

Problem Set 9

1)

Let Y be a random variable with a density function given by

$$f(y) = \begin{cases} (3/2)y^2, & -1 \leq y \leq 1, \\ 0, & \text{elsewhere.} \end{cases}$$

- a Find the density function of $U_1 = 3Y$.
- b Find the density function of $U_2 = 3 - Y$.
- c Find the density function of $U_3 = Y^2$.

2)

A parachutist wants to land at a target T , but she finds that she is equally likely to land at any point on a straight line (A, B) , of which T is the midpoint. Find the probability density function of the distance between her landing point and the target. [Hint: Denote A by -1 , B by $+1$, and T by 0 . Then the parachutist's landing point has a coordinate X , which is uniformly distributed between -1 and $+1$. The distance between X and T is $|X|$.]

3)

A fluctuating electric current I may be considered a uniformly distributed random variable over the interval $(9, 11)$. If this current flows through a 2-ohm resistor, find the probability density function of the power $P = 2I^2$.

- 4) The amount of flour used per day by a bakery is a random variable Y that has an exponential distribution with mean equal to 4 tons. The cost of the flour is $U=3Y+1$. Find the probability density function of U .

- 5) For the given joint density function, find the probability density function of $U=Y_1-Y_2$.

$$f(y_1, y_2) = \begin{cases} e^{-y_1}, & 0 \leq y_2 \leq y_1 \leq \infty \\ 0, & \text{elsewhere} \end{cases}$$

6)

Let Y be a random variable with probability density function given by

$$f(y) = \begin{cases} 2(1 - y), & 0 \leq y \leq 1, \\ 0, & \text{elsewhere.} \end{cases}$$

a Find the density function of $U_1 = 2Y - 1$.

b Find the density function of $U = Y^2$

c Find the density function of $U = 1 - 2Y$

Note: Use the method of transformation.

7)

Let the probability density function of Y be given by

$$f(x) = \begin{cases} 6x(1 - x), & 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}$$

a) Find the probability density of $Y = X^3$.

b) Find the probability density of $Y = X^4$.

8)

X is a continuous variable with the following probability density function:

$$f_X(x) = 3x^2, \text{ for } 0 < x < 1$$

i) Find the probability density of $Y = X^2$

ii) Now assume that probability density function of X is given by

$$f_X(x) = 3(1 - x)^2, \text{ for } 0 < x < 1$$

Find the probability density function of $Y = (1 - X)^3$

- 9) Suppose X_1 and X_2 are independent exponential random variables with parameter $\lambda = 1$ so that

$$f_{x_1}(x_1) = e^{-x_1}, \quad 0 < x_1 < \infty$$

$$f_{x_2}(x_2) = e^{-x_2}, \quad 0 < x_2 < \infty$$

The joint pdf is given by

$$f(x_1, x_2) = f_{x_1}(x_1)f_{x_2}(x_2) = e^{-x_1-x_2}, \quad 0 < x_1 < \infty, \quad 0 < x_2 < \infty$$

Consider the transformation $Y_1 = X_1 - X_2, Y_2 = X_1 + X_2$. Find the joint distribution of Y_1 and Y_2 .

- 10) Suppose X has the density

$$f(x) = \begin{cases} \frac{\theta}{x^{\theta+1}} & \text{for } x > 1 \\ 0, & \text{elsewhere} \end{cases}$$

Find the density of $Y = \ln X$ where θ is a positive parameter.
 $Y = \ln X, x = e^y$.