

Extra Questions on UMP and EMP

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Q.1. Seung likes three goods: x_1 , x_2 , and x_3 . He maximizes the utility function

$$u(x_1, x_2, x_3) = x_1^{\alpha_1} x_2^{\alpha_2} x_3^{\alpha_3},$$

where $0 < \alpha_i < 1$ for $i = 1, 2, 3$. The consumption good x_i has price p_i and income is M .

- (a) Solve for x_1^*, x_2^*, x_3^* as functions of $p_1, p_2, p_3, M, \alpha_1, \alpha_2, \alpha_3$.
- (b) Are the boundary conditions for x_1, x_2, x_3 satisfied?
- (c) Is good x_1 normal (for all M, p_i)? Compute and explain.

Q.2. The utility function is

$$u(x, y) = \sqrt{x} + \sqrt{y}.$$

The prices of goods are p_x, p_y and income $M > 0$.

- (1) Is the utility function increasing in x ? Is it concave in x ?
- (2) Write down the consumer's maximization problem and explain why the budget constraint holds with equality.
- (3) Write the Lagrangian.
- (4) Derive the first-order conditions w.r.t. x, y, λ .
- (5) Solve explicitly for x^* and y^* as functions of p_x, p_y, M .
- (6) Check whether these are maxima by verifying the bordered-Hessian determinant is positive at (x^*, y^*, λ) .
- (7) Guess the solutions for

$$\max u(x, y) = (\sqrt{x} + \sqrt{y})^2.$$

Q.3. For each of the utility functions find the Hicksian demand functions, $h_1(p_1, p_2, \bar{u})$ and $h_2(p_1, p_2, \bar{u})$.

- (a) $u(x_1, x_2) = x_1^\alpha x_2^{1-\alpha}$
- (b) $u(x_1, x_2) = \frac{x_1^2}{x_2+4}$
- (c) $u(x_1, x_2) = x_1 + x_2$