

METU, Department of Economics

ECON 201 – Problem Set 2

- 1- The relation “is preferred to” between commodity bundles is just one example of a binary relation. Another example is the relation “is a friend of” defined over the set of all human beings. Let xRy mean person x is a friend of person y . Is the relation R reflexive? Transitive? Complete? Explain your answer in few sentences using an example of a relationship between three individuals: Selahattin, Kemal and Meral.

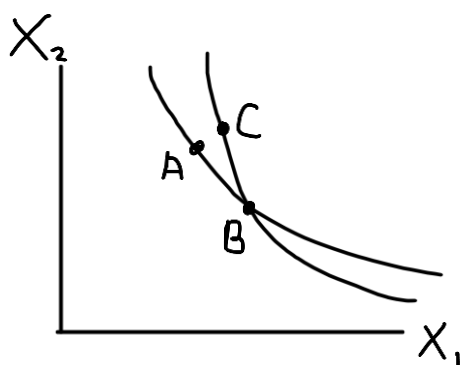
Solution:

- i. R is reflexive if we assume a person can be a friend of herself.
- ii. R is not transitive since if Selahattin is a friend of Kemal and Kemal is a friend of Meral, it does not imply Selahattin is a friend of Meral. ($sRk \wedge kRm \not\Rightarrow sRm$)
- iii. R is not complete since we can not define the “friend of” relationship between individuals who are not friends. We do not have (sRk) or (kRs).

- 2- Suppose that there are two commodities and a consumer prefers more to less of each good. If the consumer has transitive preferences, can her indifference curves cross? Why? Illustrate your answer with a graph.

Solution:

Consider the following diagram where two indifference curves are crossing with each other.



Since A and B are in the same indifference curve, $A \sim B$. Similarly since B and C are in the same indifference curve, $B \sim C$. Transitivity implies $A \sim C$. C bundle includes more of both x and y compared to bundle A. Contradiction thus indifference curves cannot cross.

- 3- Suppose that there are two goods, x and y , and that x is an inferior good. Show using the budget constraint why good y must be a normal good. Be as precise as you can.

Solution:

$$XP_x + YP_y = I$$

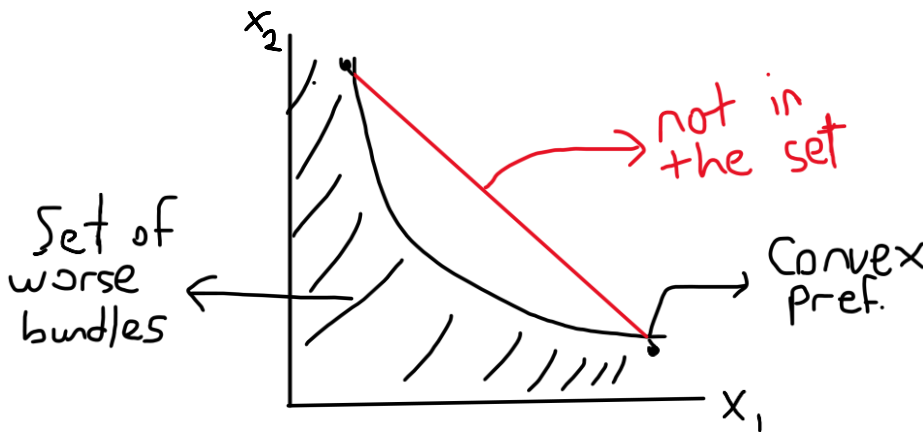
$$\frac{\partial X}{\partial I} P_x + \frac{\partial Y}{\partial I} P_y = \frac{dI}{dI} = 1$$

If X is inferior, $\frac{\partial X}{\partial I} < 0$ and $P_x > 0, P_y > 0$, then $\frac{\partial Y}{\partial I} > 0$ meaning Y is a normal good.

- 4- Comment. If preferences are convex, then for any commodity bundle x, the set of commodity bundles that are worse than x is a convex set.

Solution:

No, the set worse than bundle x is not a convex set. Remember that for a set to be convex, the line segment joining any two points taken in that set should belong to the set. Consider the following two commodity example.



- 5- Consider the following utility functions:

$$U_1(x, y) = xy$$

$$U_2(x, y) = x^2y^2$$

$$U_3(x, y) = \ln x + \ln y$$

Show that all three utility functions have the same marginal rate of substitution, MRS_{xy} . Is this a coincidence? Why, why not?

Solution:

For $U_1(x, y) = xy$, $MU_x = y$, $MU_y = x \Rightarrow MRS_{xy} = y/x$

For $U_2(x, y) = x^2y^2$, $MU_x = 2xy^2$, $MU_y = 2x^2y \Rightarrow MRS_{xy} = y/x$

For $U_3(x, y) = \ln x + \ln y$, $MU_x = 1/x$, $MU_y = 1/y \Rightarrow MRS_{xy} = y/x$

This is not a coincidence, as all three functions are monotonic transformations of each other.

$$U_3(x, y) = \ln(U_1(x, y))$$

$$U_2(x, y) = U_1(x, y)^2$$

Both $f^2(x)$ and $\ln(f(x))$ are monotonically increasing functions in the domain of non-negative real numbers as they satisfy the following condition:

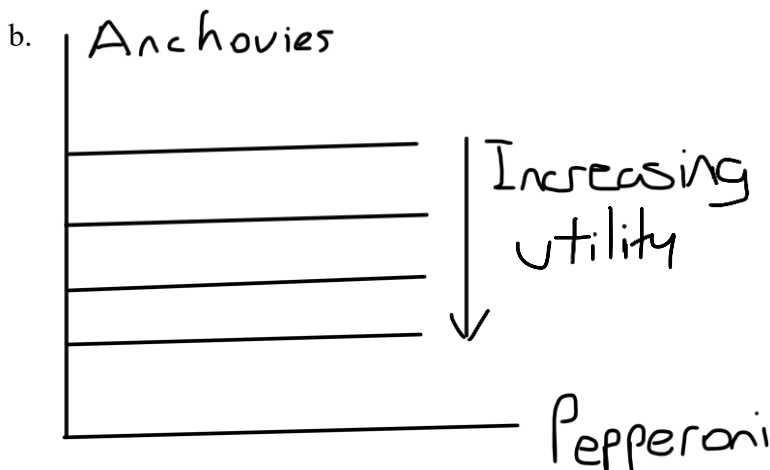
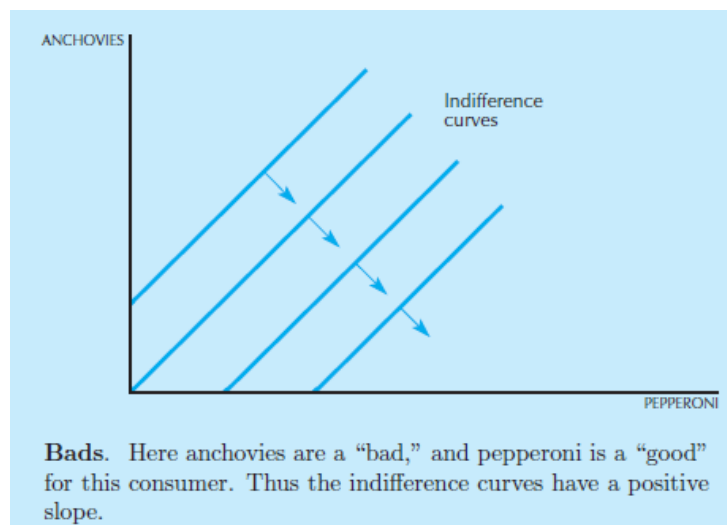
$$f(x) \geq f(y) \quad \forall x \geq y ; x, y \in [0, \infty)$$

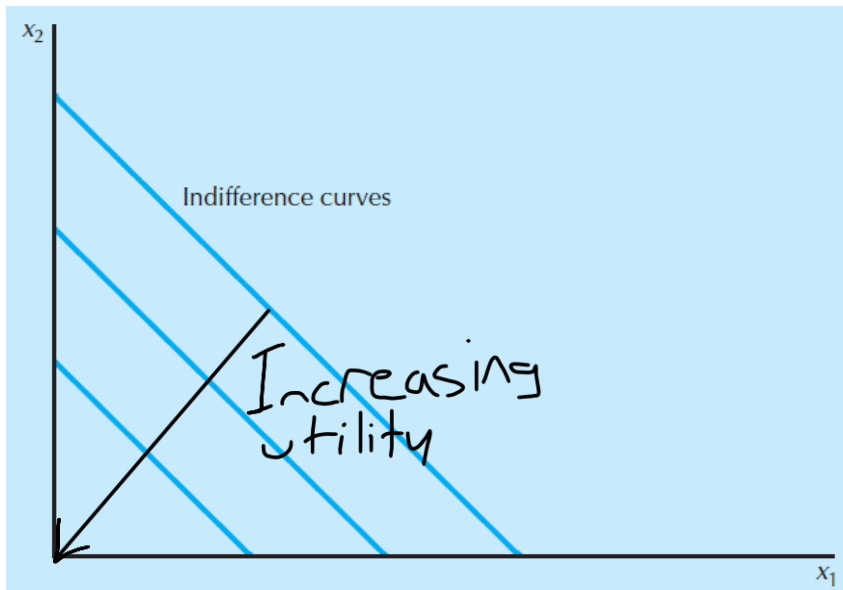
or, equivalently,

$$f'(x) \geq 0 \quad \forall x \in [0, \infty)$$

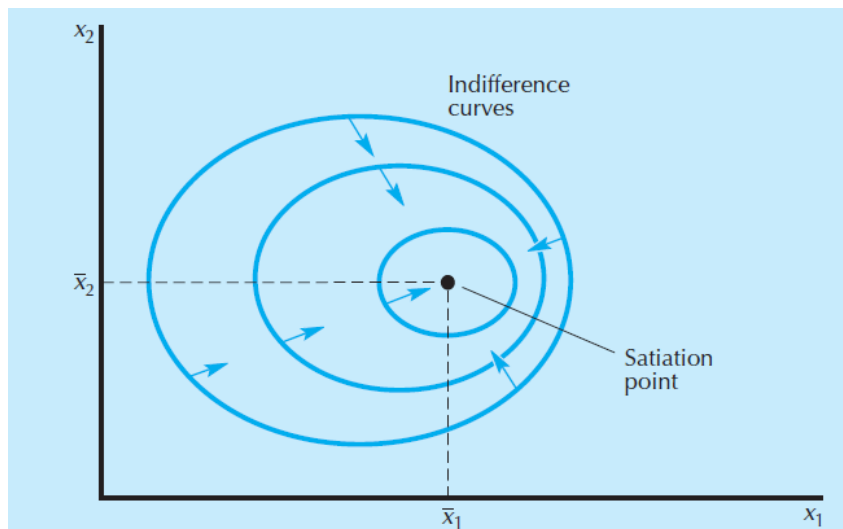
- 6- Draw graphs with quantities of pepperoni pizza on the horizontal axis and quantities of anchovy pizza on the vertical axis to illustrate the following situations. In each case draw two different indifference curves and make a little arrow pointing in the direction of greater preference.
- Hakan loves pepperoni pizza and hates anchovy pizza.
 - Hakan hates anchovy pizza and is completely indifferent about pepperoni pizza.
 - Hakan is vegan and hates both anchovy and pepperoni pizza equally.
 - Hakan only wants to eat 1 slice of anchovy pizza and 3 slices of pepperoni pizza. Any amount more or less than that exact amount gives less joy to him.

Solution:





c.



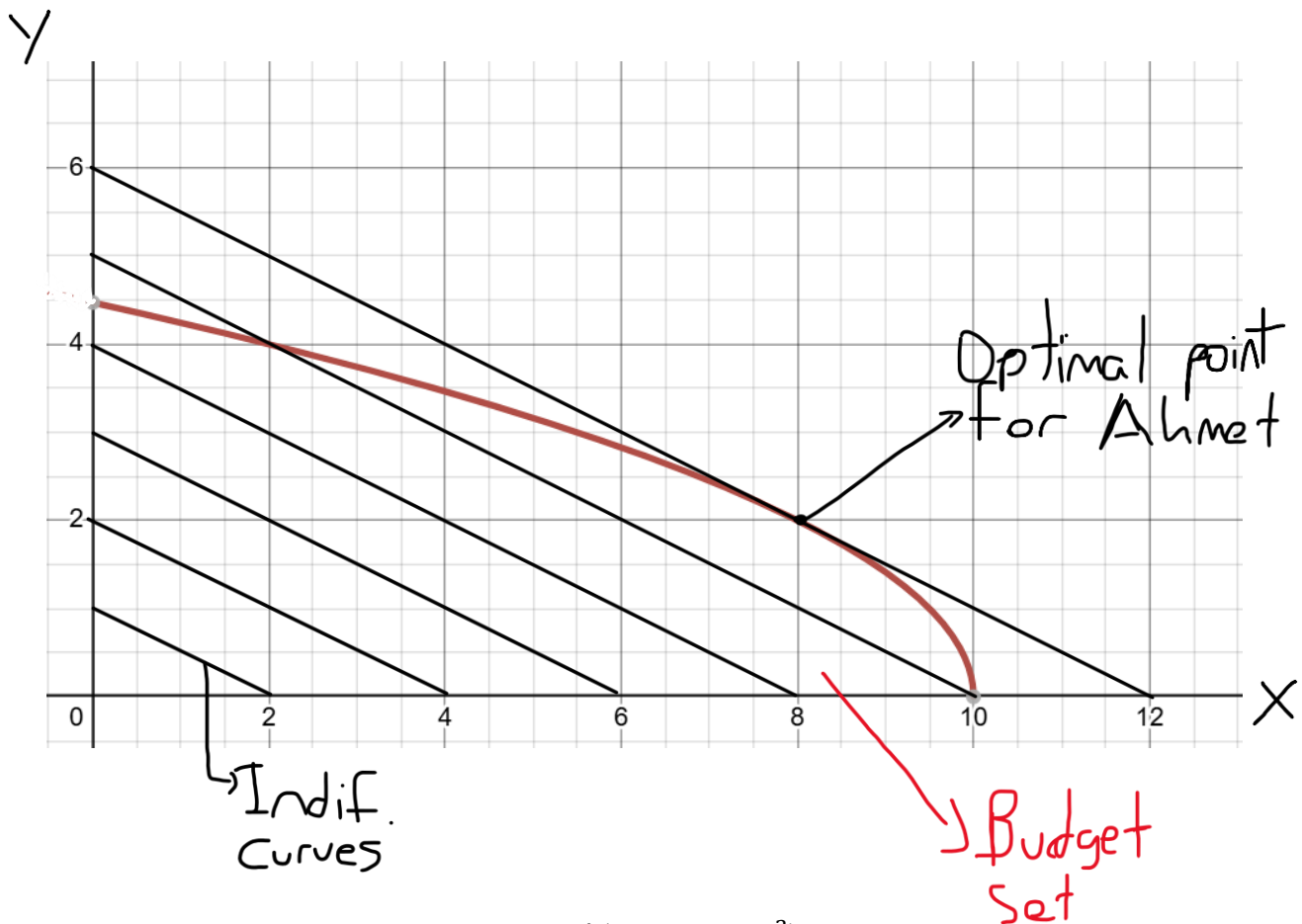
Satiated preferences. The bundle $(\bar{x}_1, \bar{x}_2)$ is the satiation point or bliss point, and the indifference curves surround this point.

d.

- 7- Ahmet consumes coffee and cheese. Coffee is sold in an unusual way. There is only one supplier, and the more coffee you buy from him, the higher the price you have to pay per unit. In fact y units of coffee will cost Ahmet y^2 dollars. Cheese is sold in the usual way at a price of 2 dollars per unit. Ahmet's income is 20 dollars and his utility function is $U(x, y) = x + 2y$, where x is his consumption of cheese and y is his consumption of coffee.
- Sketch Ahmet's budget set and shade it in.
 - Sketch some of his indifference curves and label the point that he chooses.
 - Calculate the amount of cheese and the amount of coffee that Ahmet demands at these prices and this income.

Solution:

Budget constraint: $2x + y^2 = 20$



$$L = x + 2y - \lambda(20 - 2x - y^2)$$

- i. $\frac{\partial L}{\partial x} = 1 + 2\lambda = 0 \Rightarrow \lambda = -\frac{1}{2}$
- ii. $\frac{\partial L}{\partial y} = 2 + \lambda 2y = 0 \Rightarrow y = 2$
- iii. $\frac{\partial L}{\partial \lambda} = 20 - 2x - y^2 = 0 \Rightarrow x = 8$

8- Use separate graphs to draw indifference curves for each of the following utility functions:

- a. $U(x, y) = \min\{2x + y, 2y + x\}$.
- b. $U(x, y) = \max\{2x + y, 2y + x\}$.
- c. $U(x, y) = x + \min\{x, y\}$.

Solution:

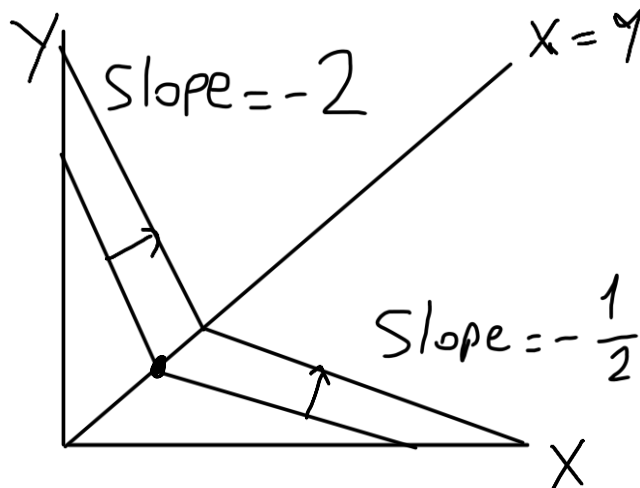
a. At the point

$$2x + y = 2y + x,$$

$$x = y$$

$$U(x, y) = 2x + y \quad \forall x < y$$

$$U(x, y) = 2y + x \quad \forall x > y$$



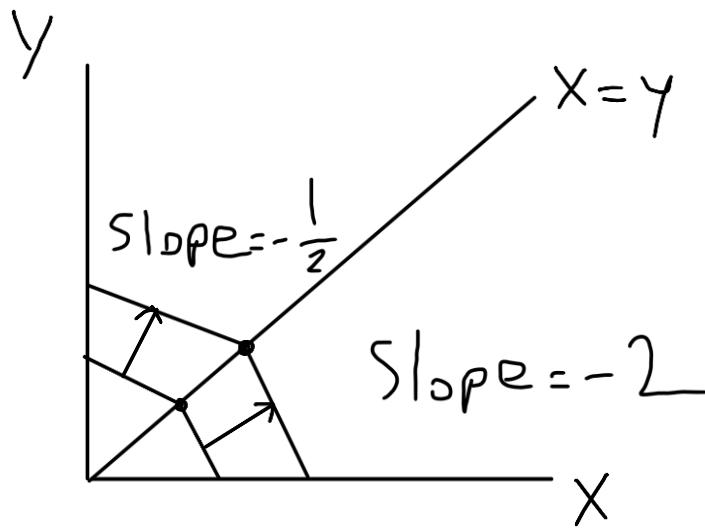
b. At the point

$$2x + y = 2y + x,$$

$$x = y$$

$$U(x, y) = 2x + y \quad \forall x > y$$

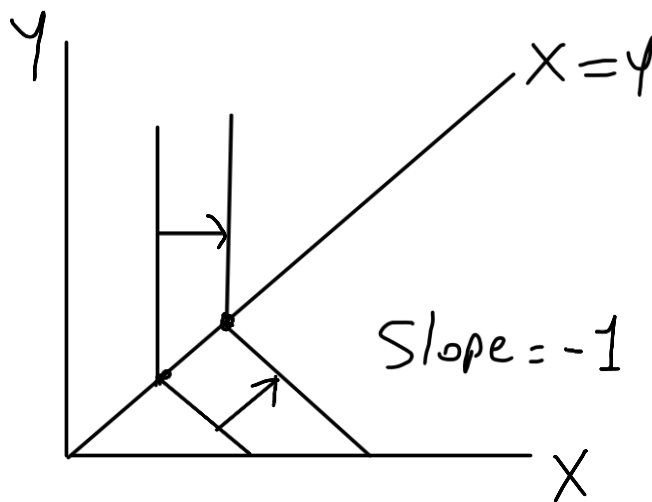
$$U(x, y) = 2y + x \quad \forall x < y$$



c.

$$U(x, y) = 2x \quad \forall x < y$$

$$U(x, y) = x + y \quad \forall x > y$$



- 9- Harry Truman has the utility function $U(x, y) = (x + 1)(y + 4)$. The price of y is \$1. Harry spends all of his income to buy 6 units of y and no x . What must the price of x be? Explain your answer and draw a diagram to illustrate it.

Solution:

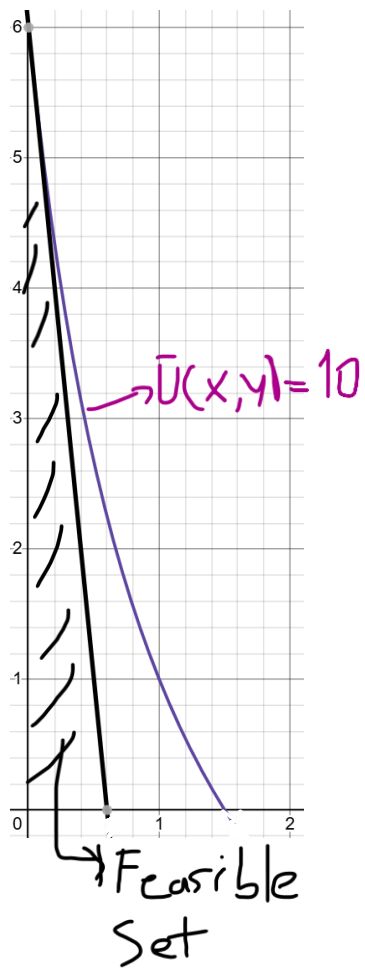
$$P_x \cdot x + P_y \cdot y = I, P_y = 1, I = 6 \Rightarrow P_x = ?$$

$$P_x \cdot x + y = 6 \Rightarrow y = 6 - P_x \cdot x$$

$$U(x, y) = (x + 1)(10 - P_x \cdot x) = 10 \cdot x + 10 - P_x \cdot x^2 - P_x \cdot x$$

$$\frac{\partial U}{\partial x} = 0 = 10 - 2 P_x \cdot x - P_x \Rightarrow x^* = \frac{10 - P_x}{2 P_x} = 0 \text{ (given)}$$

$$\Rightarrow P_x = 10$$



10- Roger consumes no commodities other than Coke and Pepsi. Roger considers 2 cans of Pepsi to be perfect substitutes for 6 cans of Coke. What is the demand of Roger for Coke and Pepsi for the following cases:

- a) His annual budget for these two commodities is described by the equation $5C + 25P = 300$.
- b) His annual budget for these two commodities is described by the equation $10C + 20P = 160$.
- c) His annual budget for these two commodities is described by the equation $10C + 30P = 600$.

Solution:

$$U(P,C) = 3P + C \Rightarrow MRS = -\frac{\partial C}{\partial P} = -\frac{\partial U / \partial P}{\partial U / \partial C} = -3$$

$$\text{a) } \frac{P_p}{P_c} = 5 > |-3| \Rightarrow P^* = 0, C^* = 300/5 = 60$$

$$\text{b) } \frac{P_p}{P_c} = 2 < |-3| \Rightarrow P^* = 160/20 = 8, C^* = 0$$

$$\text{c) } \frac{P_p}{P_c} = 3 = |-3| \Rightarrow P^* = \frac{600-10C^*}{30} \text{ where } 0 < C^* < 60 \rightarrow \text{Budget constraint overlaps with the utility function, any combination of coke and pepsi that lies on the budget constraint could be Roger's demand.}$$