

Introduction to Probability

Science of Stst. $\rightarrow$ pure stat. of math or Theory of stat
 $\rightarrow$ applied stat.

Pure Statistics $\rightarrow$ develop research tools that are applied to a specific problems in different fields including social sciences

Pure & Applied Statistic $\rightarrow$ interacts & enrich each other
App. Stst. $\rightarrow$ present new & challenging problems for pure stat

Characteristic Features of Science of stat.

- ① Stats. $\rightarrow$ is concerned with reduction of data or obtaining facts from data
- ② Stats $\rightarrow$ deals with populations or aggregate of individuals rather than individuals
- ③ Stats $\rightarrow$ deals with variation
- ④ Ststs $\rightarrow$ deals with numerically specified populations
- ⑤ Ststs $\rightarrow$ deals with populations which occur in nature and subject to large number of random forces such as births, imports, exports

- ⑥ logic used in statistical inference is inductive
- ⑦ Draw inference about population from information contained in sample

particular $\rightarrow$ general

There is always chance of being untrue

- ⑧ Statistical inference is uncertain since logic used is inductive

we can be certain with measurable degree of confidence

- ⑨ Statistical results might lead to fallacious conclusions if pulled out of context or manipulated.

Main Divisions of Stats Theory

$\rightarrow$ design of experiments

$\rightarrow$ statistical inference

statistical } is science of drawing valid conclusions
inference } from given data that are assumed to have
been so collected as to satisfy
basic requirements of stats. theory

Given data $\rightarrow$ sample from some population $\rightarrow$ we draw valid inferences about given populations (inductive)

Specify your population $\rightarrow$ choose sample
specification of math. form of population $\rightarrow$ will include
number of constants, called parameters of population
(mean, variance, median ...)
parameters $\rightarrow$ are estimated from the sample
these estimates are known as $\rightarrow$ statistics
(for population parameters)

Theory of Estimation } consists of trying to find the best
estimators for all parameters
 $\rightarrow$ subject to error

Theory of sampling distributions } help us in evaluating these errors

Term probability is a measure of ones belief in
occurrence of a future event.

The relative frequency with which they occur
in a long series of trials is often stable

Events $\rightarrow$ random or stochastic events

In stats $\rightarrow$ any process of information or
measurement is referred as an experiment

Results of experiments are outcomes of an experiment

Set theory

A set is a collection of objects

An object belongs to a set $\rightarrow$ element of this set

A set can be shown in two ways

$\hookrightarrow$ Roster Method $\rightarrow A = \{a, b, c\}$

$\hookrightarrow$ Rule Method $\rightarrow B = \{x \in \mathbb{N} \mid x = 1, 2, 3, \dots, 10\}$

Two sets are equal $A = B$ if every element belongs to A also belongs to B vice versa

Ex: $A = \{1, 2, \emptyset, 3\}$
 $B = \{1, 2, 3, \emptyset\}$ equal but not identical

A is a subset of B ($A \subset B$) if every element of A also belongs to B

$$A = \{\emptyset\}$$

$$B = \{\emptyset, b, c\} \rightarrow A \subset B$$

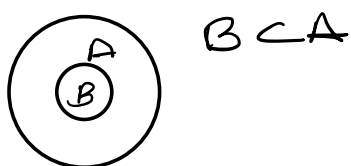
$A = B$ if and only if $A \subset B$ & $B \subset A$

$\emptyset \rightarrow$ Null set is a set with no elements $\emptyset = \{\}$

is a subset of any set

Suppose A is a subset of U ($A \subset U$), then
 set $A = \{x : x \notin A \text{ and } x \in U\}$ contains all elements
 that belong to A but also to U

$\bar{A} \rightarrow$ complement with respect to U
 All sets are subsets of $U \rightarrow$ universal set



every set is equal to itself & subset of itself
 $A = A$, $\forall A$ Two sets are disjoint as mutually
 $A \subset B$, $\forall A$ exclusive if $A \cap B = \emptyset$

Set Operators

The union of A & B ($A \cup B$) is set that consists
 of all elements that belong to A or B or both.

$$A \cup B = \{x, x \in A \text{ or } x \in B\}$$

Ex: $A = \{1, 3\}$ $B = \{1, 2\}$

$$A \cup B = \{1, 2, 3\}$$

$A \cup B$ & $B \cup A$ are identical

Union operator is commutative

$$(A \cup B) \cup C = A \cup (B \cup C) = A \cup B \cup C$$

Union operator is associative

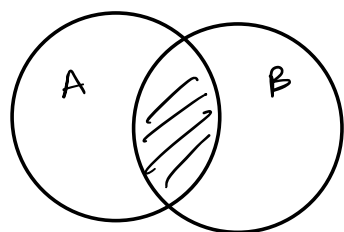
Intersection of A & B ($A \cap B$) is sets that consists of
 all elements belongs to A and B

$$A \cap B = \{x \mid x \in A \text{ and } x \in B\}$$

$$A = \{0, 1\}, B = \{1, 3\}$$

$$C = \{0, 1, 3\}$$

$$A \cap B = \{1\}, A \cap C = \{0, 1\}, B \cap C = \{1, 3\}$$



Intersection operation is commutative
If $A \cap B = \emptyset \rightarrow$ sets are mutually exclusive or disjoint

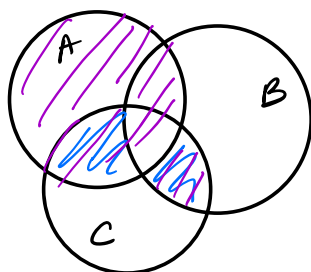
$$(A \cap B) \cap C = A \cap (B \cap C)$$

$\hookrightarrow$ Also associative

Both union & intersection operations are associative
 $\hookrightarrow$ same is not necessarily true if both operators appear in same expression

Such as $(A \cup B) \cap C$

$A \cup (B \cap C)$



n-tuples

an n-tuple is an ordered array of n components written $(x_1, x_2, \dots, x_n)$

Two n-tuples are different even if they contain some components if they are written in different order

$$(1, 2) \neq (2, 1) \quad (1, 3, 1) \neq (1, 1, 3)$$

Cartesian product $A \times B$ of A & B as set of all possible 2-tuple (x_1, x_2)

where $x_1 \in A$ & $x_2 \in B$

$$A \times B = \{ (x_1, x_2) \mid x_1 \in A, x_2 \in B \}$$

Ex:

$$A = \{0, 1, 2, 3\} \quad B = \{2, 5\} \quad C = \{0\}$$

$$A \times B = (0, 2) (0, 5) (1, 2) (1, 5) (2, 2) (2, 5) (3, 2) (3, 5)$$

$$A \times B \neq B \times A$$

$$B \times C = (2, 0) (5, 0)$$

Cartesian product operation is not commutative

$A \times B \times C \rightarrow 3\text{-tuples}$

$$A \times B \times C = \{ (0, 2, 0) (0, 5, 0) (1, 2, 0) (1, 5, 0) (2, 2, 0) (2, 5, 0) (3, 2, 0) (3, 5, 0) \}$$

$$A \times B \times C \neq A \times C \times B$$

$$(A \times B) \times C = A \times (B \times C) \Rightarrow \underline{\underline{\text{associative}}}$$

Discrete Case

An experiment is process by which an observation is made.

Results of experiments $\rightarrow$ outcomes of experiment
When experiments is made $\rightarrow$ it results with one or more outcomes $\rightarrow$ events, denoted by capital letters

Consider events associated with single toss of a die.

A: observe an odd number

B: observe a number greater than 3

C: observe > 2 or $= 3$

if you observe A, you can observe E_1, E_3, E_5

E_1 : observe 1
 E_2 : " 2
 E_3 : " 3
 E_4 : " 4
 E_5 : " 5
 E_6 : " 6

A $\rightarrow$ E_1
A $\rightarrow$ E_3
A $\rightarrow$ E_5 } compound events

$\{E_1, \dots, E_6\}$ cannot be decomposed
 $\downarrow$
simple events

A simple event is an event that cannot be decomposed

Each simple event $\rightarrow$ corresponds to one sample point

Generally capital letter E $\rightarrow$ simple events

set of all possible sample points (outcomes) of an experiment called sample space, S

Sample space for one flip of a coin $\rightarrow S = \{H, T\}$

A discrete sample space is one that contains either a finite or a countable number of discrete sample points (infinite though countable number of elements)

A coin is flipped until head appears

$$S = \{H, TH, TTH, TTTH, TTTTH, \dots\}$$

If a sample space consists of continuum such as all points in a line segment, is said to be continuous

An event in a discrete S , is collection of sample points, any subset of S

Therefore, probabilistic model for an experiment with a discrete S , can be conducted by assigning a numerical prob. to each simple event in S .

$$S = \{1, 2, 3, 4, 5, 6\}$$

$$E_1 \rightarrow 1/6$$

On analysing relative frequency concept of prob, 3 conditions must hold

- ① Relative frequency of occurrence of any event must be greater or equal to 0. a negative relative frequency does not make sense
- ② Relative frequency whole sample space must be unity

- ③ If two events are mutually exclusive relative frequency of their union is sum of their respective relative frequencies

Suppose S is a sample space associated with an experiment to any event A in S

We assign a number $P(A)$ with following axioms must hold

① $P(A) \geq 0$

② $P(S) = 1$

- ③ If $A_1, A_2, A_3, \dots$ form a sequence of pair mutually exclusive events in S , then $A_i \cap A_j = \emptyset$ if $i \neq j$

$$P(A_1 \cup A_2 \cup A_3 \cup \dots) = \sum_{i=1}^{\infty} P(A_i)$$

$$P(A_1 \cup A_2 \cup A_3 \cup \dots) = \sum_{i=1}^{\infty} P(A_i) \text{ for finite sequence}$$