$P > AC \Rightarrow \pi > 0$$

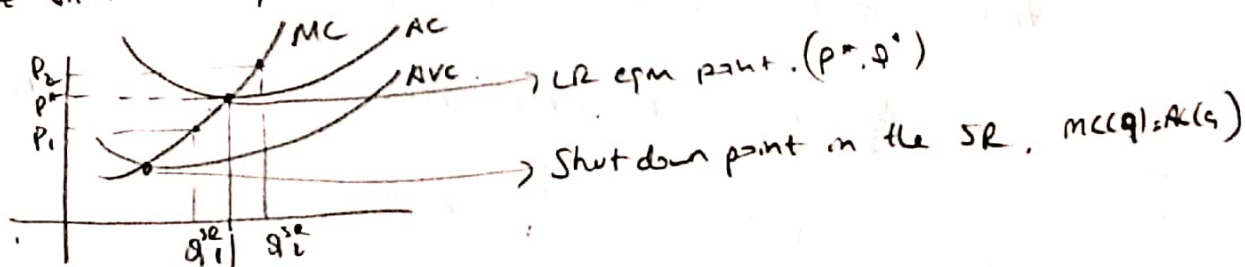
Note

In the SR,

$$P = MC(q) > AC(q) \Rightarrow$$

$$P = MC(q) < AC(q)$$

The shut-down point is where $P(q) = AVC(q)$



In the LR $q_{LR}^* \quad P = MC(q) = AC(q) \rightarrow$ where AC is min.

In the LR, firms exit from the market $P(q) < AC(q), \pi < 0$

Supply decrease until q^* reaches $P(q) = AC(q)$

If $\pi > 0 \Rightarrow$ firms enter until $\pi = 0$

If $\pi < 0 \Rightarrow$ firms exit until $\pi = 0$

Q2 $P_1 = 104 - 10q_1 - 2q_2$ $MC = 6$

$$\pi_1 = P_1 \cdot q_1 - TC_1$$

$$\max_{q_1} \pi_1 = (104 - 10q_1 - 2q_2) q_1 - TC_1$$

FOC: $104 - 20q_1 - 2q_2 - 6 = 0 \Rightarrow MR(q_1^*) = MC(q_1^*)$

$$\Rightarrow q_1 = \frac{98 - 2q_2}{20} \Rightarrow \text{Best Response fnc of 1.}$$

$$P_2 = 42 - 2q_2 - q_1$$

$$\pi_2 = P_2 \cdot q_2 - TC_2$$

$$\max_{q_2} \pi_2 = (42 - 2q_2 - q_1) q_2 - TC_2$$

$$42 - 4q_2 - q_1 - 6 = 0$$

$$\Rightarrow q_2 = \frac{36 - q_1}{4} \Rightarrow \text{Best Response fnc of 2.}$$

Nash-Cournot Eqn

$$q_1 = \frac{98 - 2 \cdot \left(\frac{36 - q_1}{4} \right)}{20} = 98 - 18 + 0.5q_1 = 20q_1$$

$$80 = 19.5q_1$$

$$q_1 \hat{=} 4$$

$$q_2 \hat{=} 8$$

$$\Rightarrow P_1 = 104 - 10 \cdot 4 - 2 \cdot 8 = 48$$

$$P_2 = 42 - 2 \cdot 8 - 4 = 30$$

Q.3

$$P = 200 - 5Q \quad MC = 20$$

a) $P = 200 - 5(q_1 + q_2)$

$$\max_{q_1} \pi_1 = [200 - 5(q_1 + q_2)] q_1 - TC$$

FOC $\Rightarrow 200 - 10q_1 - 5q_2 = 20$

$$q_1 = \frac{200 - 20 - 5q_2}{10}$$

$$q_1 = 18 - 0.5q_2$$

$$\max_{q_2} \pi_2 = [200 - 5(q_1 + q_2)] q_2 - TC$$

$$q_2 = 18 - 0.5q_1$$

Nash - Cournot

$$q_1 = 18 - 0.5(18 - 0.5q_1) \Rightarrow q_1 = q_2 = 12$$

$$P = 200 - 5(q_1 + q_2) \Rightarrow P = 80$$

b) Stackelberg (Firm 1 moves first)

Substitute Firm 2's best response for Firm 1's profit function. Firm 1 max π respect to response of Firm 2.

$$\max_{q_1} \pi_1 = [200 - 5(q_1 + 18 - 0.5q_1)] q_1 - 20q_1$$

FOC $90 - 5q_1 = 0 \Rightarrow q_1 = 18$
 $q_2 = 9 \quad P = 65$

Q.4

$$m = 15$$

$$q_1 = 120 - 6p_1 - 2p_2$$

$$q_2 = 120 - 6p_2 - 2p_1$$

Nash-Bertrand Eqn:

Set prices to max profit

$$\max \Pi_1 = (p_1 - m) q_1(p_1, p_2)$$

$$\max_{p_1} \Pi_1 = (p_1 - 15)(120 - 6p_1 - 2p_2)$$

$$\frac{\partial \Pi}{\partial p_1} = 120 - 12p_1 + 2p_1 \cdot p_2 = 0 \Rightarrow p_1 = 17.5 + \frac{p_2}{6}$$

Symmetric demand fncs and same MC's, so best responses are symmetric too. $p_2 = 17.5 + \frac{p_1}{6}$

$$p_1 = 17.5 + \frac{17.5 + \frac{p_1}{6}}{6} \Rightarrow p_1^* = p_2^* = 21$$
$$q_1^* = q_2^* = 36$$

Q.5

$$AC = MC = 5$$

$$Q = 53 - P$$

a)

$$\max_Q \pi = TR(Q) - TC(Q)$$

$$\frac{\partial TR}{\partial Q} - \frac{\partial TC}{\partial Q} = 0 \Rightarrow MR(Q) = MC(Q^*) \Rightarrow \text{at profit max}$$

$$P = 53 - Q$$

$$TR = P \cdot Q = (53 - Q)Q$$

$$MR = 53 - 2Q = 5 \Rightarrow \boxed{Q_M^* = 24}$$

$$Q = 53 - P \Rightarrow P^* = 29$$

$$\pi = (P - AC)Q = (29 - 5)24 = 576$$

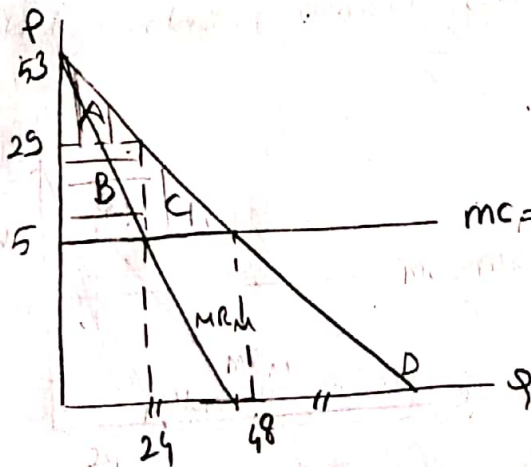
b)

$$MR(Q) = MC(Q) \Rightarrow \text{profit maximization.}$$

$$P = MC = 5$$

$$Q = 53 - P \Rightarrow \boxed{Q_C^* = 48}$$

c)



$$\text{Competitive CS: } \frac{(53 - 5)48}{2} = 1152 \quad (A+B+C)$$

$$\text{Monopoly CS: } (A) \frac{(53 - 29) \cdot 24}{2} = 288$$

$$\text{Profits (B)} \Rightarrow 24 \cdot 24 = 576$$

$$\text{DWL (C)} \Rightarrow \frac{1}{2} \cdot 24 \cdot 24 = 288$$

$$\text{CS in competitive Eqm} = \underbrace{CS}_{A+B+C} + \underbrace{\text{Profits}}_B + \underbrace{\text{DWL in Monopoly}}_C$$

Q.6

a) $Q = aP^e$ (where e is the price elasticity of demand)

528

$$\frac{\partial Q}{\partial P} \cdot \frac{P}{Q} = a \cdot e P^{-1} \cdot \frac{P}{a \cdot P^e} = e \rightarrow \text{constant elasticity} = e$$

$$TR = P(Q) \cdot Q$$

$$MR = \frac{\partial TR}{\partial Q} = P + Q \cdot \frac{\partial P}{\partial Q} = P \left(1 + \frac{Q}{P} \cdot \frac{\partial P}{\partial Q} \right) = P \left(1 + \frac{1}{e_{Q,P}} \right)$$

in competitive eqm,

this part = 0, since $e \rightarrow -\infty$

firm in CE faces perfectly elastic demand curve.

$$MR = P \left(1 + \frac{1}{e_{Q,P}} \right)$$

Profit maximization condition in all market types is that

$$MR(Q) = MC(Q)$$

$$\Rightarrow MR = MC = P \left(1 + \frac{1}{e} \right)$$

$$\Rightarrow \frac{P - MC}{P} = -\frac{1}{e_{Q,P}}$$

$$\frac{P - (m + t)}{P} = -\frac{1}{e} \Rightarrow P(1 + e) = e(m + t)$$
$$P = \frac{e(m + t)}{1 + \frac{1}{e}}$$

$$\Rightarrow \frac{dP}{dt} = \frac{1}{1 + \frac{1}{e}}$$

Monopoly never operates in the inelastic portion of its demand

curve, so $E < -1 \Rightarrow \frac{\partial P}{\partial t} > 1$ (so consumer pays

$$\text{ii) } t=2, e=-2$$

$$\frac{dP}{dt} = 2 \cdot \frac{1}{2} \Rightarrow dP = 4 =$$

more than twice of tax incidence. That's, price increase more than tax)

6-b)

$$\left. \begin{array}{l} MC = aQ + b \\ MR = c - dQ \end{array} \right\} \pi \max \Rightarrow MC(Q^*) = MR(Q^*)$$

MC after tax $MC = aQ + b + t$

$$aQ + b + t = c - dQ$$

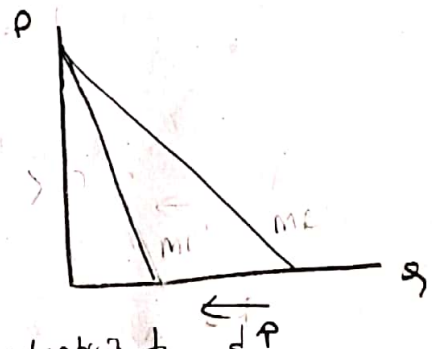
$$Q^* = \frac{c - b - t}{a + d}$$

$$TR = cQ - \frac{d}{2}Q^2 = Q \underbrace{\left(c - \frac{d}{2}Q \right)}_P$$

$$\boxed{P = c - \frac{d}{2}Q} \Rightarrow P^* = c - \frac{d}{2} \cdot \frac{c - b - t}{a + d}$$

$$\frac{dP}{dt} = \frac{d}{2(a+d)}$$

$$\left(\frac{dP}{dt} \right)'_d = \frac{2(a+d) - 2d}{2(a+d)} = 1 - \frac{d}{a+d} > 0$$



As $d \uparrow \rightarrow$ (slope of price-Q curve) $\rightarrow$ elasticity $\downarrow$

Consumers pay more of tax incidence as gets more inelastic.

b) $a=4 \quad b=2 \quad c=18 \quad d=4 \quad t=4$

$$\frac{dP}{dt} = \frac{d}{2(a+d)} = \frac{4}{2 \cdot 8} = \frac{1}{4}$$

$$dP = \frac{1}{4} \cdot dt$$

$$dP = 1 \quad \text{when } dt = 4.$$

Consumers pay % 25 of tax incidence..

Q.7

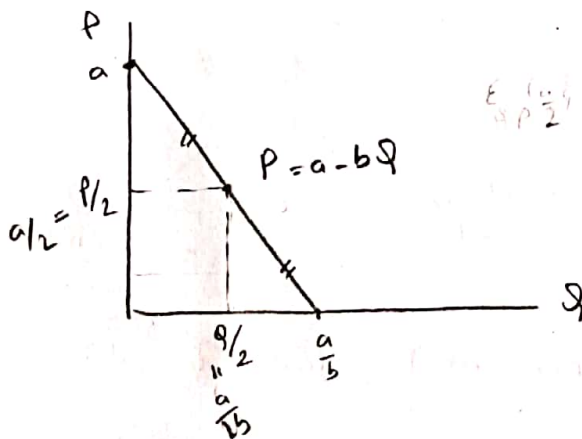
$$MR = P \left(1 + \frac{1}{e_{q,p}} \right) = MC \Rightarrow \frac{P-MC}{P} = \frac{-1}{e_{q,p}}$$

If $e = -1$ $MR=0$ $MC=0$

If $e > -1$ $MR < 0$ $MC < 0$

If $e < -1$ $MR > 0$

So, monopoly will not operate to the right of midpoint of a linear inverse demand curve where elasticity of demand is unitary.



$$e_{q,p} = \frac{\partial Q}{\partial P} \cdot \frac{P}{Q}$$

$$e_{q,p} \left(\frac{a}{2}, \frac{a}{2b} \right) = \frac{1}{-b} \cdot \frac{a/2}{a/2b} = -1$$

unit elasticity at the midpoint.

Q.8

$$Q = 53 - P \quad MC = AC = 5$$

a) $P = 53 - Q$

$$TR = (53 - Q)Q$$

$\Rightarrow$ Profit max

$$\max_Q \pi = TR(Q) - TC(Q)$$

Q

FOC: $MR(Q) = MC(Q)$

$$53 - 2Q = 5 \Rightarrow Q^* = 24$$

$$P^* = 53 - Q^* \Rightarrow P^* = 29$$

$$\pi^* = TR(Q^*) - TC(Q^*)$$

$$= P^* \cdot Q^* - AC(Q^*) \cdot Q^*$$

$$= Q^* (P - AC)$$

$$= 24(29 - 5) = 576$$

b) $P = 53 - q_1 - q_2$

$$P \cdot q_1 = (53 - q_1 - q_2) q_1$$

$$\max_{q_1} \pi_1 = (53 - q_1 - q_2) q_1 - TC(q_1)$$

$$\Rightarrow MR(q_1) = MC(q_1)$$

$$\Rightarrow 53 - 2q_1 - q_2 - 5 = 0$$

$$\Rightarrow q_1 = \frac{24 - q_2}{2}$$

$$P \cdot q_2 = (53 - q_1 - q_2) q_2$$

$$\max_{q_2} \pi = (53 - q_1 - q_2) q_2 - TC(q_2)$$

$$MR(q_2) = MC(q_2)$$

$$\Rightarrow 53 - q_1 - 2q_2 = 5$$

$$\Rightarrow q_2 = \frac{24 - q_1}{2}$$

Substitute one of q 's into other

$$q_1 = \frac{24 - \frac{24 - q_1}{2}}{2}$$

$$q_1 = 8 = q_2$$

$$P = 53 - q_1 - q_2$$

$$P^* = 53 - 8 - 8$$

$$P^* = 37$$

$$\pi_1 = q_1(P - AC)$$

$$= 8(37 - 5) = 256$$

$$\pi_2 = q_2(P - AC)$$

$$= 8(37 - 5) = 256$$

$$\text{Industry profit } \pi_1 + \pi_2 = 512$$

$$1) \quad P = 53 - P$$

$$q_1 = 91$$

$$53 - P$$

$$q_1 = 53 - P - 12$$

$$q_1 = 41 - P$$

$$\text{From 1st - The 1st firm } P = 70 - 2q_1$$

$$\text{and } 53 - P = 41 - P$$

$$q_1$$

$$12 - 2q_1$$

$$P = 48 - q_1 = 43 - q_1$$

$$12 - 2q_1 = 43 - q_1$$

$$q_1 = 31$$

$$P = 17$$

$$q_2 = 22$$

$$P = 10$$

$$q_1 = 15$$

$$P = 38$$

$$q_2 = 15$$

$$P = 38$$

$$q_2 = 15$$