

①

$$u(x_1, x_2) = \ln x_1 + \ln x_2$$

- (a) well behaved preferences $\Rightarrow$ Benjamin spends all of his income, choosing a bundle on the budget line.

So; $MRS_{xy} = -P_1/P_2 \leadsto$ He chooses the bundle where the budget line is tangent to the indifference curve furthest from the origin.

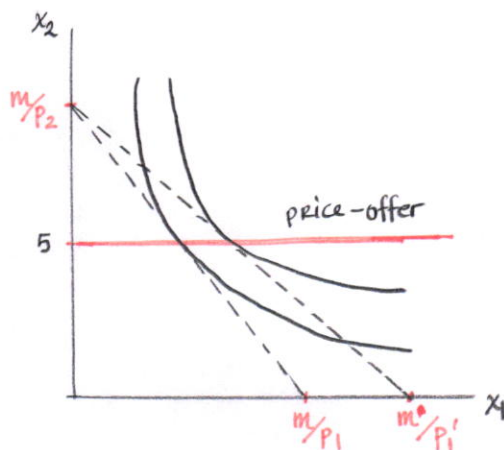
$$-\frac{1/x_1}{1/x_2} = -P_1/P_2 \Rightarrow \frac{x_2}{x_1} = \frac{P_1}{P_2} \quad x_2 = \frac{x_1 P_1}{P_2}$$

Substitute x_2 in BC: $P_1 x_1 + P_2 \cdot \frac{x_1 P_1}{P_2} = m$

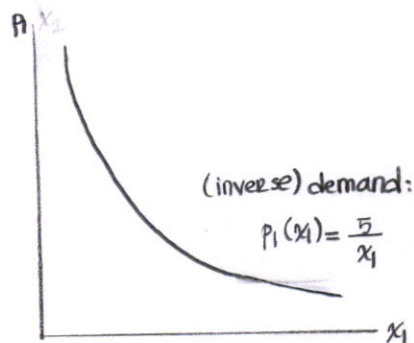
$$x_1 = \frac{m}{2P_1} \quad \& \quad x_2 = \frac{m}{2P_2}$$

- (b) To find the price-offer curve (which lies in our x_1 - x_2 commodity space) with $P_2=1$ and $m=10$, we plug these numbers into demand $x_2 = \frac{m}{2P_2} \Rightarrow x_2 = \frac{10}{2 \cdot 1} = 5$. So, regardless of the price of x_1 , with $P_2=1$ and $m=10$, 5 units of x_2 are always purchased.

Plotting this out in the commodity space:



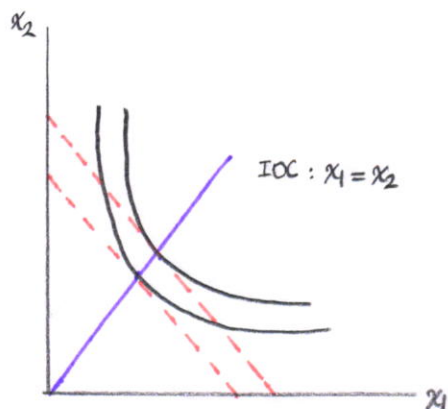
We found demand function $x_1 = \frac{m}{2P_1}$ above, and with $m=10$ we have $x_1 = \frac{10}{2P_1}$ (or, equivalently $x_1(P_1) = \frac{m}{2P_1}$). To plot the demand in the P_1 - x_1 space, we use the inverse of the demand function:

$$P_1(x_1) = \frac{10}{2x_1} = \frac{5}{x_1}$$


- (c) Looking at the demand function for x_1 , we can see that as P_1 increases (decreases), the amount of x_1 demanded decreases (increases), so x_1 is an ordinary good. This is consistent with the fact that the inverse demand function we graphed above is downward sloping.

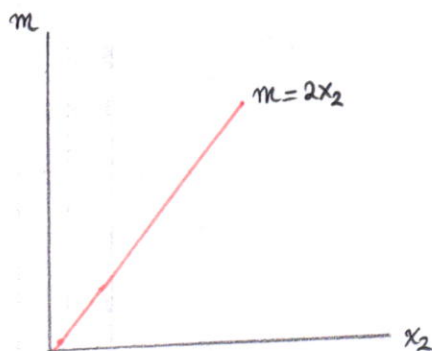
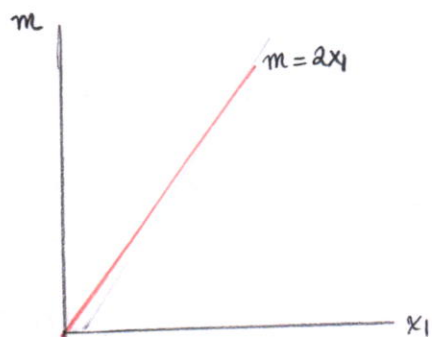
(a)

- Fixing $p_1=1$ and $p_2=1$, the income offer curve shows us the optimal bundles in the commodity space for different levels of m . Graphically, we see where the tangencies are (since these are well-behaved preferences) b/w the indifference curves and budget lines associated with different levels of income.



- Analytically, for the income offer curve we turn to our demand functions, which with these prices turn into $x_1 = \frac{1}{2}m$ and $x_2 = \frac{1}{2}m$. Solving these two eqns. (setting $m=2x_1$ for the first eqn. & plugging it into the second), we get that our optimal bundles as income changes will lie along the line $x_2 = x_1$.

- The Engel curve for each good, movies x_1 and songs x_2 , plots income m against quantity demanded, holding the price of both goods fixed. Here we can again use the demand curves $x_1 = \frac{1}{2}m$ and $x_2 = \frac{1}{2}m$. Similar to when we're graphing demand, we solve for what's on the vertical axis (here, m) so we graph $m=2x_1$ and $m=2x_2$.



- The two commodities are normal rather than inferior since, as we can see in the demand functions analytically and the Engel curve graphically, as income increases, the quantity demanded for both goods increases.

2

• Trevor's preferences can be represented by utility function: $u(x_1, x_2) = \min\{2x_1, x_2\}$

(a) He chooses a bundle along the optimal proportion line $\Rightarrow 2x_1 = x_2$

Trevor spends all of his income: $p_1x_1 + p_2x_2 = m$

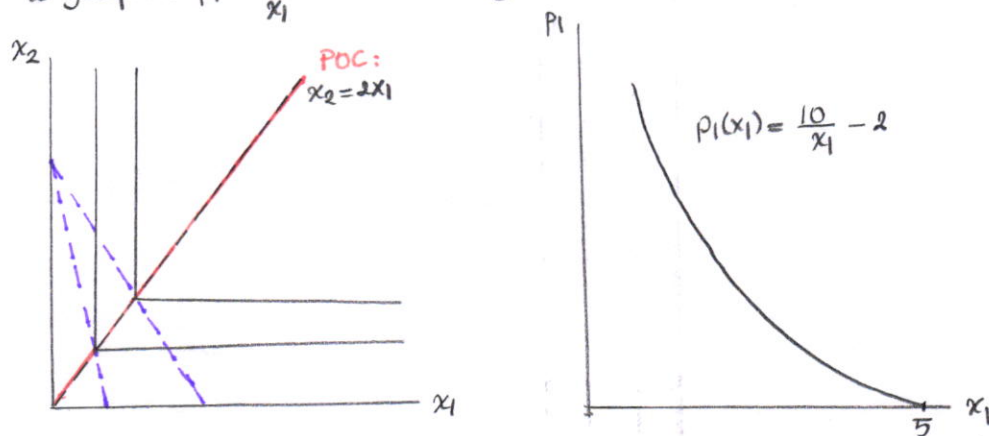
solving these equations for x_1 and x_2 we get

$$x_1 = \frac{m}{p_1 + 2p_2} \quad \text{and} \quad x_2 = \frac{2m}{p_1 + 2p_2}$$

(b) For $p_2 = 1$ and $m = 10$, we have the price-offer curve $x_2 = 2x_1$. (on the left)

Demand for x_1 is $x_1 = \frac{10}{p_1 + 2}$. we solve this for p_1 to get the inverse demand curve

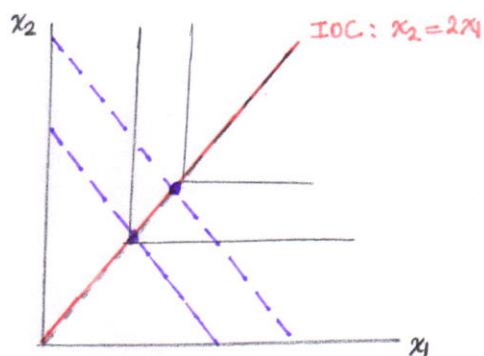
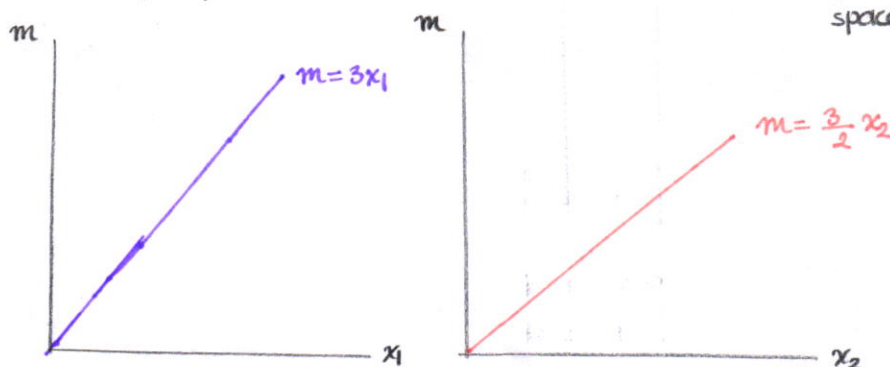
to graph: $p_1 = \frac{10}{x_1} - 2$. (on the right)



(c) milk, x_1 is an ordinary good for Trevor. Looking at the demand function, we see that as p_1 increases (decreases), quantity demanded x_1 decreases (increases).

(d) Given $p_1 = p_2 = 1$, the optimal choices given m (which define the Engel curve) are given by:

$$x_1 = \frac{m}{p_1 + 2p_2} = \frac{m}{3} \quad \text{and} \quad x_2 = \frac{2m}{p_1 + 2p_2} = \frac{2m}{3} \quad \text{We graph the Engel curves for each good in } x_1\text{-}m \text{ and } x_2\text{-}m \text{ space:}$$



• The income-offet curve coincides with the optimal proportion line (solve each of the demand functions to m and set them equal to each other and solve for x_2).

3

$$u(x_1, x_2) = 2x_1 + x_2$$

(a) Our demand functions for x_1 and x_2 will be depend on what the price ratio is relative to the MRS (the slope of the indifference curves, which is constant for perfect substitutes like these).

Here, $MRS_{x_1, x_2} = -\frac{MU_1}{MU_2} = -\frac{2}{1}$. If the budget line is less steep than the slope of the indifference curves, i.e. if $|-p_1/p_2| < |MRS_{12}|$, then Kate consumes only x_1 (Red Delicious). Otherwise, if the budget line is more steep than the indifference curves, $|-p_1/p_2| > |MRS_{12}|$, Kate consumes only Jonagolds x_2 .

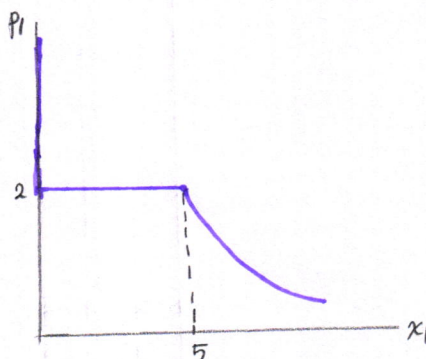
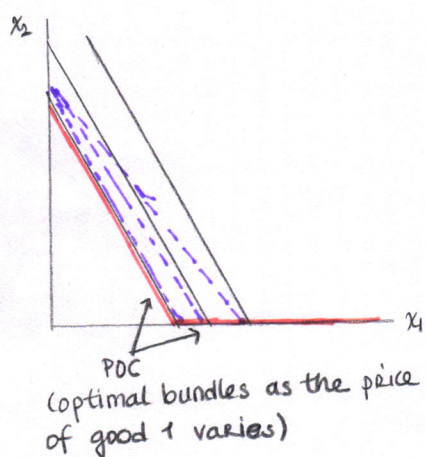
$$\text{so, we have that } |-p_1/p_2| < |MRS_{12}| \Rightarrow x_1 = \frac{m}{p_1}, x_2 = 0$$

$$|-p_1/p_2| > |MRS_{12}| \quad x_1 = 0, x_2 = \frac{m}{p_2}$$

$$|-p_1/p_2| = |MRS_{12}| \quad x_1 = \alpha \in [0, m/p_1] \text{ \& } x_2 = \frac{m - \alpha p_1}{p_2}$$

(b) with $m=10$ and $p_2=1$, we have $|-p_1/p_2| = p_1$. since $|MRS_{12}| = 2$

- For $p_1 > 2$, demand for $x_1 = 0$ and $x_2 = m/p_2 = 10$
- For $p_1 < 2$, demand for $x_1 = m/p_1$ and $x_2 = 0$
- For $p_1 = 2$, $MRS = -p_1/p_2$ everywhere, so any bundle along the budget line is optimal.



(c) Since the demand for x_1 does not increase as its price p_1 increases, x_1 is an ordinary good.

(d) Fixing $p_1=1$ and $p_2=1$, for this price ratio we know that Kate will always choose $x_1 = \frac{m}{p_1} = m$ Red Delicious apples and $x_2 = 0$ Jonagold apples.

