

ECON 203.01 - Problem Set 7

Spring 2026

1. Give an example of:

- excludable and rival good
- nonexcludable and rival good
- excludable and nonrival good
- nonexcludable and nonrival good

In each example explain why the goods belongs to the given category. Which of the four categories is called a pure public good?

2. Consider a 2 person 2 good economy where there is a private good x and a public good y . Agent 1 has an endowment of 10 units of the private good and Agent 2 has an endowment of 20 units of the private good. Initially there is no public good in the economy. In order to produce y units of the public good, y^2 units of the private good should be used. That is, the cost function is $C(y) = y^2$. Utility functions of the agents are as follows:

$$u_1(x_1, y) = x_1 + 6y$$

$$u_2(x_2, y) = x_2^2 y$$

- (a) Find the Pareto efficient allocation where 4 units of public good is produced.
- (b) Find the equilibrium allocation (i.e. find the private consumption of both agents as well as the public good level at equilibrium).
3. Consider a 4 person 2 good economy where there is a private good x and a public good y . Each agent has an endowment of 25 units of the private good. Initially there is no public good in the economy but each unit of the public good can be produced using 1 unit of the private good. Find the set of Pareto efficient allocations for the economy in which utility functions of the agents are as follows:

$$u_1(x_1, y) = x_1 + y/2 \qquad u_2(x_2, y) = x_2 + \sqrt{y}$$

$$u_3(x_3, y) = x_3 + 2\sqrt{y} \quad u_4(x_4, y) = x_4 + 3\sqrt{y}$$

Find the set of pareto efficient allocations.

4. Consider a 2 person 2 good economy where there is a private good and a public good. Agent 1 has a private endowment of 6 unit and agent 2 has a private endowment of 4 units. Initially there is no public good in the economy but each unit of the public good can be produced using 1 unit of the private good. That is, the cost function is $C(y) = y$. Utility functions of the agents are as follows:

$$u_1(x_1, y) = \min\{x_1, y\} \quad \text{and} \quad u_2(x_2, y) = \min\{x_2, y\}$$

- (a) Find the equilibrium allocation (i.e. find the private consumption of both agents as well as the public good level at equilibrium).
- (b) Sketch the set of Pareto efficient allocations on a Kolm triangle.
5. The town of Economia has two residents: Ann and Bob. The town currently funds its fire department solely from the individual contributions of these residents. Each of the two residents has a utility function over private goods (X) and total firemen (M), of the form: $U = 2\ln(X) + \ln(M)$. The total provision of firemen hired, M, is the sum of the number hired by each of the two persons: $M = M_A + M_B$. Ann and Bob both have income of 200, and the price of the private good and a firemen is 1. They are limited to providing between 0 and 200 firemen. For the purposes of this problem, you can treat the number of firemen as a continuous variable (it could be man-years).
- (a) How many firemen are hired if the government does not intervene? How many are paid by Ann? By Bob?
- (b) What is the socially optimal number of firemen? If your answer differs from (a), explain why.
- (c) Suppose the government is not happy with the private equilibrium, and it decides to provide 20 more firemen. It taxes Ann and Bob equally to pay for the new hires. What is the total number of firemen? How does your answer compare to part (a)? Have we achieved the social optimum? Why or why not?
- (d) Suppose now that the government is still not happy, so it decides to provide 75 new firemen. It taxes Ann 50 to pay for them, and it taxes Bob 25. What is the new total number of firemen? How many are provided by Ann, and how many by Bob? How does this compare to the level of provision in (c), and why?
6. There are two major owners of real estate in 100.Yil area in Ankara, called Alfon Inc (A) and Beton Inc (B). Both firms specialize in renting their apartments to METU students. Each year firm A and firm B decides how much to spend on the common areas in 100.Yil, such as playgrounds and bike paths. x^A and x^B are spendings for Alfon and Beton respectively and $x = x^A + x^B$ is the total spending for both firms. The common area is used by all members of the community, and hence x is non-excludable. The

market rent received by firm A depends on the quality of the property and its surrounding common area. The profit then is given by:

$$\pi^A = \ln(x^A + x^B) - x^A$$

The customers of firm B value the common area hence the profit of firm B is given by:

$$\pi^B = 2\ln(x^A + x^B) - x^B$$

- (a) Suppose $x^B = 0.5$. Find the optimal level of investment in a common area by firm A, x^A . Mark this point in space (x^A, x^B) .
- (b) Find analytically the best response function for firm A (an optimal investment x^A as a function of x^B) and plot the entire function in your graph from (a). Make sure you also show the optimal x^A for $x^B > 1$.
- (c) Find the best response function for firm B. Add it to the graph.
- (d) Find in your graph the Nash equilibrium for two firms. What is the total amount of money invested in common area?
- (e) Is the predicted outcome associated with free riding? If yes, by which firm? Explain why this firm is free riding?
- (f) What is the Pareto efficient joint level of investment in the common area? Is it greater, smaller or equal to the one observed in the market (the NE)? Explain intuitively why is it so?

7. Suppose that five homeowners live on the shore of Lake Magnavista: Amy, Bev, Cat, Dee, and Eve. In order to deal with such public good problems as deciding on the water level in the lake and how to control mosquitoes in the summer, they have formed a homeowners' association (HOA for short). Concerning the mosquitoes, the five women's preferences are all described by utility functions of the form:

$$u(x, y_i) = y_i - \frac{1}{2}(\alpha_i - x)^2$$

where x denotes the number of tankful of mosquito spray that are sprayed each week during summer, and y_i denotes the amount of money homeowner i has available to spend on other goods. The values of their preference parameters α_i are $\alpha_A = 30$, $\alpha_B = 27$, $\alpha_C = 24$, $\alpha_D = 21$, $\alpha_E = 18$. There are several local firms that will spray to control mosquitoes. The firms all charge the same price, $p = \$40$ per tankful they spray.

- (a) What is the Pareto amount of bug spray for homeowners?
- (b) If the homeowners each contact separately with bug-spray firms to spray, by taking the others' purchases as given, what will be the amount of bug spray for each?
- (c) Suppose instead, HOA will charge each homeowner only a share of the \$40 price. What are the Lindahl taxes that each homeowner is charged by the HOA at the PO level?

8. Suppose that there are three roommates: Buket, Ceren and Demet. They are trying to decide whether or not to hire Ayşe Teyze for cleaning the house. Since all three roommates will be able to take the advantage of a clean house, it will be a public good, rather than a private good. Valuations are given as: $v_B = 60$ TL, $v_C = 120$ TL, and $v_D = 50$ TL. Also, the cost of cleaning is 210 TL, and they will split the cost equally.
- (a) Is it efficient for them to have home cleaned by Ayşe Teyze?
 - (b) Which person is pivotal? What is the externality created on others as a result of cleaning? Who pays the tax and how much?
 - (c) Show that telling the truth is optimal.
9. Suppose that five teaching assistants Rengin, Burcu, Duygu, Zeki, and Aysenur, try to decide whether to buy a coffee machine for their office. Since all five will be able to take advantage of the coffee machine, it will be a public good, rather than a private good. However their valuations differ and given as: $v_R = 110$ TL, $v_B = 60$ TL, $v_D = 125$, $v_Z = 70$, and $v_A = 100$ TL. Also, the price of the coffee machine is 450 TL, and they will split the cost equally.
- (a) Is it efficient for them to buy a coffee machine?
 - (b) Which person is pivotal? What is the externality created on others as a result of buying the machine? Who pays the tax and how much?
 - (c) Show that telling the truth is optimal.