

2025-2026
ECON 205

Problem Set 9

- 1) Two messages are to be sent. The time (min) necessary to send each message has an exponential distribution with parameter $k = 1$, and the two times are independent of each other. It costs \$2 per minute to send the first message and \$1 per minute to send the second. Obtain the density function of the total cost of sending the two messages. **[Hint: First obtain the cumulative distribution function of the total cost, which involves integrating the joint pdf.]**

- 2) A type of elevator has a maximum weight capacity Y_1 , which is normally distributed with mean 5000 pounds and standard deviation 300 pounds. For a certain building equipped with this type of elevator, the elevator's load, Y_2 , is a normally distributed random variable with mean 4000 pounds and standard deviation 400 pounds. For any given time that the elevator is in use, find the probability that it will be overloaded, assuming that Y_1 and Y_2 are independent.

- 3) A machine produces spherical containers whose radii vary according to the probability density function given by

$$f(r) = \begin{cases} 2r, & 0 \leq r \leq 1 \\ 0, & \text{elsewhere} \end{cases}$$

Find the probability density function for the volume of the containers.

(Hint : Volume of a sphere is $V = \frac{4}{3} \pi R^3$)

- 4) Suppose that X_1 and X_2 have joint probability density function

$$f(x_1, x_2) = \begin{cases} 2 & \text{for } 0 < x_1 < x_2 < 1 \\ 0, & \text{otherwise} \end{cases}$$

Compute the joint probability density function of random variables $Y_1 = \frac{x_1}{x_2}$ and $Y_2 = X_2$.

- 5) Let Y_1 and Y_2 be independent random variables with moment-generating functions $m_{Y_1}(t)$ and $m_{Y_2}(t)$, respectively. If a and b are constants and $U=aY_1+bY_2$, show that the moment-generating function for U is $m_U(t) = m_{Y_1}(at) * m_{Y_2}(bt)$.
- 6) Let Y_1 and Y_2 be independent Poisson random variables with means λ_1 and λ_2 , respectively. Find the probability mass function of Y_1+Y_2 .
- 7) Let Z_1 and Z_2 be independent standard normal random variables and $U_1=Z_1$ and $U_2=Z_1+Z_2$. Derive the joint density of U_1 and U_2 .
- 8) Suppose that Y_1 and Y_2 are independent, standard normal random variables. Find the density function of $U = Y_1^2 + Y_2^2$.

9)

The length of time that a machine operates without failure is denoted by Y_1 and the length of time to repair a failure is denoted by Y_2 . After a repair is made, the machine is assumed to operate like a new machine. Y_1 and Y_2 are independent and each has the density function

$$f(y) = \begin{cases} e^{-y}, & y > 0, \\ 0, & \text{elsewhere.} \end{cases}$$

Find the probability density function for $U = Y_1/(Y_1 + Y_2)$, the proportion of time that the machine is in operation during any one operation–repair cycle.