

2025-2026

ECON 205

**Problem Set 5**

- 1) Let the joint probability density function of X and Y be:

$$f(x, y) = \begin{cases} \frac{3}{8}y, & 0 \leq x \leq y, 0 \leq y \leq 2 \\ 0, & \text{otherwise} \end{cases}$$

Compute the covariance between X and Y.

- 2) Suppose that  $Y_1$  and  $Y_2$  are random variables and show that

$$V(Y_1 + Y_2) = V(Y_1) + V(Y_2) + 2\text{Cov}(Y_1, Y_2).$$

- 3) Let  $Y_1$  and  $Y_2$  have the joint probability function given by

$$f(y_1, y_2) = \begin{cases} ky_1y_2, & 0 \leq y_1 \leq 1, 0 \leq y_2 \leq 1 \\ 0, & \text{elsewhere} \end{cases}$$

- a) Find the value of k that makes this a probability density function.  
 b) Find the cumulative distribution function for  $Y_1$  and  $Y_2$ .  
 c) Find  $P(Y_1 \leq \frac{1}{2}, Y_2 \leq \frac{3}{4})$

- 4) Find the marginal densities for  $Y_1$  and  $Y_2$ .

- a)  $f(y_1, y_2) = \begin{cases} 4y_1y_2, & 0 \leq y_1 \leq 1, 0 \leq y_2 \leq 1 \\ 0, & \text{elsewhere} \end{cases}$   
 b)  $f(y_1, y_2) = \begin{cases} 6(1 - y_2), & 0 \leq y_1 \leq y_2 \leq 1 \\ 0, & \text{elsewhere} \end{cases}$

5)

The random variable  $X$  has a range of  $\{0,1,2\}$  and the random variable  $Y$  has a range of  $\{1,2\}$ . The joint distribution of  $X$  and  $Y$  is given by the following table.

| $p(x,y)$ |   | $y$ |     |
|----------|---|-----|-----|
|          |   | 1   | 2   |
| $x$      | 0 | 0.2 | 0.1 |
|          | 1 | 0.0 | 0.2 |
|          | 2 | 0.3 | 0.2 |

- i) Write down tables for marginal distributions of  $X$  and  $Y$ .
- ii) Compute  $E(X)$  and  $E(Y)$ .
- iii) Compute  $E(XY)$

6)

Let  $X$  and  $Y$  be jointly continuous random variables with joint pdf

$$f(x,y) = \begin{cases} cx + 1, & x, y \geq 0, x + y \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

- i) Show the range of  $(X, Y)$ , in the  $x$ - $y$  plane.
- ii) Find the constant  $c$ .
- iii) Find the marginal pdf's  $f_X(x)$  and  $f_Y(y)$ .
- iv) Find  $P(Y < 2X^2)$ .

- 7) Suppose you roll two fair, six-sided dice, one of which is red and the other of which is green. Define the following random variables:

$X$  = The number shown on the red die

$$Y = \begin{cases} 0 & \text{if the two dice show the same number} \\ 1 & \text{if the number on the green die is bigger than the number on the red die.} \\ 2 & \text{if the number on the red die is bigger than the number on the green die.} \end{cases}$$

- a) Write down a table showing the joint probability mass function for  $X$  and  $Y$ .  
b) Find the marginal probability mass function for  $Y$ , and compute its expected value.

8)

Assume we have the following form;

$$f(x, y) = \begin{cases} 6yx^2, & 0 < y < 1 \text{ and } 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}$$

Is  $f(x, y)$  a joint probability density function? If so calculate the following:

- a) The marginal densities of  $X$  and  $Y$ .  
b)  $P(X + Y > 1)$   
c)  $P\left(\frac{1}{3} < Y < \frac{1}{2}\right)$ .