

Problem Set 8

1)

- a. Use the rules of expected value to show that

$$\text{Cov}(aX + b, cY + d) = ac\text{Cov}(X, Y)$$

- b. Use part (a) along with the rules of variance and standard deviation to show that

$$\text{Corr}(aX + b, cY + d) = \text{Corr}(X, Y)$$

when “a” and “c” have the same sign.

- c. What happens if “a” and “c” have opposite signs?

2)

A quality control plan calls for randomly selecting three items from the daily production (assumed large) of a certain machine and observing the number of defectives. However, the proportion p of defectives produced by the machine varies from day to day and is assumed to have a uniform distribution on the interval $(0, 1)$. For a randomly chosen day, find the unconditional probability that exactly two defectives are observed in the sample.

3)

The National Fire Incident Reporting Service stated that, among residential fires, 73% are in family homes, 20% are in apartments, and 7% are in other types of dwellings. If four residential fires are independently reported on a single day, what is the probability that two are in family homes, one is in an apartment, and one is in another type of dwelling?

4)

The weights of a population of mice fed on a certain diet since birth are assumed to be normally distributed with $\mu = 100$ and $\sigma = 20$ (measurement in grams). Suppose that a random sample of $n = 4$ mice is taken from this population. Find the probability that

exactly two weigh between 80 and 100 grams and exactly one weighs more than 100 grams.

5)

A statistics class takes two exams X (Exam 1) and Y (Exam 2) where the scores follow a bivariate normal distribution with parameters:

- $\mu_X = 70$ and $\mu_Y = 60$ are the marginal means
- $\sigma_X = 10$ and $\sigma_Y = 15$ are the marginal standard deviations
- $\rho = 0.6$ is the correlation coefficient

Suppose we select a student at random. What is the probability that

- a. the student scores over 75 on Exam 2?
- b. the student scores over 75 on Exam 2, given that the student scored $X=80$ on Exam 1?
- c. the sum of his / her Exam 1 and Exam 2 scores is over 150?
- d. the student did better on Exam 1 than Exam 2?

6) Let X = height (inches) and Y = weight (lbs) for an American male. Suppose X and Y have a bivariate normal distribution, the mean and sd of heights are 70 in and 3 in, the mean and sd of weights are 170 lbs and 20 lbs, and $\rho = .9$.

- a. Determine the distribution of Y given $X = 68$, i.e., the weight distribution for 5'8" American males.
- b. Determine the distribution of Y given $X = 70$, i.e., the weight distribution for 5'10" American males. In what ways is this distribution similar to that of part (a), and how are they different?
- c. Calculate $P(Y < 180 | X = 72)$, the probability that a 6-foot-tall American male weighs less than 180 lb.

- 7) For a few years, the SAT consisted of three components: writing, critical reading, and mathematics. Let W = SAT Writing score and X = SAT Critical Reading score for a randomly selected student. According to the College Board, in 2012 W had mean 488 and standard deviation 114, while X had mean 496 and standard deviation 114. Suppose X and W have a bivariate normal distribution with $\text{Corr}(X, W) = .5$.
- An English department plans to use $X + W$, a student's total score on the nonmath sections of the SAT, to help determine admission. Determine the distribution of $X + W$.
 - Calculate $P(X + W > 1200)$.
 - Suppose the English department wishes to admit only those students who score in the top 10% on this Critical Reading + Writing criterion. What combined score separates the top 10% of students from the rest?
- 8) Measurements for the length and width of a rectangular plastic covers for CDs are rounded to nearest mm (so they are discrete). Let X denote the length and Y denote the width. The possible value of X are 129, 130, and 131 mm. The possible values of Y are 15 and 16 mm. We show the probability for each pair in the following table:

		y	
		15	16
x	129	0.12	0.08
	130	0.42	0.28
	131	0.06	0.04

- Find the conditional probability that CD cover has a length of 130mm given the width is 15mm.
- What is the conditional probability that X has a maximum width of 130mm given $Y = 16$?
- Find $E(X|Y = 15)$.
- Find variance $V(X|Y = 15)$ and $\sigma_{X|Y=15}$.